The turbulent dynamo.
(helicity, closures and large-scale fields)

Steve Tobias (University of Leeds)

The Turbulent Dynamo (2019): JFM Perspectives, Tobias S.M.
arXiv:1907.03685

See also: The Self-Excited Dynamo (2019), Moffatt & Dormy, CUP
Dynamo Theories (2019), Rincon, F. arXiv: 903.07829.

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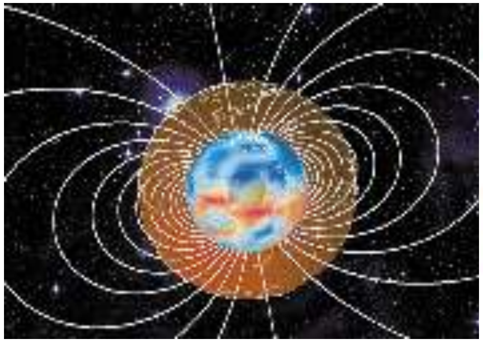
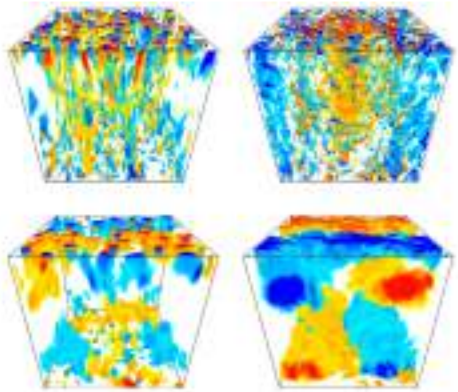
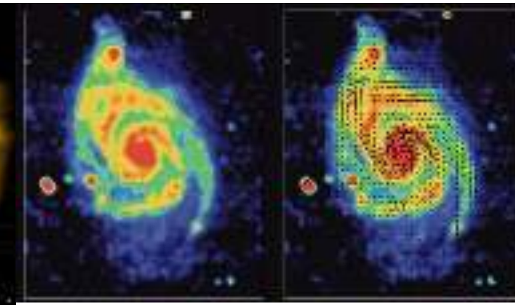
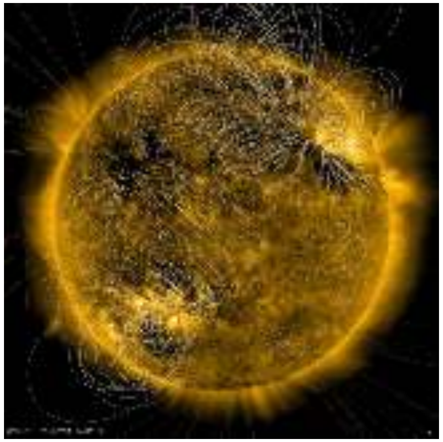


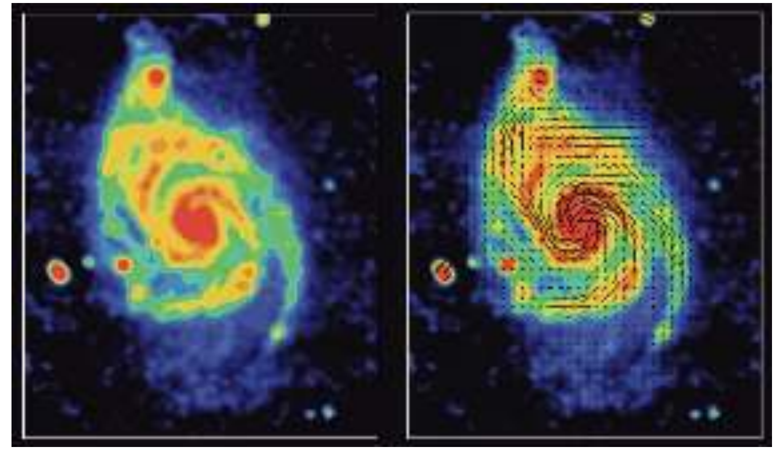


- Earth/planets?
- Rapid rotator (Jerome, Keith)
  - Low Ekman number E
- Liquid Metal (Stephan)
  - Low magnetic Prandlt number Pm
- Force balances
  - Geostrophic
  - Magnetostrophic
  - At what scale?

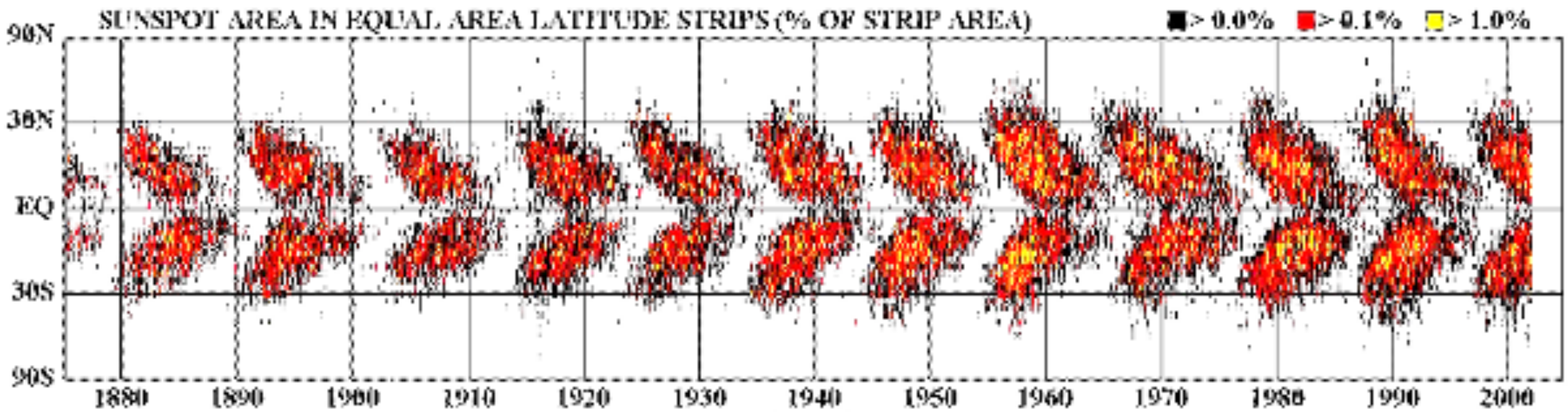


- Stars
  - High Rm
    - Very far from critical dynamo
  - Very turbulent
    - Extremely High Re
- Galaxies
  - High Rm
  - Not so turbulent





- How can an astrophysical object such as a star or galaxy generate a systematic (large-scale) magnetic field *at high Rm*? How can it overcome its tendency to be dominated by fluctuations at the small scales?
- How does conservation of magnetic helicity place constraints on field generation?
- Understanding breaking of constraints is key.
- Can we derive a (statistical?) theory that describes these interactions and maintains quadratic invariants in the limit of no dissipation.



Solar cycle:

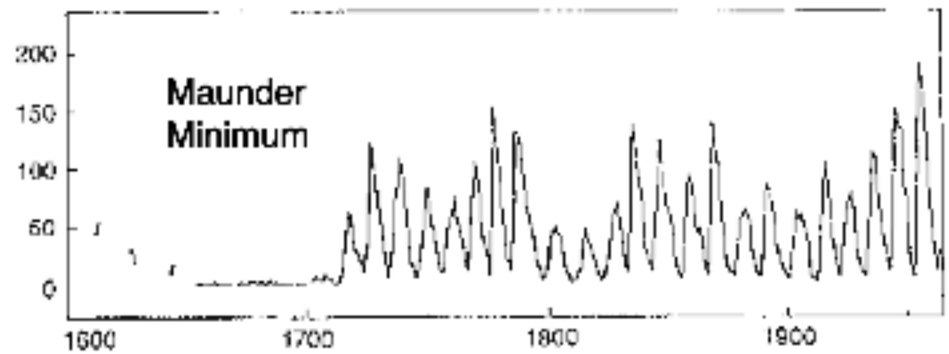
"Large-scale" in space

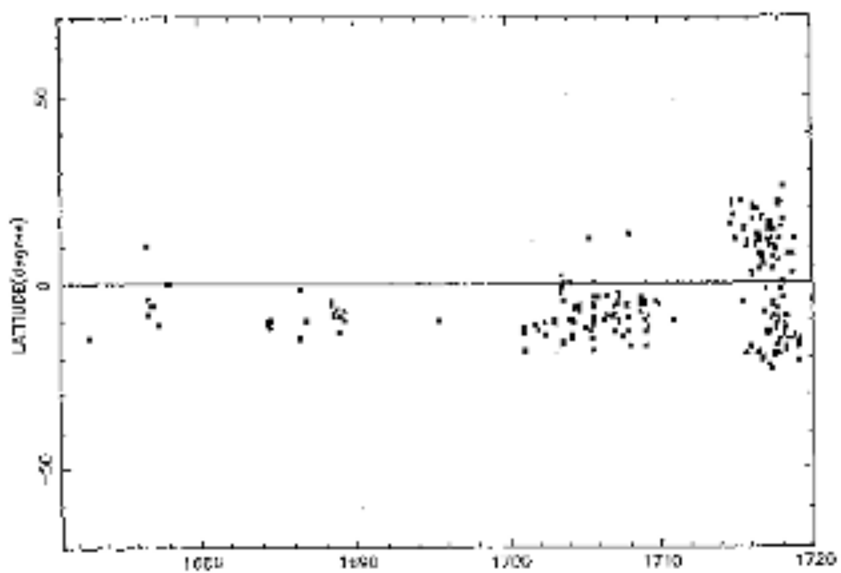
Systematic in time

Spatio-temporal ordering

Large-scale wave?

High Rm





Dynamo Theory

- Dynamos involve the self-consistent solution of the induction and momentum equations of MHD

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \mathbf{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{u} + \frac{\mathbf{F}}{\rho},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}) - \boldsymbol{\nabla} \times (\eta \boldsymbol{\nabla} \times \mathbf{B}),$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0.$$

- A dynamo solution is one where the field remains finite for large times.
- Often split into two problems.
  - Can a velocity be found for which **B** grows? Kinematic (if so what is form of the field, large-scale or small-scale) *kinematic*
  - How does this generated field interact with the velocity in the momentum equation. (what is the amplitude of the generated field?) *dynamic*

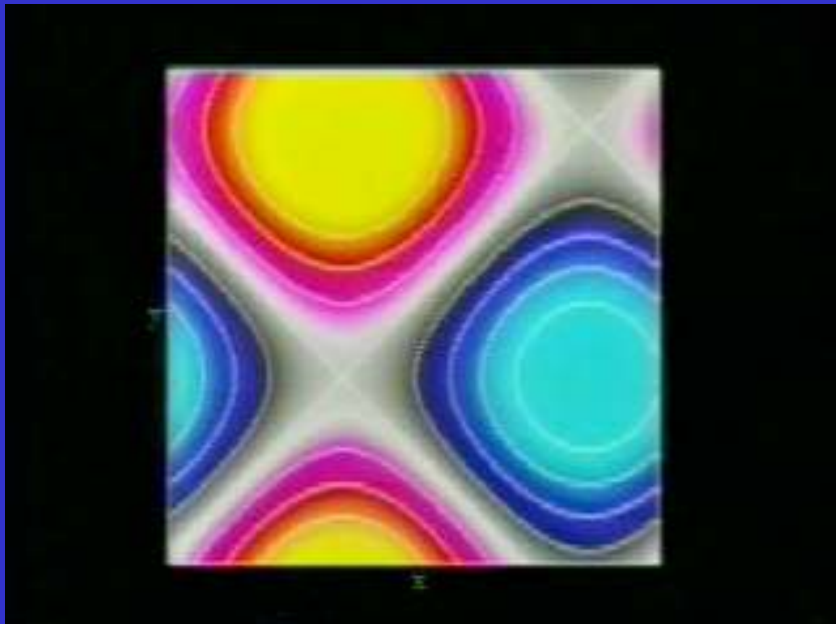
Why not just simulate ?

