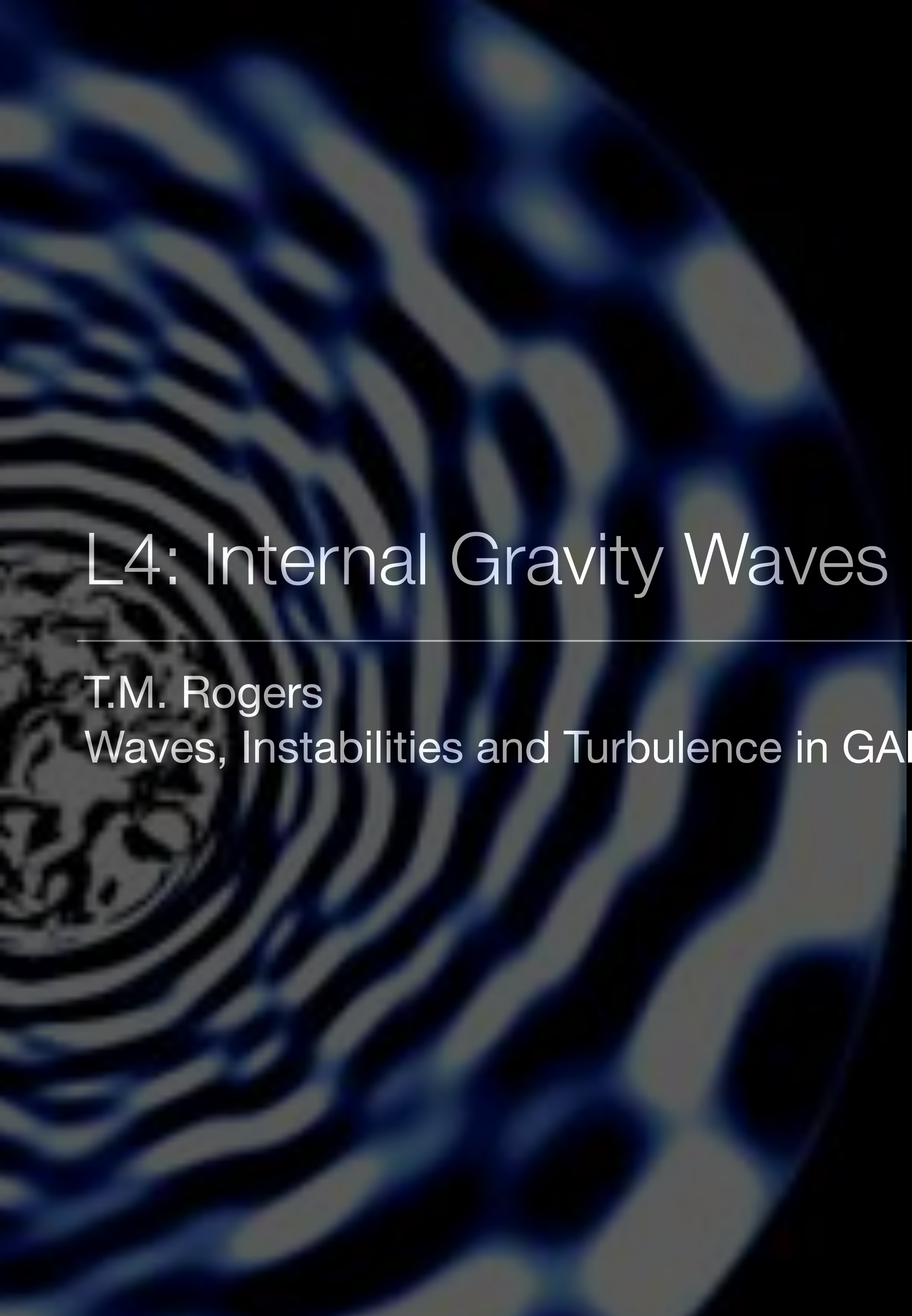