- Imagine you had access to all the computational resources you needed?
- How much power would you need to simulate a star?
- Conservative estimates (Kapyla) suggest the required power would be 10^{22}W!
- This is the power output of a M9V main sequence red dwarf…

Small-Scale Dynamos

- Small-scale dynamos rely on *chaotic stretching* and reinforcement of the field (see e.g. Childress & Gilbert 1995)
  - More coherent (in time) the velocity the better the stretching (usually)
- ***Any sufficiently chaotic flow will tend to generate magnetic field on the resistive scale.***
- Interesting questions do remain…
  - e.g. Low Pm problem.
    - What happens when magnetic field dissipates in inertial range of the turbulence? (see Stephan's talks)
  - Coherent structures versus random flows (high Kubo vs low Kubo)

Small-scale Dynamos at a single scale



Galloway & Proctor, (1992) Nature

$$\mathbf{u}(x,y,t) = (\psi_y, -\psi_x, w)$$

$$\psi = \sqrt{3/2}\,(\sin(x + \epsilon\cos\omega t) + \cos(y + \epsilon\sin\omega t))$$

$$w = \psi$$

$$\mathbf{B} = \hat{\mathbf{B}}(x,y)e^{ik_z z + \sigma t}$$

- For a velocity field imposed at a finite scale
  - Competition between stretching and diffusion.
  - If stretching strong enough and coherent enough get exponential growth of field.
  - Field is usually amplified at small scales
    - Resistive scale

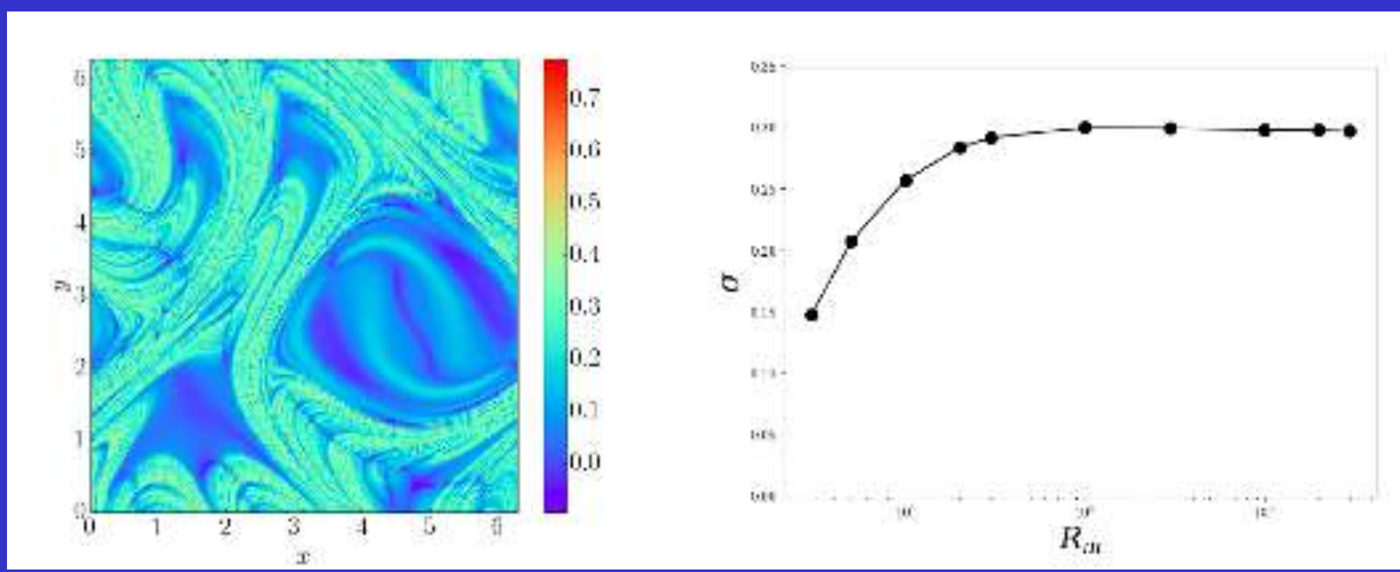
$$l_B \sim Rm^{-1/2}$$

Note:This flow lacks reflexional symmetry (helical)
And should be a good large-scale dynamo

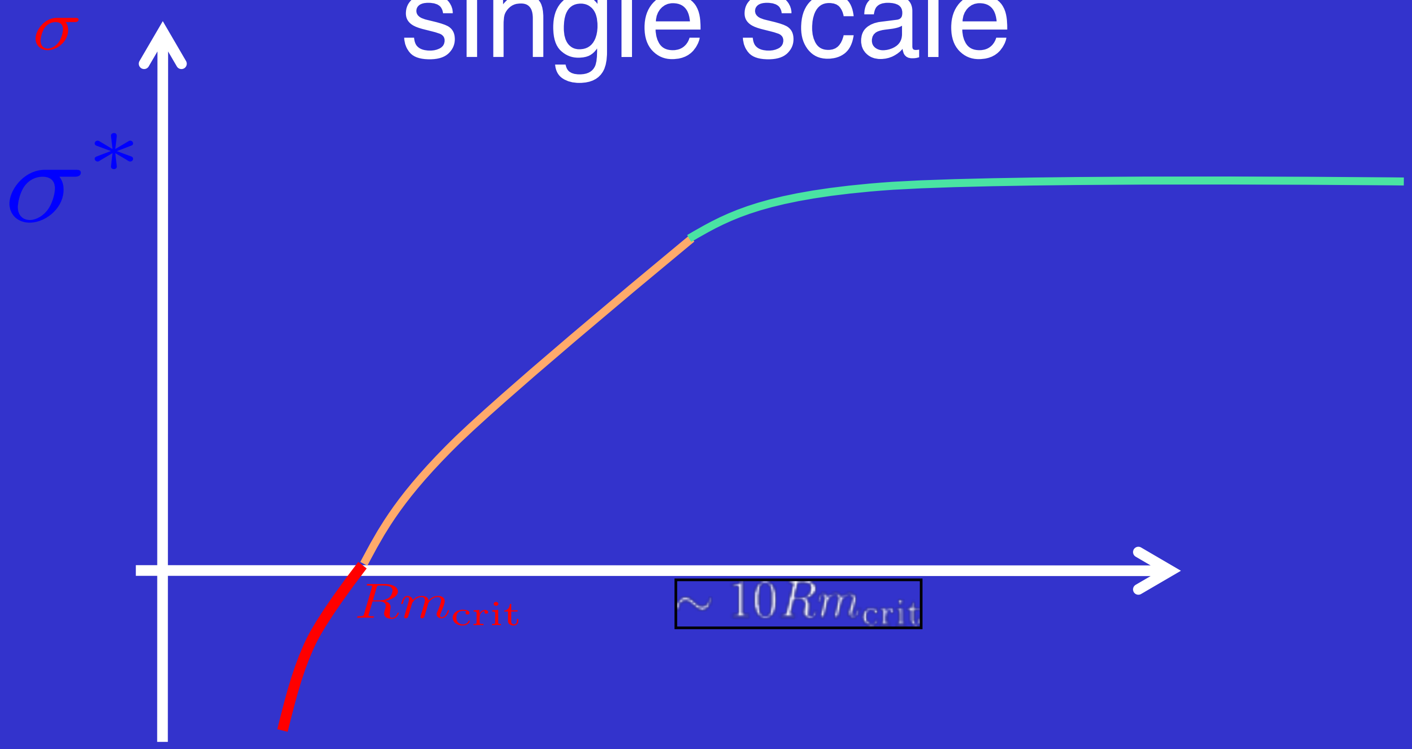
Small-scale Dynamos at a single scale

- Field is amplified on local turnover time of the flow
  - Independent of diffusion as Rm gets large (fast)
- Relies on
  - Chaotic stretching of magnetic fieldlines by velocity
    - Measured by the finite time Lyapunov exponent
  - Not too much cancellation
    - measured by the cancellation exponent (Ott et al 1992, Du & Ott 1993)

Galloway & Proctor, (1992) Nature

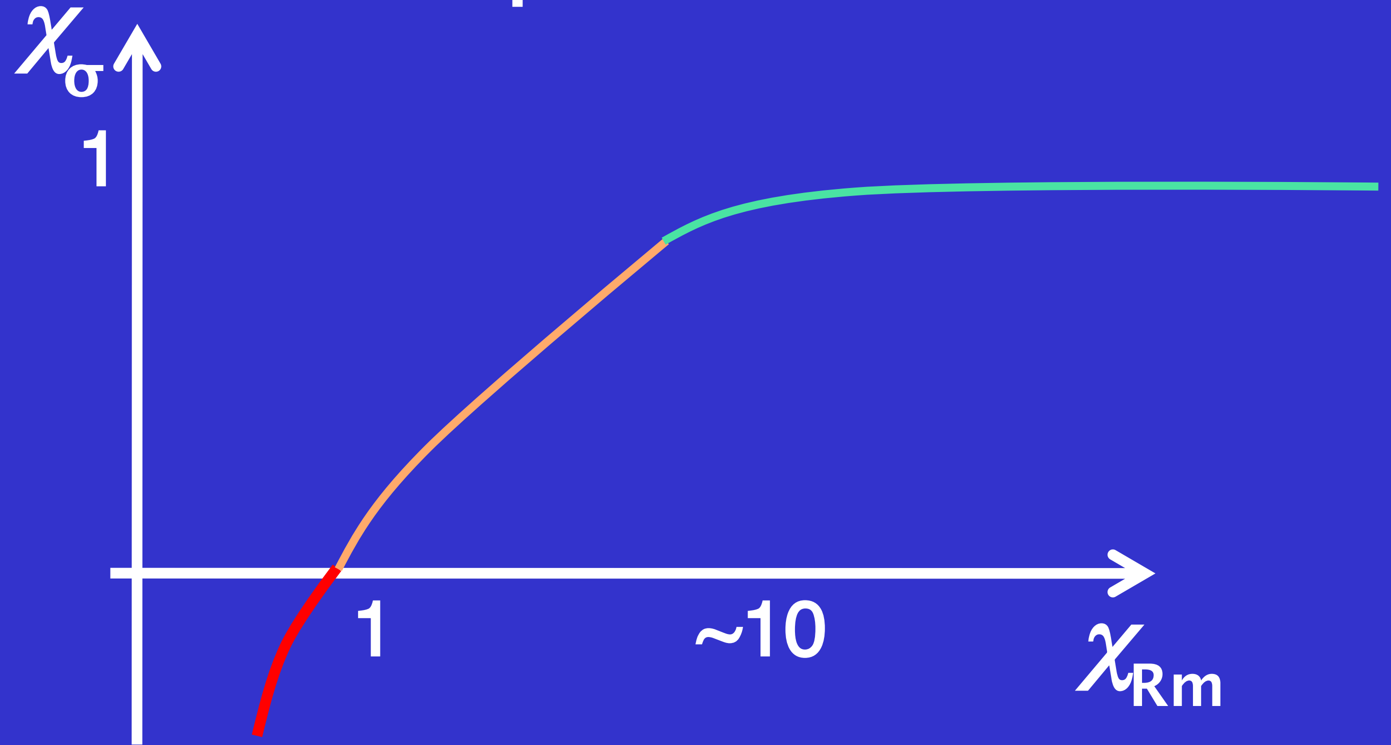


Growth-rates for dynamos at a single scale



- A *fast* dynamo is has an asymptotic growth rate as Rm gets large
- We define high Rm to be well into "the green zone"
    - Certainly the case for astrophysical flows
    - Not usually true for numerical simulations (if we are talking about small-scale flows)

Two important Ratios



$\chi_\sigma = \sigma/\sigma_\star$ $\chi_{Rm} = Rm/Rm_{crit}$

- In a cascade of eddies at large Kubo number the growth-rate is determined by the small-scale stretching of the eddy with the fastest turnover time that has $X > 1$ (Tobias & Cattaneo 2008)

Large-Scale Dynamos

- Large-scale dynamos rely on *lack of reflexional symmetry (parity/symmetry breaking)*
  - Note field may also be generated on a “large-scale” via a large-scale flow.
- This relies on correlations between the small-scale flow and magnetic field leading to the generation of net emf.
  - *Phase* between field and flow is therefore important
  - Makes this a more sensitive type of dynamo than the small-scale dynamo which will exist in any sufficiently turbulent flow.

Mean Field Electrodynamics

(see e.g Krause & Raedler 1980, Brandenburg & Subramanian 2005, Moffatt & Dormy 2019, Tobias 2019)

$$\mathbf{u} = \overline{\mathbf{U}} + \mathbf{u}', \quad \mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}'$$

$$\overline{\frac{\partial \mathbf{B}}{\partial t}} = \overline{\nabla \times (\mathbf{u} \times \mathbf{B})} + \eta \overline{\nabla^2 \mathbf{B}},$$

$$\begin{aligned} \overline{\mathbf{u} \times \mathbf{B}} &= \overline{\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{U}} \times \mathbf{b}' + \mathbf{u}' \times \overline{\mathbf{B}} + \mathbf{u}' \times \mathbf{b}'}, \\ &= \overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{u}' \times \mathbf{b}'}. \end{aligned}$$

$$\boxed{\mathcal{E} = \overline{\mathbf{u}' \times \mathbf{b}'}}$$

Mean Field Electrodynamics

(see e.g Krause & Raedler 1980, Brandenburg & Subramanian 2005, Moffatt & Dormy 2019, Tobias 2019)

$$\frac{\partial \mathbf{b}'}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \mathbf{b}') + \nabla \times (\mathbf{u}' \times \overline{\mathbf{B}}) + \nabla \times \mathcal{G} + \eta \nabla^2 \mathbf{b}',$$

$$\mathcal{G} = \mathbf{u}' \times \mathbf{b}' - \overline{\mathbf{u}' \times \mathbf{b}'}.$$

$$\mathcal{E}_i(\mathbf{x}, t) = \int \int K_{ij}(\mathbf{x}, t; \boldsymbol{\xi}, \tau) \overline{B}_j(\mathbf{x} + \boldsymbol{\xi}, t + \tau)\, d^3\boldsymbol{\xi}\, d\tau,$$

$$\overline{B}_i(\mathbf{x} + \boldsymbol{\xi}) = \overline{B}_i(\mathbf{x}) + \xi_j \frac{\partial \overline{B}_i}{\partial x_j}(\mathbf{x}) + \frac{1}{2}\xi_j \xi_k \frac{\partial^2 \overline{B}_i}{\partial x_j \partial x_k}(\mathbf{x}) + \cdots$$

$$\mathcal{E}_i = \alpha_{ij} \overline{B}_j + \beta_{ijk} \frac{\partial \overline{B}_j}{\partial x_k} + \cdots$$

1st Order Smoothing

$$\boldsymbol{\mathcal{G}} = 0$$

$$\frac{\partial \mathbf{b}'}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \mathbf{b}') + \nabla \times (\mathbf{u}' \times \overline{\mathbf{B}}) + \eta \nabla^2 \mathbf{b}',$$

$Rm \ll 1$

$$\eta \nabla^2 \mathbf{b}' = -\nabla \times (\overline{\mathbf{U}} \times \mathbf{b}') - \nabla \times (\mathbf{u}' \times \overline{\mathbf{B}}),$$

$u_{rms}\tau_c/\ell \ll 1 \quad Rm \gg 1$

$$\frac{\partial \mathbf{b}'}{\partial t} \approx \nabla \times (\overline{\mathbf{U}} \times \mathbf{b}') + \nabla \times (\mathbf{u}' \times \overline{\mathbf{B}}).$$

1st Order Smoothing

$$\boldsymbol{\mathcal{G}} = 0$$

statistically steady state   isotropic turbulence

$$\overline{\boldsymbol{\mathcal{E}}}(t) = \int_0^t \left(\hat{\alpha}(t-t')\overline{\mathbf{B}(t')} - \hat{\eta}_T(t-t')\overline{\boldsymbol{\nabla} \times \mathbf{B}(t')} \right) dt'.$$

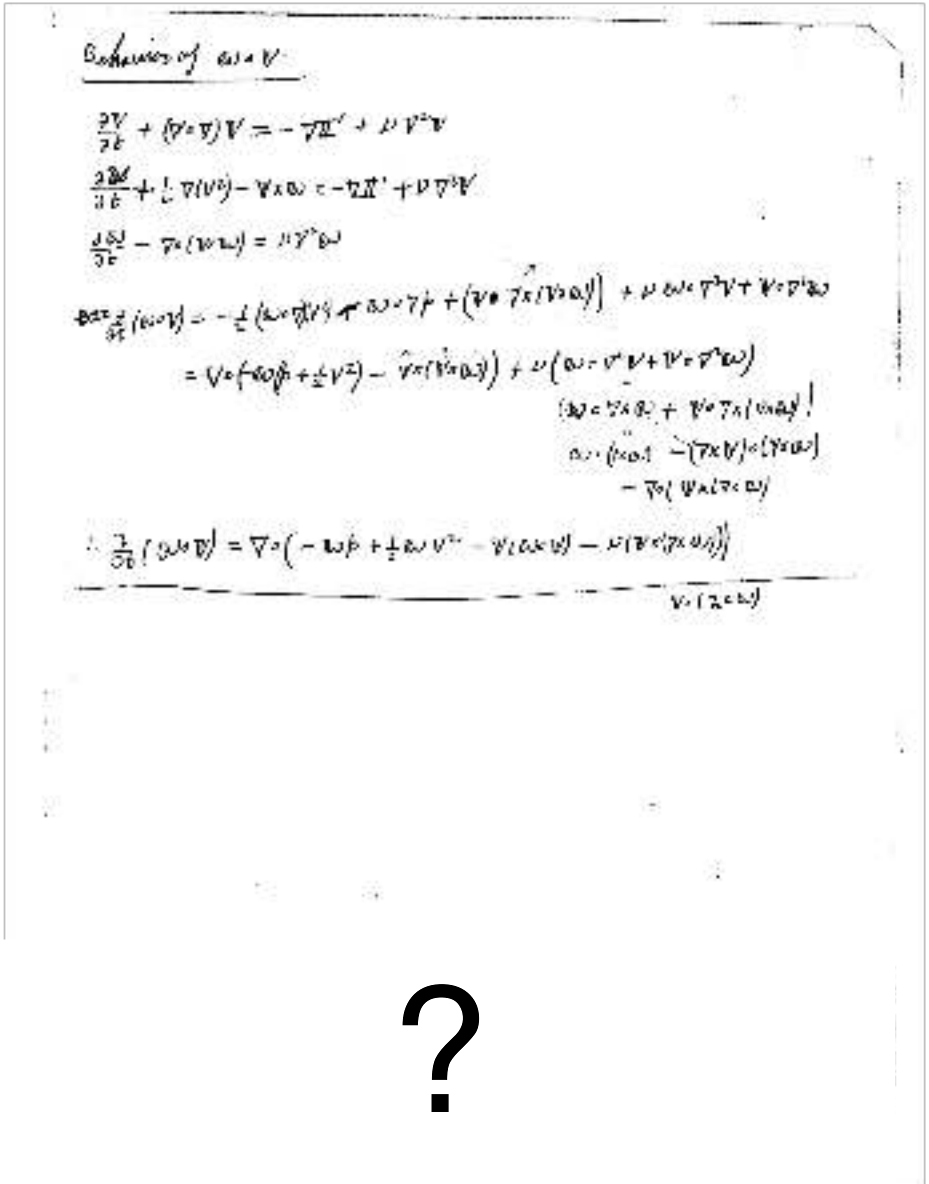
slowly varying mean field

$$\alpha = -\frac{1}{3}\int_0^t \overline{\boldsymbol{u}'(t)\cdot\boldsymbol{\omega}'(t')}dt' \approx -\frac{1}{3}\tau_c\,\overline{\boldsymbol{u}'\cdot\boldsymbol{\omega}'}$$

$$\eta_T = \frac{1}{3}\int_0^t \overline{\boldsymbol{u}'(t)\cdot\boldsymbol{u}'(t')}dt' \approx -\frac{1}{3}\tau_c\,\overline{\boldsymbol{u}'\cdot\boldsymbol{u}'}$$

Helicity broken reflectional symmetry important

A Historical Aside...

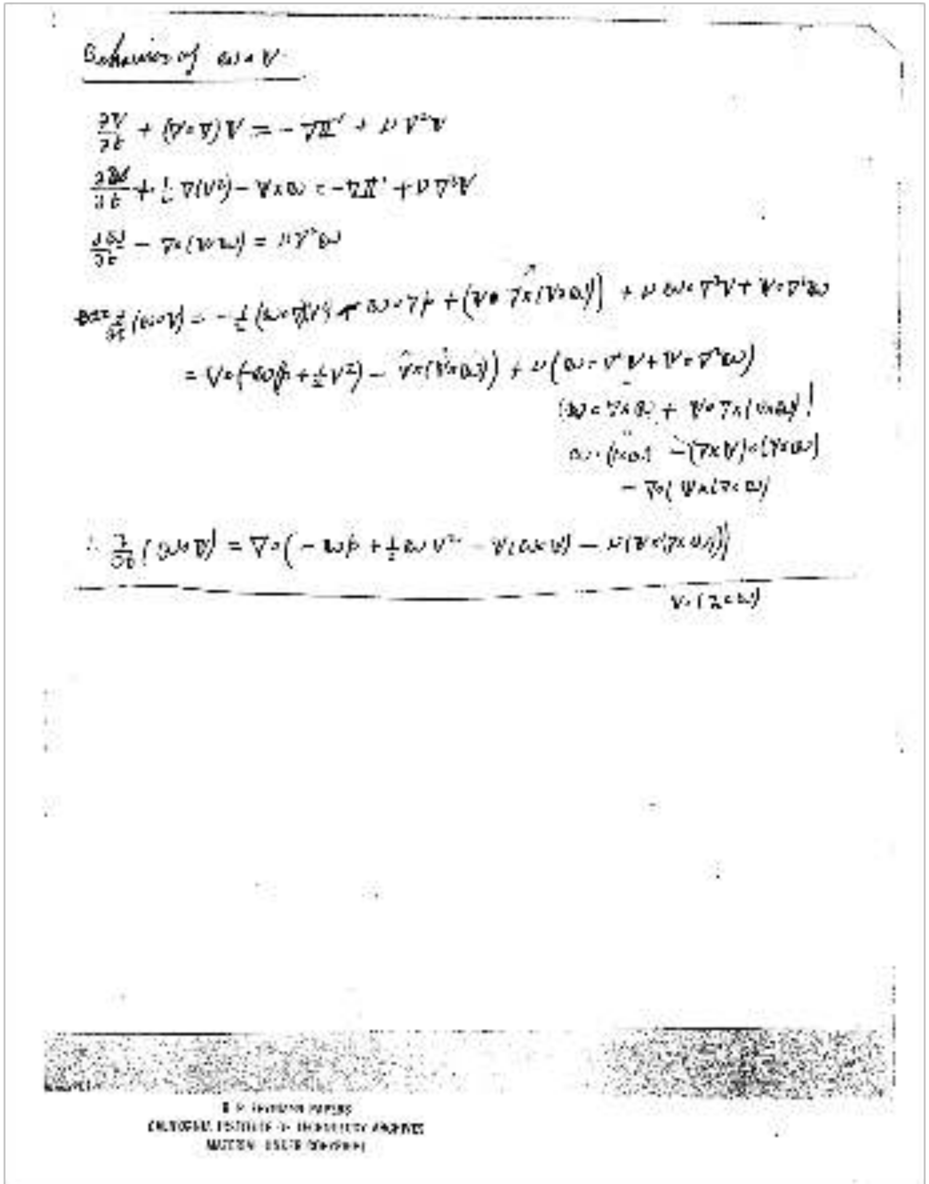


- Conservation of Helicity
  - Often attributed to Moreau (1961) Moffatt (1969)
  - Important results concerning topological nature of

    $$\int \boldsymbol{X}\cdot\nabla\times\boldsymbol{X}\,\mathrm{d}^3x$$

  - See e.g. Berger & Field (1984)
- Simplest result concerning conservation of kinetic helicity for incompressible Euler was discovered earlier.

A Historical Aside...



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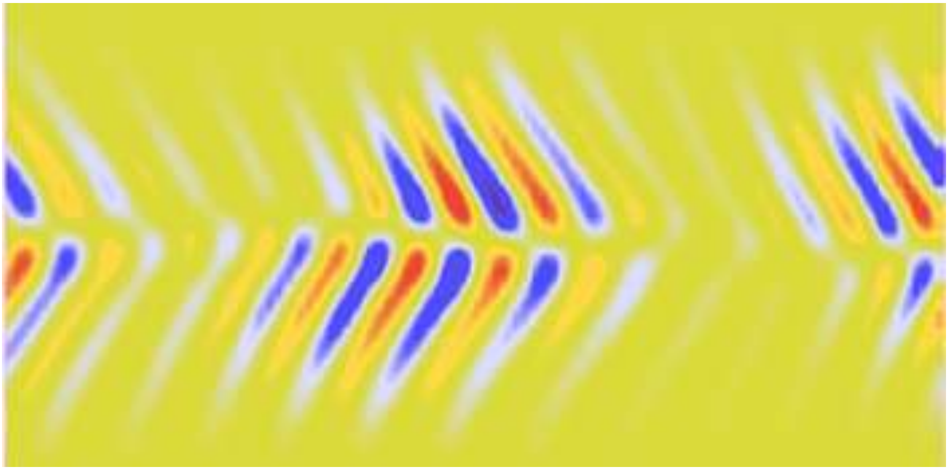
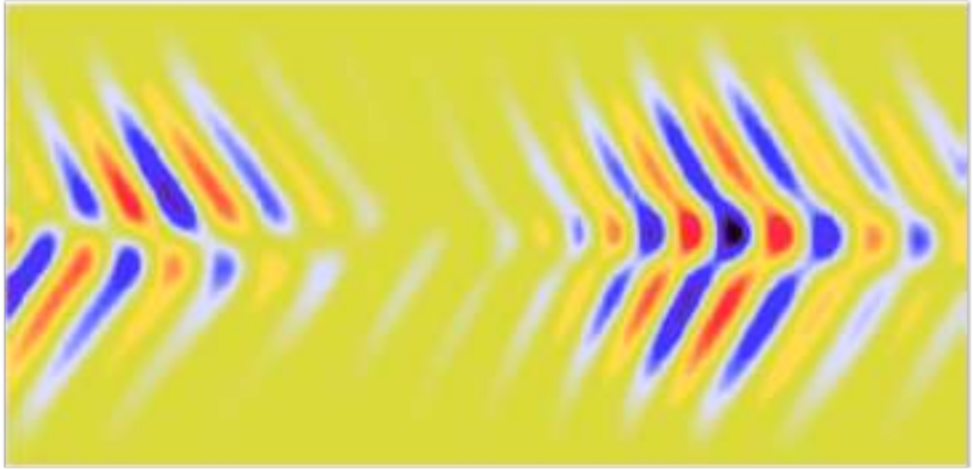
  - See e.g. Berger & Field (1984)
- Simplest result concerning conservation of kinetic helicity for incompressible Euler was discovered earlier.
- Feynman, Folder 76.14 “Turbulence” (courtesy L. Kadanov, G. Eyink)

Why are mean field models popular?





Tobias 1998

- Solution of mean field models for plausible solar differential rotation and turbulent transport coefficients can give solar-like behaviour
- Interaction of oscillatory dipole and quadrupole modes can lead to modulation and symmetry breaking.
- Construction of low-order models are possible (Knobloch et al 1998)

Large-scale versus small-scale: Kinematic considerations

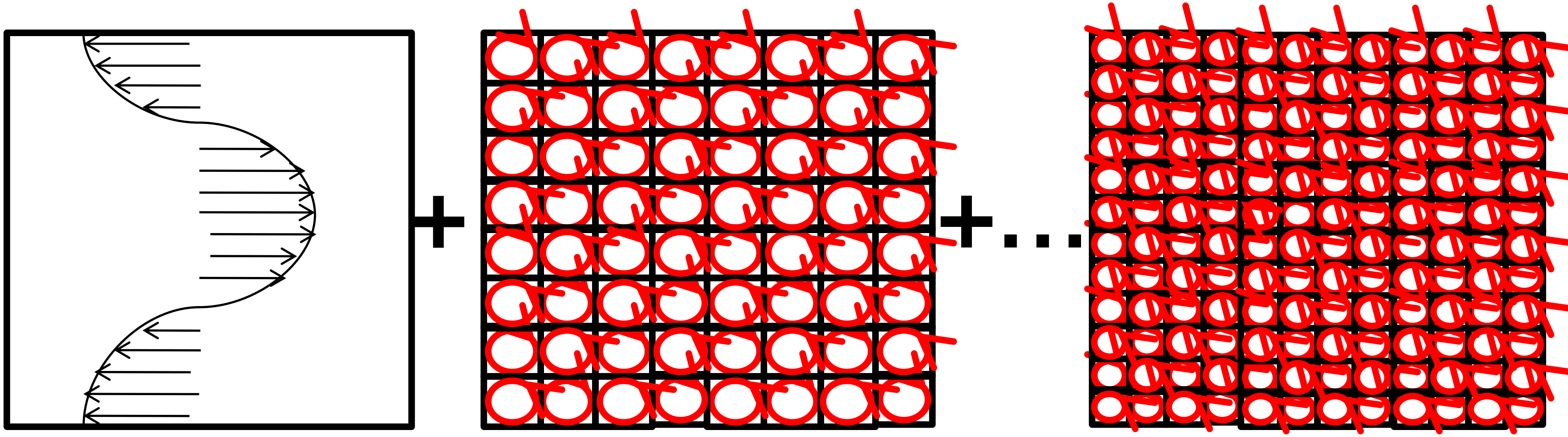
- Some small-scale dynamos (e.g. Galloway Proctor) have the ingredients required to be a large-scale dynamo
  - Lack of reflexional symmetry in the flow
  - Leads to the generation of a mean EMF

$$\mathcal{E} = \langle \mathbf{u}' \times \mathbf{b}' \rangle$$

- However at high R_{Rm} (in the green zone - and even in the amber zone) the large-scale magnetic field generated by this EMF is completely dominated by the small-scale fluctuations provided by the small-scale dynamo (cf Cattaneo & Hughes 2006, Tobias & Cattaneo 2015)
- One idea is to use a shear flow to “boost” the EMF (and indeed the dynamo growth) via one of many effects (shear-current effect etc) (see e.g. Yousef et al 2008, Käpylä & Brandenburg 2009, Sridhar & Singh 2010, Hughes & Proctor 2013)
- *An alternative is to use a shear to control the fluctuations*

High Rm effects of shear

Tobias & Cattaneo (2013, Nature) Cattaneo & Tobias (2014 ApJ)



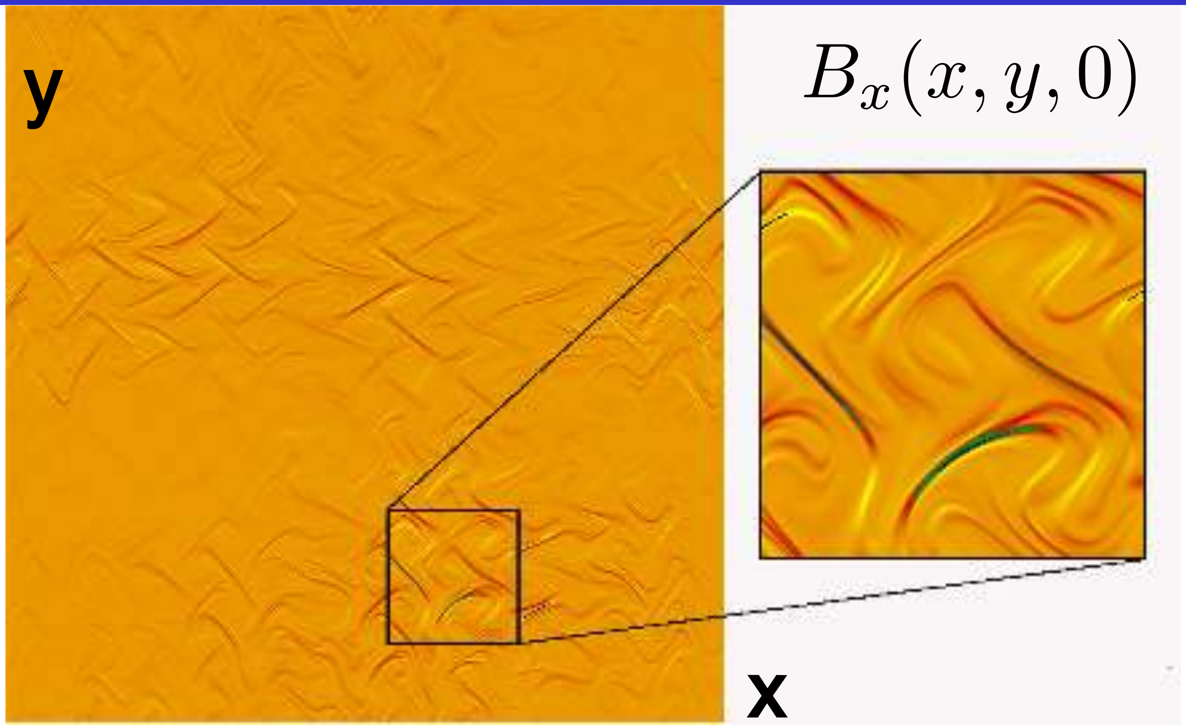
$$Rm_s \approx 0 \to 10^5$$

$$Rm_k = \frac{U_k}{k\eta} \approx 2500$$

- Need to get to very high Rm (so growth-rate is asymptotic for small-scale flow.)
- Very hard to do in 3D flow.
  - Resolutions up to 4096^2
- Use multi-scale generalisation of the CT2005 flow (2.5 D)
  - Velocity amplitude decreases with scale; shear rate and turnover frequency increase with scale
  - *Scale dependent renewal time*
    - *comparable with local turnover time (in asymptotic regime)*
    - *much shorter than local turnover time (poor dynamo)*

No shear: long correlation time

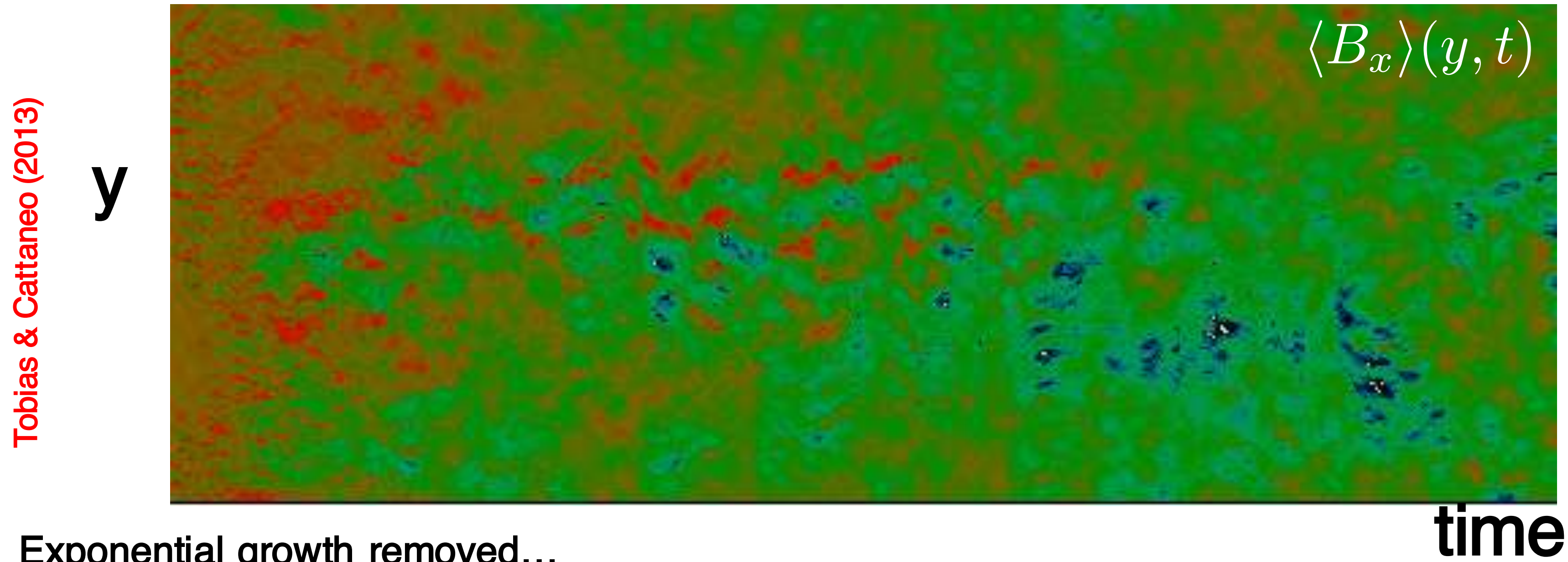
Tobias & Cattaneo (2013)



Computations using UK MHD Consortium Machine at University of Leeds

- ***Great small-scale dynamo (no real surprise)***
- Filamentary field with
  - Length comparable to scale of velocity
  - Width controlled by diffusion $\propto Rm^{-1/2}$
- Overall pattern changes on the turnover time
  - Comparable with correlation time

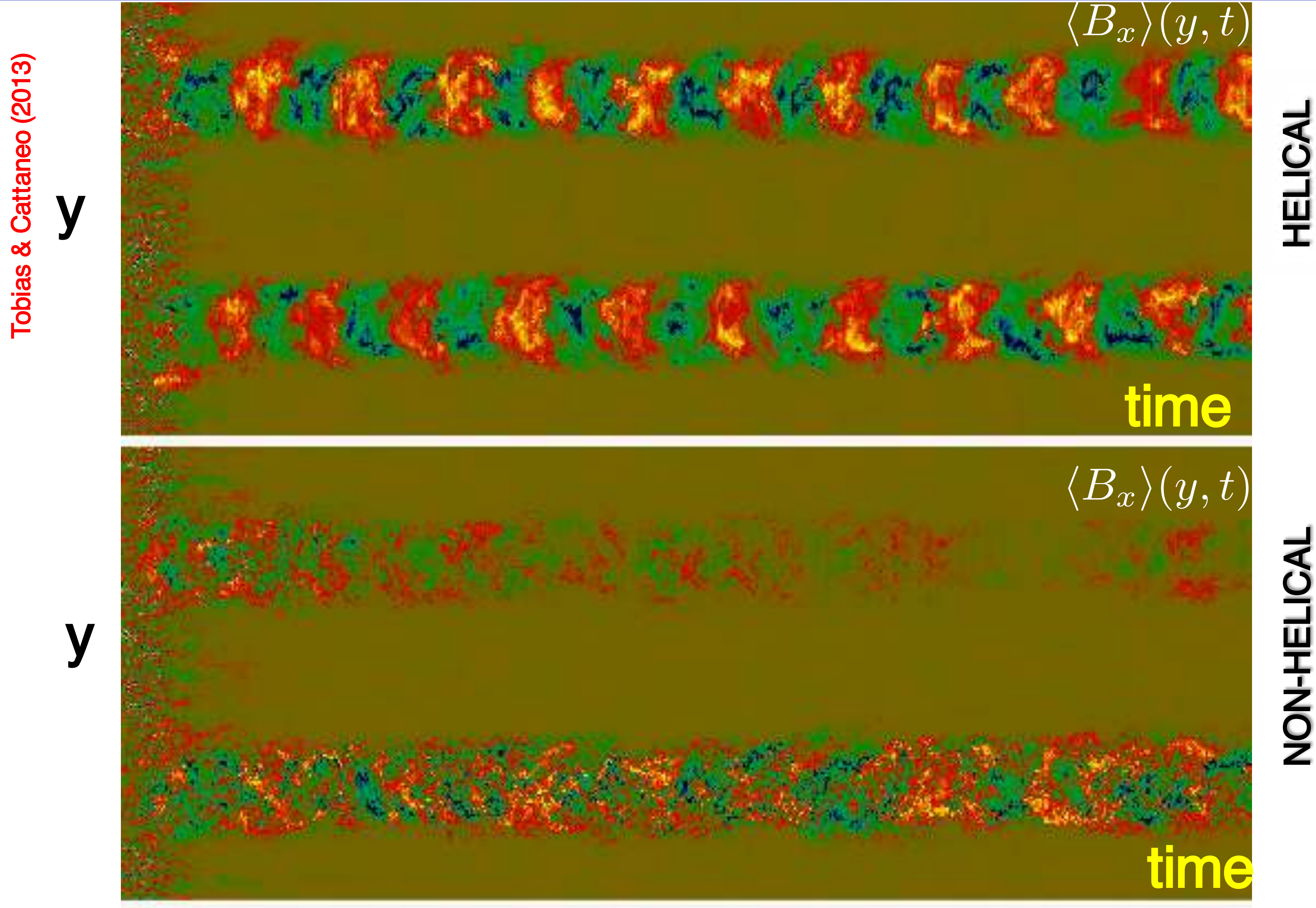
No shear: Long correlation time



Exponential growth removed…

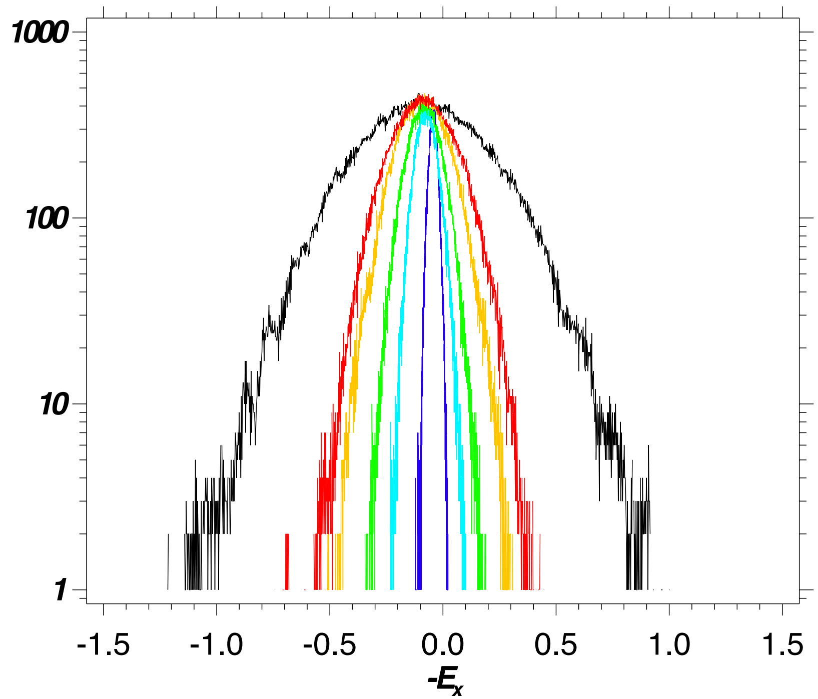
- No systematic large-scale behaviour
  - E.g. average B_x over x and plot as a function of y and t
- Can also construct a velocity field with ***<u>no net helicity</u>*** when averaged over time
  - This has comparable growth-rate as a small-scale dynamo
  - Similar stretching, similar cancellation, similar pictures…

With shear…



Exponential growth removed…moved to middle of the domain for clarity…

Suppression of fluctuations by shear



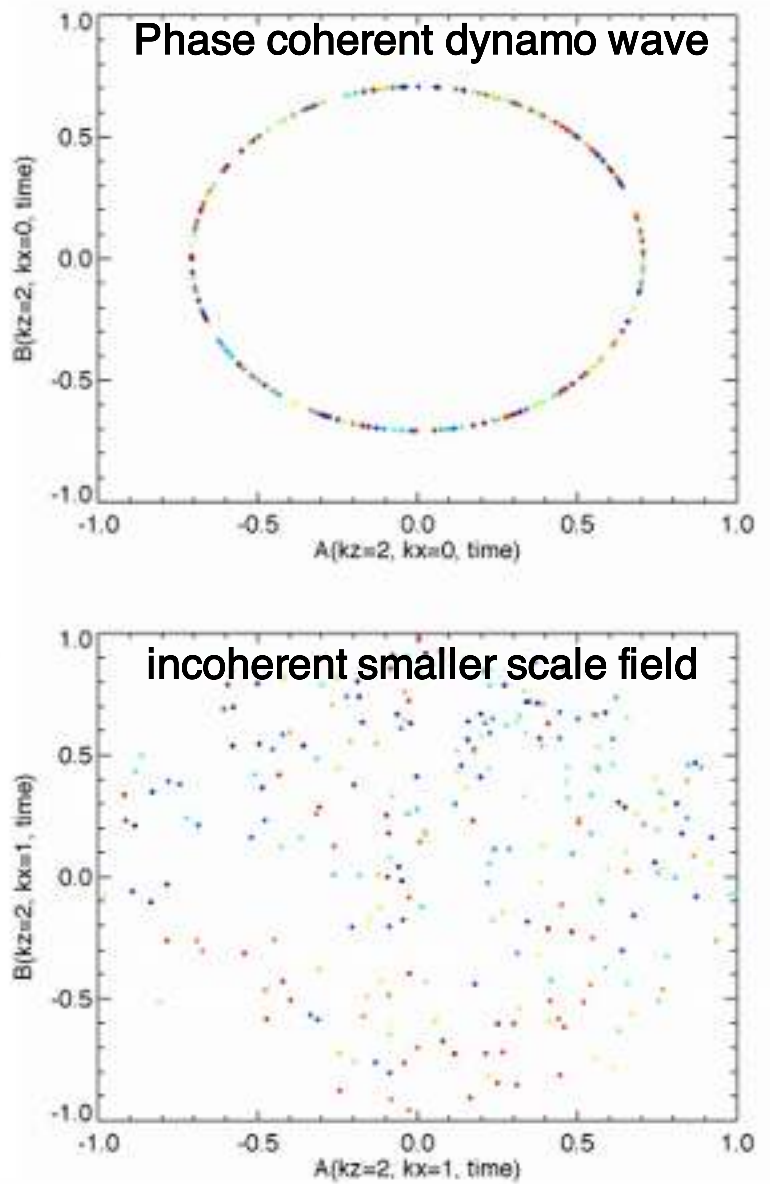
Variance of EMF is a simple function of shear rate; (also depends on Rm)

(Tobias & Cattaneo 2014)

What does mean field theory get right in the kinematic regime? (Nigro et al MNRAS 2017)

- Derivation of mean-field equations is done by a filtering procedure.
- Would like the application of the filter to the eigenfunctions of the full equations to correspond to the eigenfunctions of the filtered equations.
- Would also like the growth-rate to match…
  - However, this is controlled by the stretching of the eddies with the fastest turnover time and $Rm>Rm_c$ (Tobias & Cattaneo 2008)
- However, in presence of shear
  - In the case where shear breaks the isotropy of the filtered equations and also the isotropy of the statistics of the velocity
  - Symmetry breaking acts so as to perturb frequency of solutions away from zero and is controlled by the change in symmetry of large scales.
  - In this case MFT does get this frequency correct
  - Gives hope for statistical theories in the nonlinear regime where the small-scale growth has saturated.

Nigro et al (2017)



Catastrophic Quenching?

Vainshtein & Cattaneo (1992),

- Assume we can generate a growing kinematic magnetic field.
- At what level does this saturate in the nonlinear regime
- What is the form of the quenching of the turbulent transport coefficients.
- Assume begins to equilibrate when mean field energy comes into equipartition with turbulent kinetic energy. Then…

$$\alpha = \frac{\alpha_0}{1 + B_0^2/\mathcal{B}^2},$$

Catastrophic Quenching?

Vainshtein & Cattaneo (1992),

- However that assumes magnetic energy is dominated by large scales.
- If there is a mean field, turbulence will amplify this into small scales, until ratio is large. Can only get rid of small scales via diffusion so ratio will depend on Rm.
- Transport coefficients become quenched when energy in small-scale field is in equipartition, i.e.

$$\alpha = \frac{\alpha_0}{1 + Rm^p B_0^2 / \mathcal{B}^2},$$

- Formal proof for turbulent diffusion in 2D.
- Bad news for mean-field dynamos.

Conservation of magnetic helicity

$$\nabla \cdot \mathbf{B} = 0, \quad \longrightarrow \quad \mathbf{B} = \nabla \times \mathbf{A}$$

Defined up to a gauge:

$$\mathbf{A} \to \mathbf{A} + \nabla\psi$$

$$\frac{\partial \mathbf{A}}{\partial t} = (\mathbf{u} \times \mathbf{B}) - \eta \nabla \times \mathbf{B} + \nabla\phi$$

Various choices of gauge are possible, e.g.

Coulomb gauge:

$$\nabla \cdot \mathbf{A} = 0; \quad \nabla^2\phi = -\nabla \cdot (\mathbf{u} \times \mathbf{B})$$

Winding Gauge:
(Prior & Yeates 2014)

$$\nabla_H \cdot \mathbf{A} = 0; \quad \nabla_H^2\phi' = -\nabla \cdot (\mathbf{u} \times \mathbf{B})$$

$$\phi' = \phi - \eta\frac{\partial A_z}{\partial z}$$

$$H = \int_V \mathbf{A} \cdot \mathbf{B}\, dV, \quad \frac{dH}{dt} = -2\eta\mu \int_V \mathbf{j} \cdot \mathbf{B}\, dV + F_s,$$

Volume dissipation

Surface Flux

Celebrating 25 years of GD94!

VOLUME 72, NUMBER 11 PHYSICAL REVIEW LETTERS 14 MARCH 1994

Self-Consistent Theory of Mean-Field Electrodynamics

A. V. Gruzinov and P. H. Diamond
Department of Physics, University of California at San Diego, La Jolla, California 92093-0319
(Received 6 December 1993)

Combine 3 exact results

$$\boldsymbol{\mathcal{E}} \cdot \boldsymbol{B}_0 = -\frac{1}{\sigma}\langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle + \langle \boldsymbol{e} \cdot \boldsymbol{b} \rangle = \alpha B_0^2,$$

$$\frac{\partial \langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle}{\partial t} = -2\langle \boldsymbol{e} \cdot \boldsymbol{b} \rangle + \langle \boldsymbol{\nabla} \cdot (\boldsymbol{b}\phi) \rangle - \langle \boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{e}) \rangle,$$

$$\frac{dH_M}{dt} = -2\eta Q_H + F_s,$$

With one result from a nMHD closure (EDQNM)

$$\alpha = -\frac{\tau_c}{3}\langle \boldsymbol{u} \cdot \boldsymbol{\omega} - \boldsymbol{b} \cdot \boldsymbol{j} \rangle,$$

Pouquet, Frisch & Leorat (1976)

In a closed system (F_S=0) to yield…

In presence of mean current

$$\alpha = \frac{\alpha_K + \eta_t R_m \langle \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} \rangle / B_{eq}^2}{1 + R_m \langle \overline{\boldsymbol{B}}^2 \rangle / B_{eq}^2}$$

- Physically this can be understood that if you want to grow large-scale magnetic helicity, you must get rid of the small-scale magnetic helicity.
- Can only do this in a closed system on a diffusive timescale
- Helicity fluxes can potentially alleviate the effects of catastrophic α-quenching.
  - Losing helicity through the boundary at a rate independent of resistivity introduces irreversability
  - *on what timescale can you do this?*

Nonlinear formulation with helicity fluxes

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \eta \, \nabla \times \mathbf{J},$$

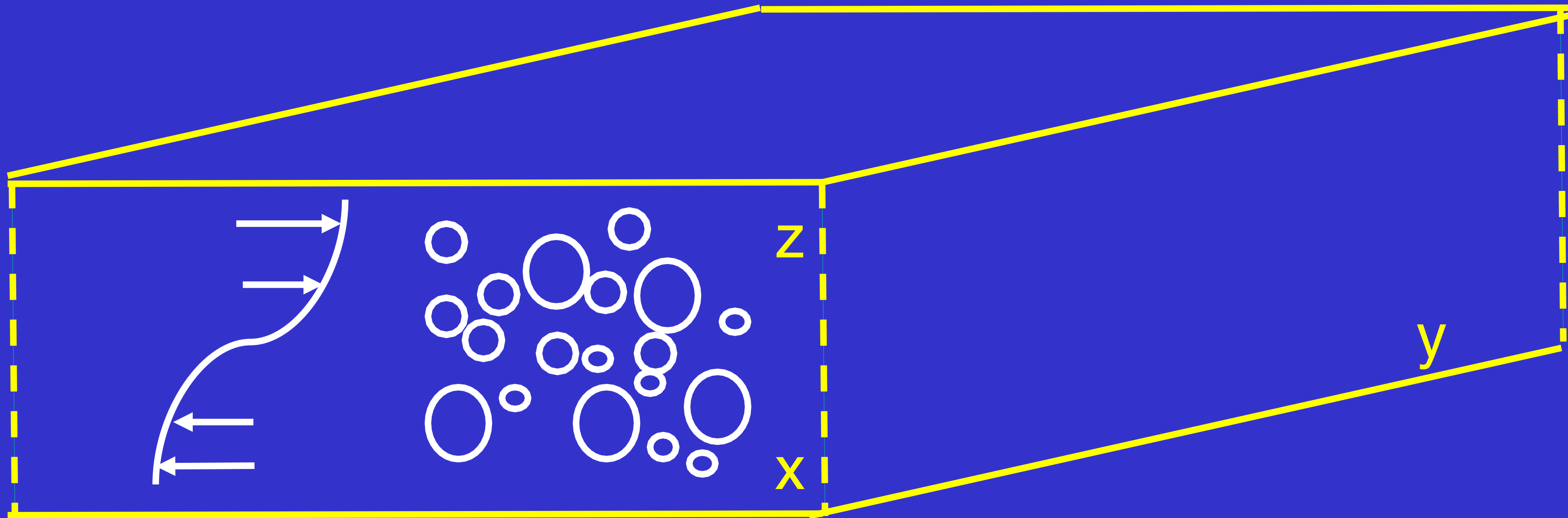
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \mathbf{p} + \mathbf{J} \times \mathbf{B} + \nu \, \nabla^{2} \mathbf{u} + \mathbf{f},$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{B} = \mathbf{0},$$

$$\mathbf{f} = A(k)\mathbf{g}(\mathbf{x}, \mathbf{t}) + S(\cos z, 0, 0).$$

$$\frac{du}{dz} = \frac{dv}{dz} = w = 0,$$

$$B_x = B_y = \frac{dB_z}{dz} = 0.$$



Helicity Considerations

$$\frac{dH}{dt} = W_H + F_I + W_D + F_D + F_G,$$

$$W_H = -\mathcal{V}\langle \mathbf{u} \times \mathbf{B} \rangle \cdot \langle \mathbf{B} \rangle_H,$$

$$F_I = \int_S (\mathbf{B} \cdot \mathbf{n})[\mathbf{A} \cdot \mathbf{u} + \Phi'] \, dS,$$

$$W_D = -2\, \eta \mathcal{V} \langle \mathbf{B} \cdot \mathbf{J} \rangle,$$

$$F_D = \eta \int_S (\mathbf{A} \times \mathbf{J}) \cdot \mathbf{n} \, dS,$$

$$F_G = \eta \int_S (\mathbf{B} \cdot \mathbf{n}) \frac{\partial}{\partial z} A_z \, dS.$$

Results

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \eta \, \nabla \times \mathbf{J},$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \mathbf{p} + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{u} + \mathbf{f},$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{B} = \mathbf{0},$$

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Choose S=0,1,5

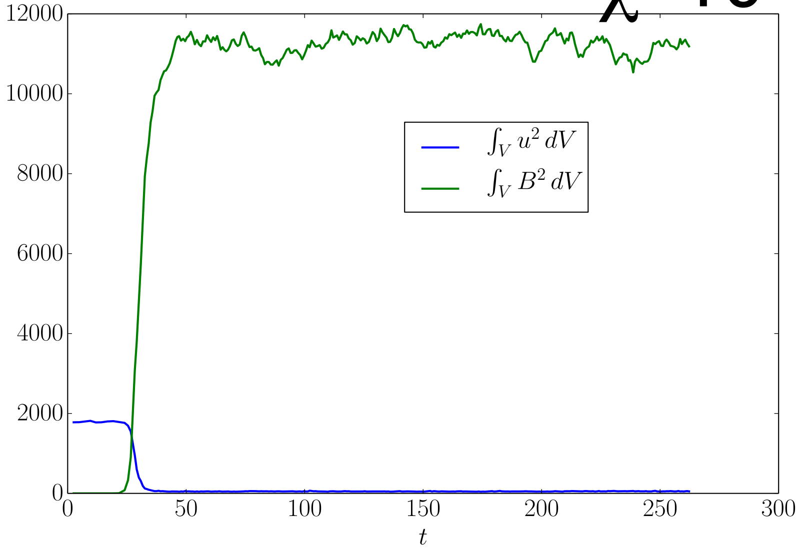
k~6

Re~1

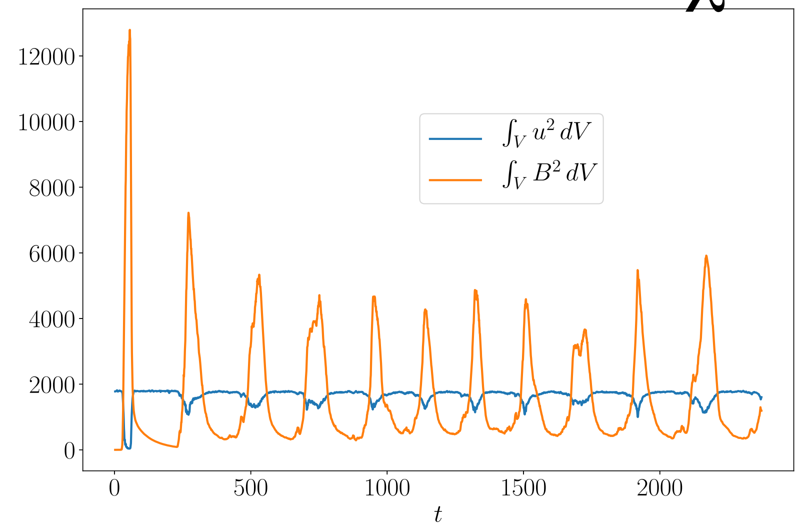
$\chi = Rm/Rm_c \sim 10,20,40$

Results for large shear: S=5

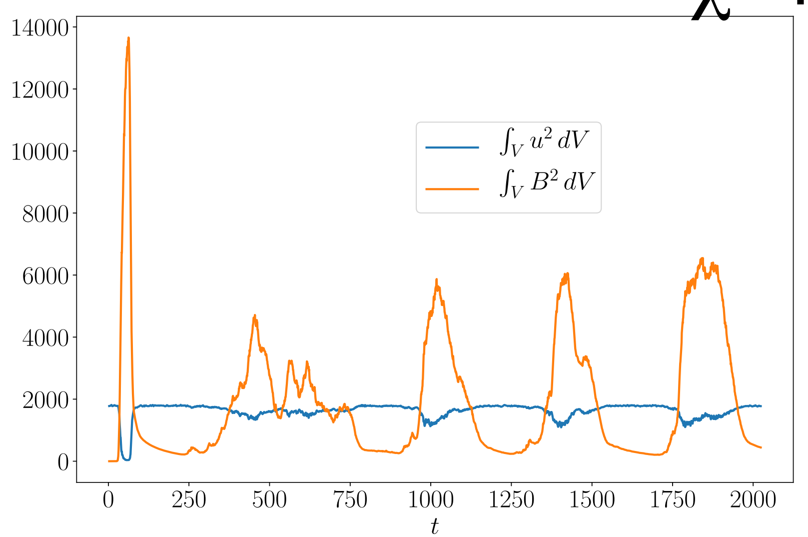
χ=10



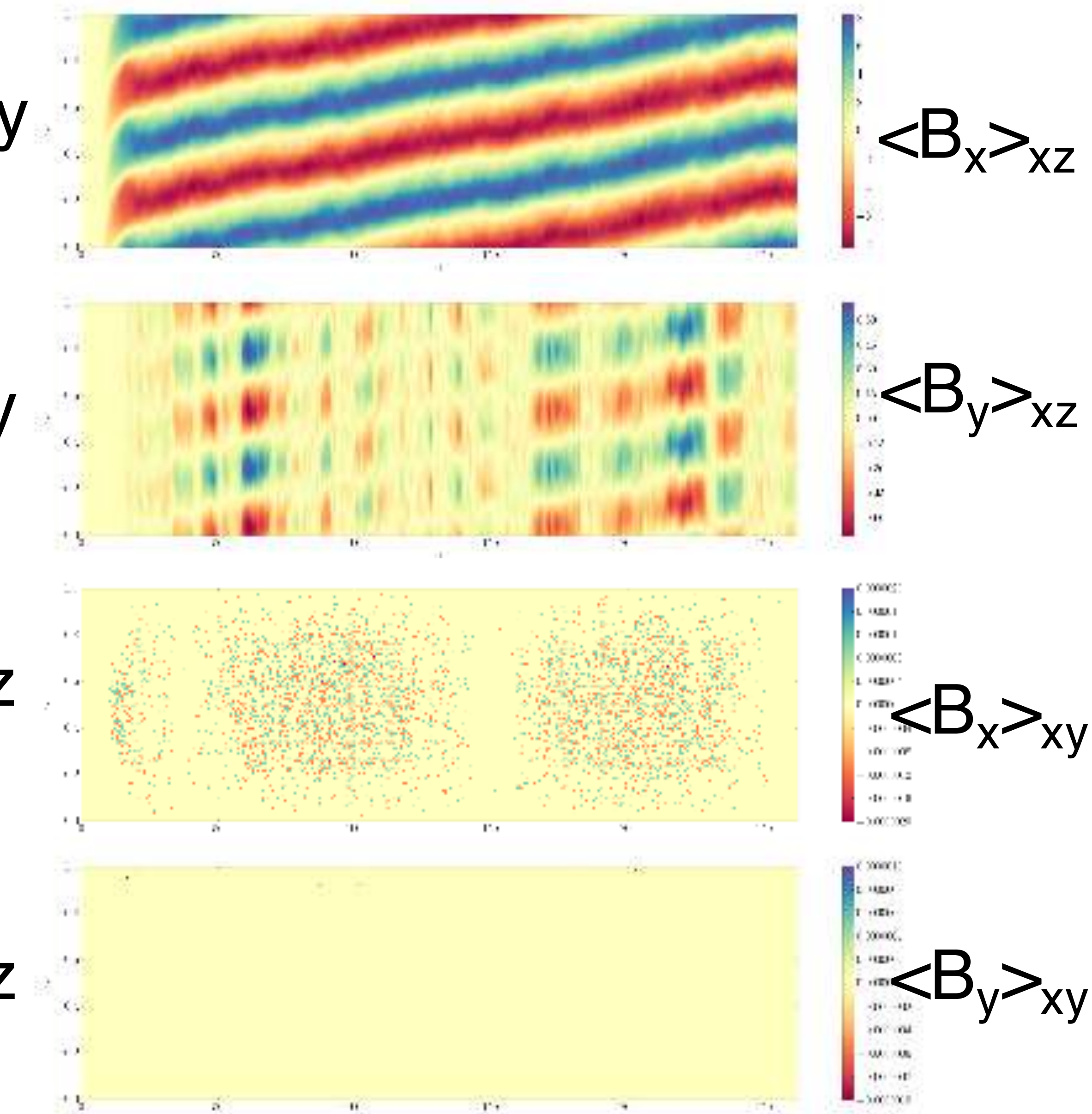
χ=20

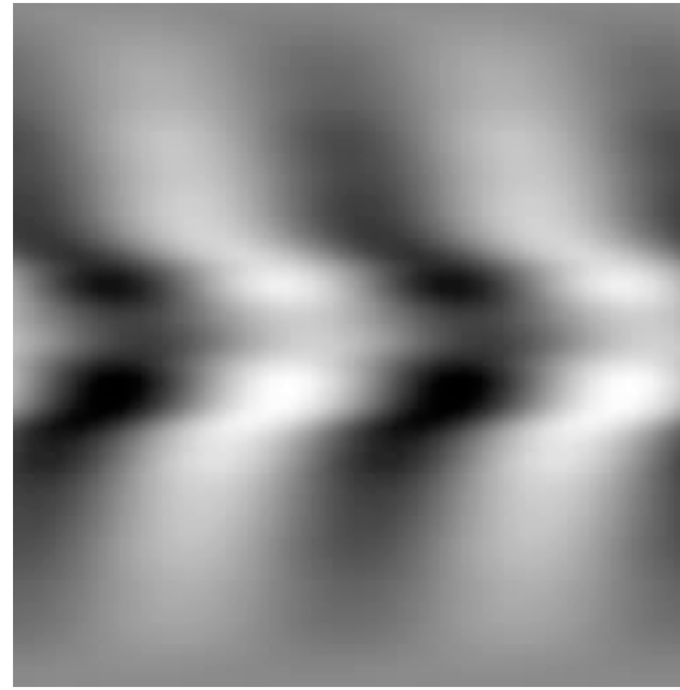


χ=40



Results for large shear: low Rm χ=10

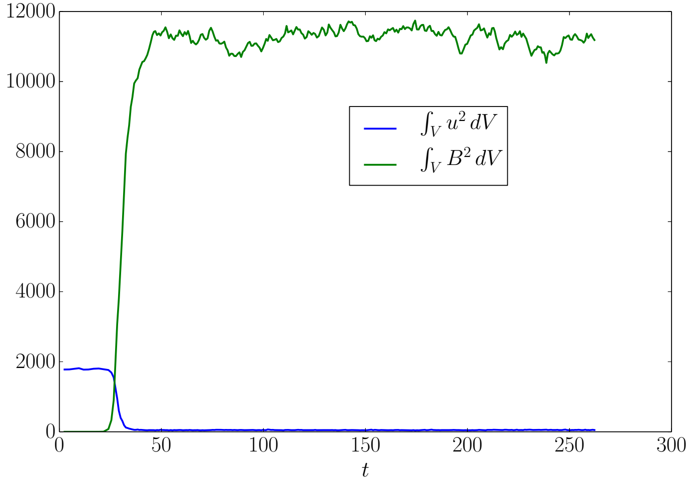




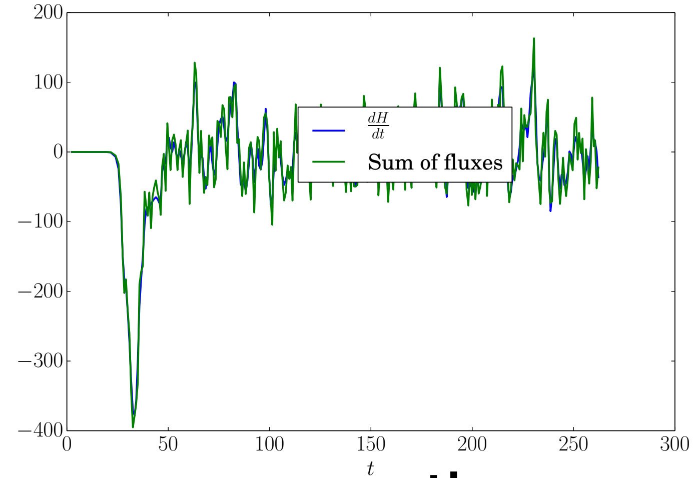
Results for large shear: low Rm χ=10

- **Short kinematic phase**
  - large growth rate comparable with the turnover frequency of the little eddies
  - Dominated by propagating waves with ky=2 and high phase velocity(approx 0.13).
- **Saturated Phase**
  - Invariant manifold.
  - Velocity is not y-independent but, nevertheless, no mean By or Bx can be generated.
  - symmetry between B(odd) and u(even). In this state the
  - Lorentz force has a big projection on ky=0 and it can eciently wipe out the shear. Velocity remains nearly y-independent.
  - the rate of doing work is large
- High magnetic energy, little shear, slow propagation velocity, ky=2 for the dynamo wave solution.

Results for large shear: low Rm χ=10

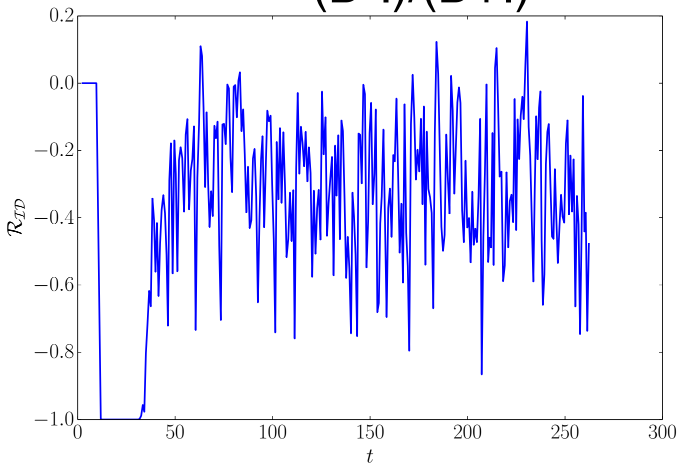


time



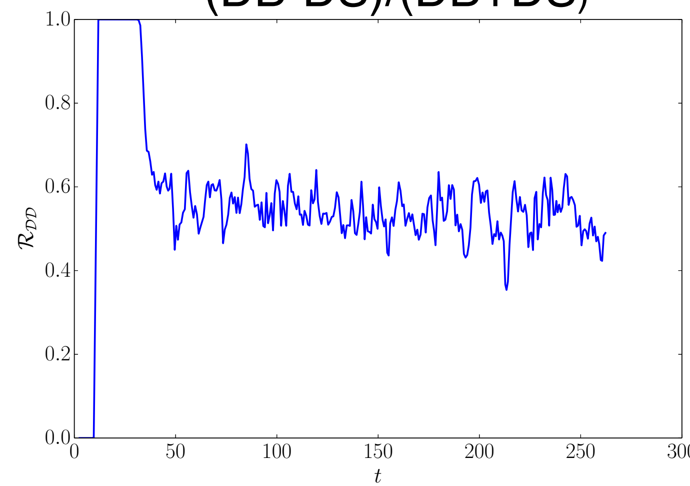
time

(D-I)/(D+I)



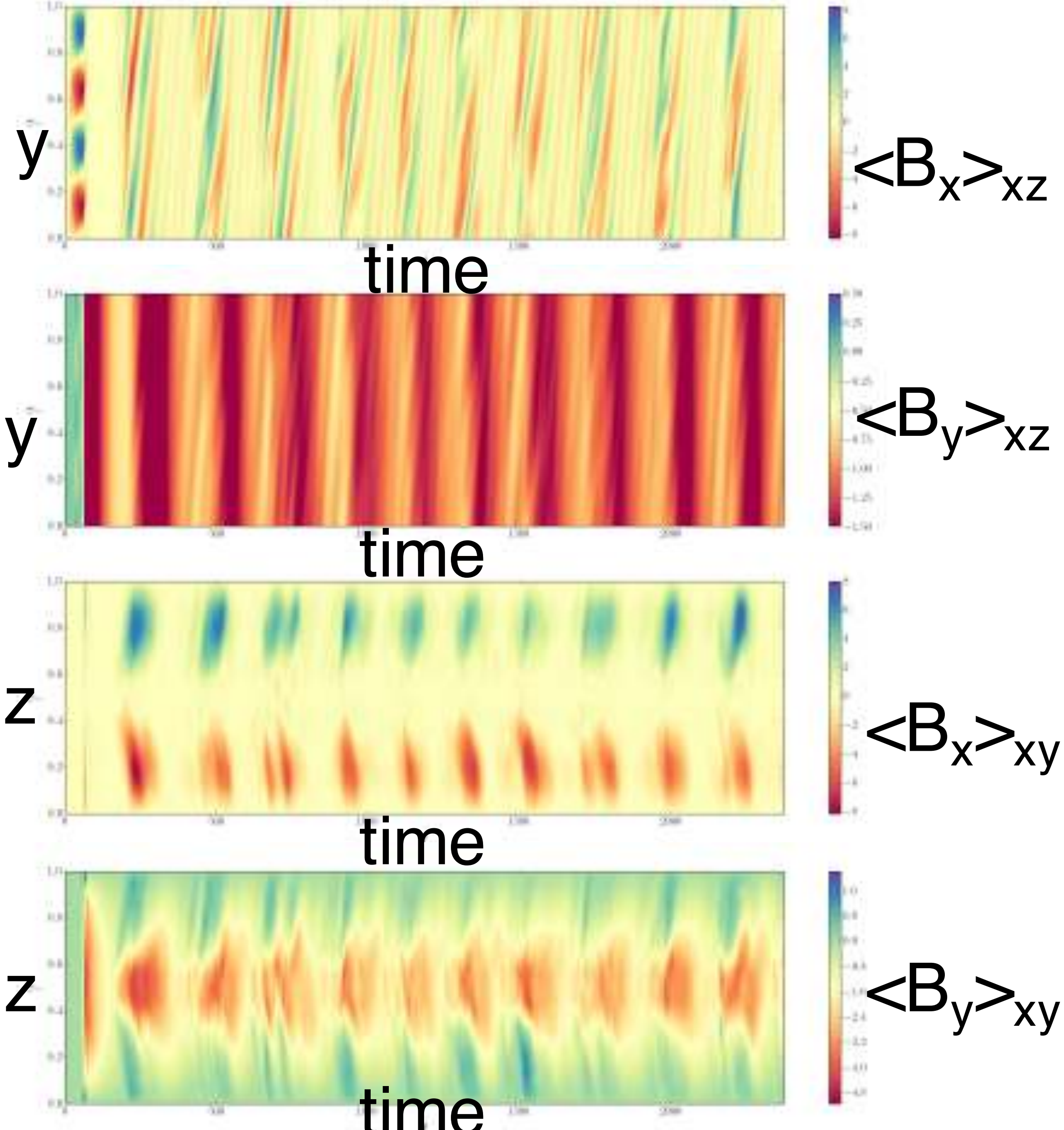
time

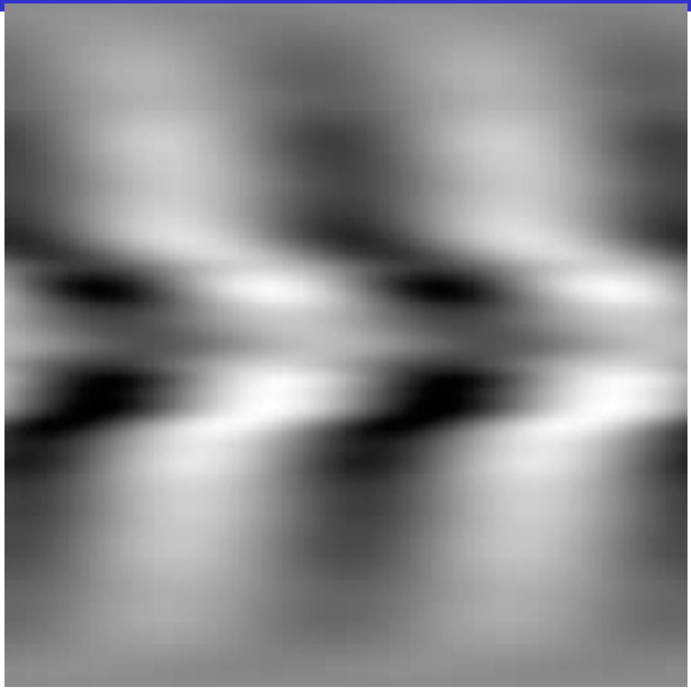
(DB-DS)/(DB+DS)



time

Results for large shear: moderate Rm $\chi=20$

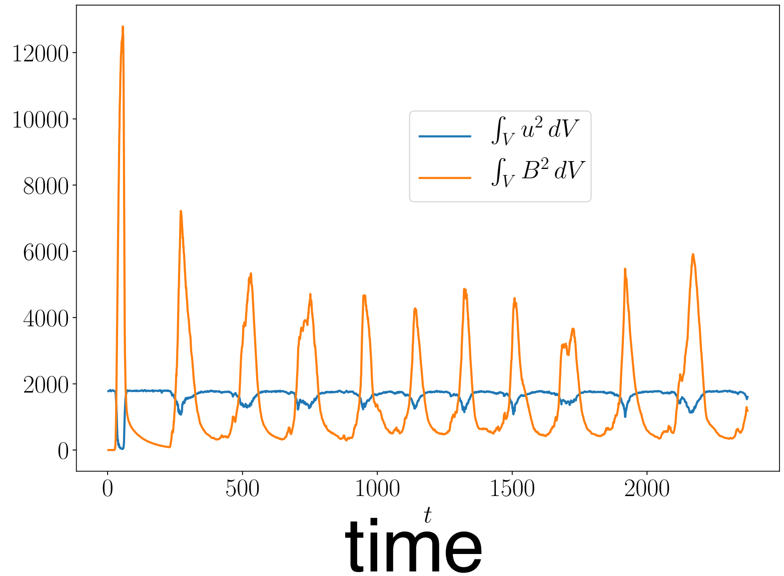


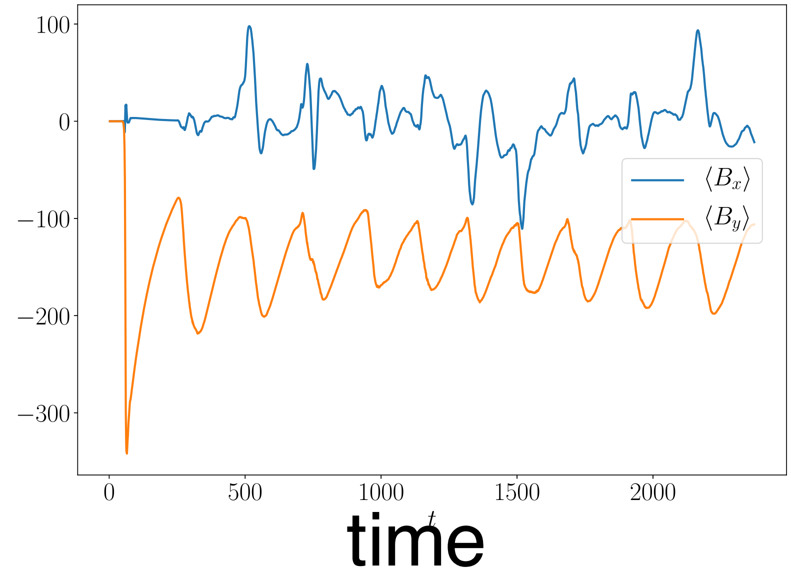


Results for large shear: moderate Rm χ=20

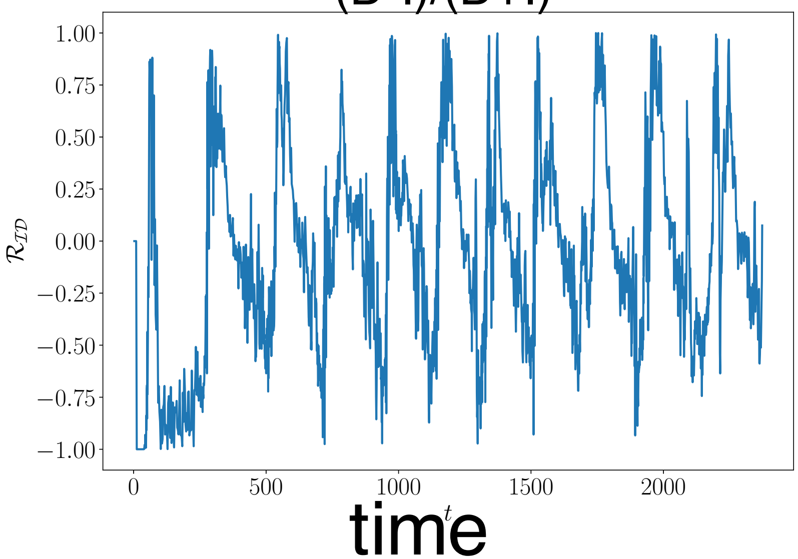
- Initially similar evolution
- Invariant manifold becomes unstable (eventually)
- Wave solution is re-established, but now with ky=1.
- Lorentz force loses most of its projection onto ky=0,
  - No longer able to suppress the shear, and the latter comes back
  - Being out of invariant manifold implies that the dynamo can generate <By> and <Bx>,
  - Cycles appear superposed on the wave solution in which the velocity is mostly in the y-direction and then mostly not.
  - Beginning of the cycle
    - velocity is close to being y-independent, there is a lot of alignment between F and U and sustains the growth of the field-everything grows.
  - Middle of cycle
    - Lorentz force still has a projection on Ky=0 and begins to suppress the shear. thiLorentz force stops projecting mostly on Ky=0, thus it loses the ability to reduce the shear.
  - End of cycle
    - The velocity moves away from y-independence.
    - B moves to lower wavenumbers (i.e. larges cales). We know that J.B and JXB both decrease at the same rate (i.e. no change of angle)

Results for large shear: moderate Rm χ=20

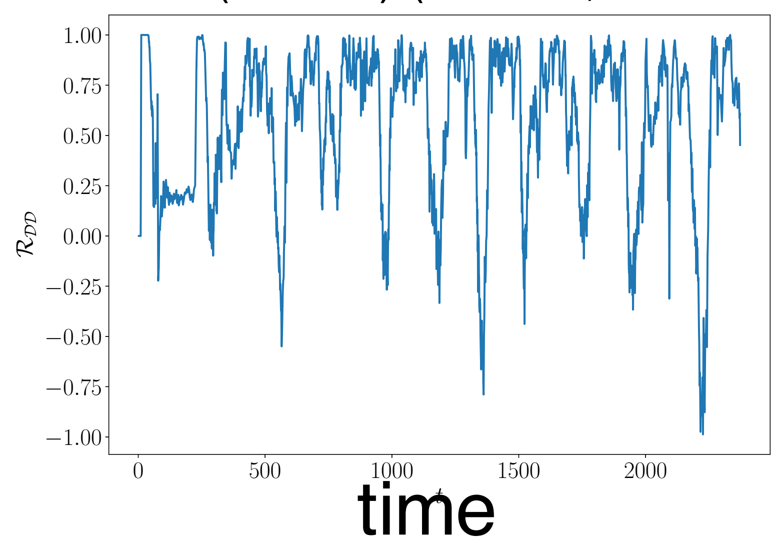




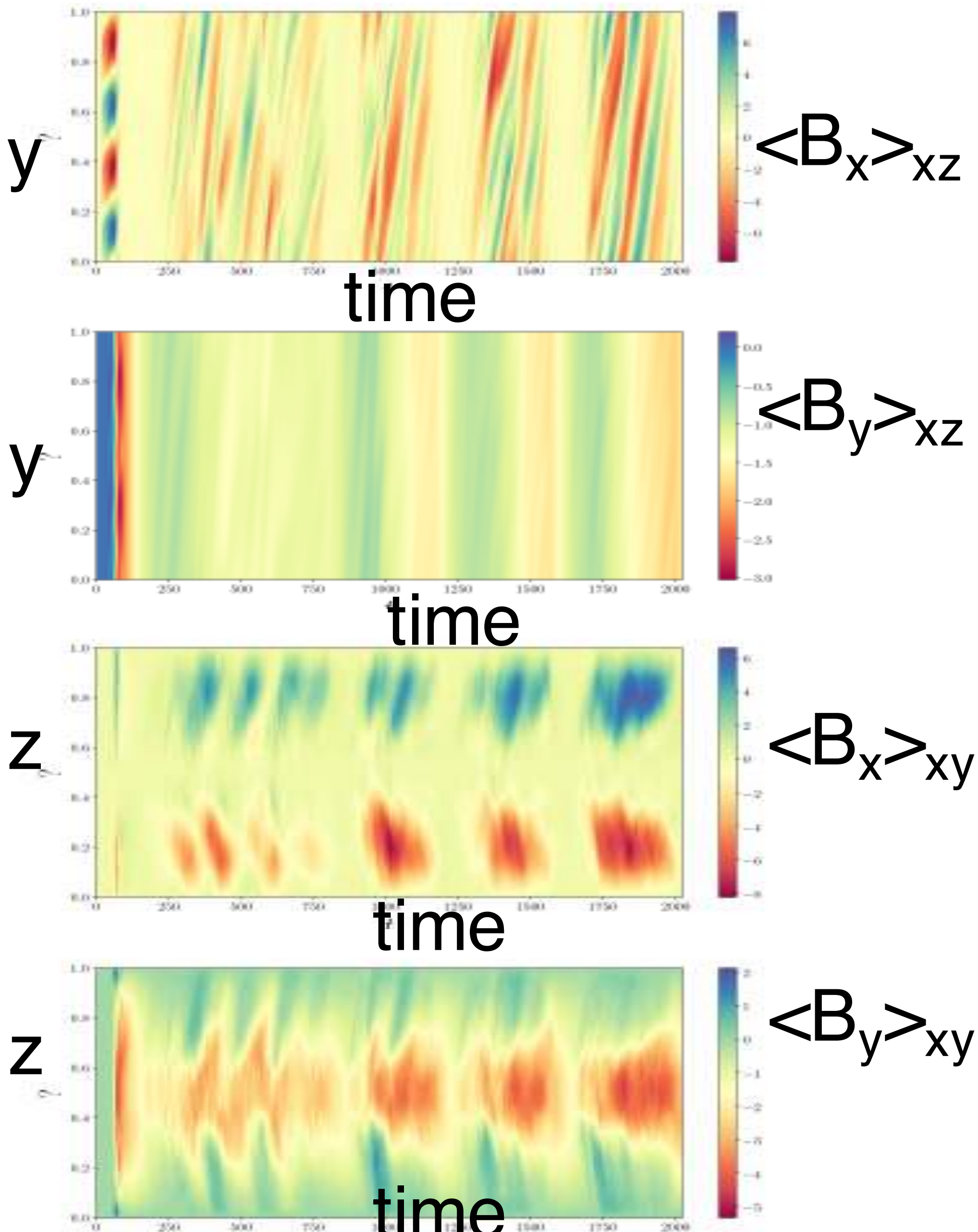
(D-I)/(D+I)



(DB-DS)/(DB+DS)

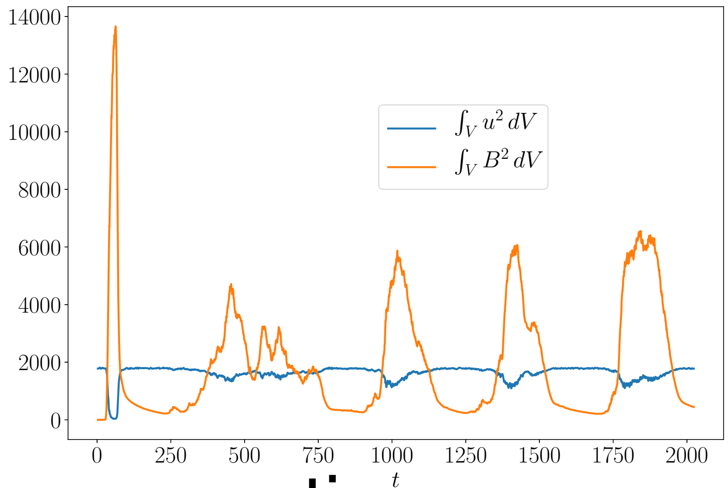


Results for large shear: large Rm χ=40

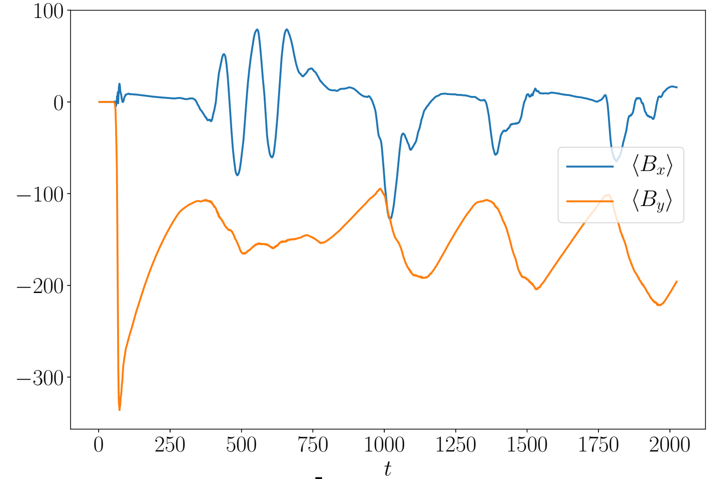


- Similar evolution to χ=20
- Same dynamics but on a Longer timescale

Results for large shear: a bit higher Rm χ=40

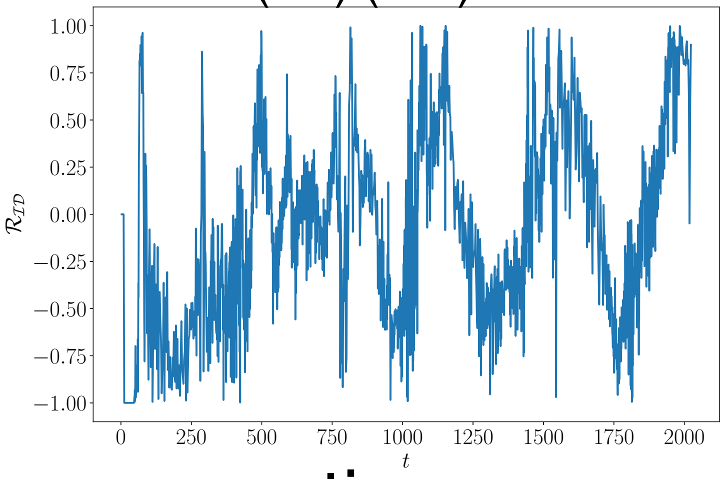


time



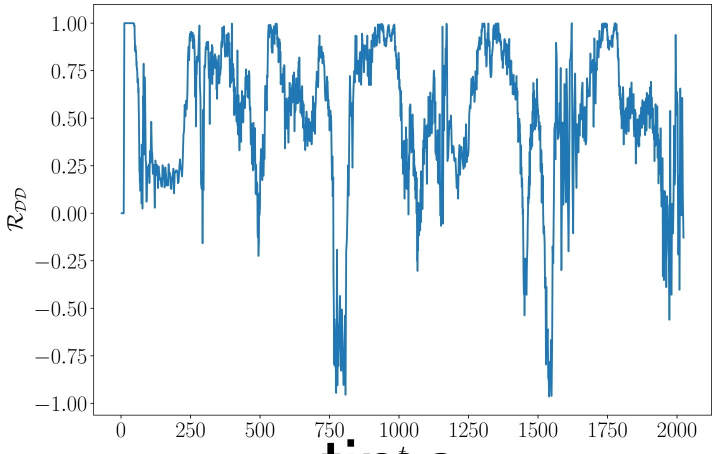
time

(D-I)/(D+I)



time

(DB-DS)/(DB+DS)



time

Helicity Fluxes

- In kinematic regime: diffusive body flux wins
- At cycle peak ideal flux starts to dominate
- Rm is not high enough for ideal flux to dominate over diffusive flux (higher Rm runs in progress/needed)
- At fixed shear doubling Rm
  - halves the diffusive flux (not surprisingly!)
  - Lasts for twice as long (perhaps surprisingly)
  - Gets the job done - but on a long timescale?
- Does ideal flux ever take over from diffusive flux?
  - Hubbard & Brandenburg (2010), Del Sordo et al (2013)
  - Diffusive flux decreases faster than ideal flux with Rm
    - But low Rm, X= Rm/Rm_c

What to do…what to do?

- Transport is usually evaluated via closure models that are to first order homogeneous and isotropic (see Yokoi 2019)
  - Quasi-Normal Models
  - EDQNM
  - DIA
  - TSDIA
- It is hard to build in conservation of quadratic invariants (such as kinetic helicity and energy in hydro) and (cross helicity, magnetic helicity and energy in MHD).
- An alternative is to derive and solve evolution equations for the statistics (Direct Statistical Simulation, see e.g. Marston et al 2019, in Zonal Jets, eds Galperin & Read) that
  - Don't assume homogeneity and isotropy
  - Ensure conservation of global quadratic invariants (via triad decimation in pairs Kraichnan 1985)
- Treat magnetic field and velocity on an equal footing…

Conclusions

1. **Small-scale dynamos put energy at the resistive scale**
   - **Rely on exponential stretching.**
   - **Fast dynamos work as Rm → infinity**
2. **Large-scale dynamos rely on breaking of reflectional symmetry**
   - **presence of pseudo-scalar kinetic helicity/PT symm breaking**
3. **Shear can help large-scale win out over small-scale kinematically**
   - **It suppresses the small-scale dynamo at high Rm**
4. **Catastrophic quenching can be understood in terms of helicity conservation**
   - **Wrong to think helicity conservation causes catastrophic quenching**
5. **Not clear that helicity fluxes alleviate slow resistive growth of essentially kinematic dynamos**
   - **need more efficient dynamos or simulations to get to higher Rm**
6. **Maybe the answer is to examine essentially nonlinear dynamos (T., Cattaneo & Brummell 2011).**
   - **velocity fluctuations and magnetic field perturbations emerge from an instability of a large-scale field (see e.g. Riols et al 2013).**
   - **Then they should keep correlated even at high Rm.**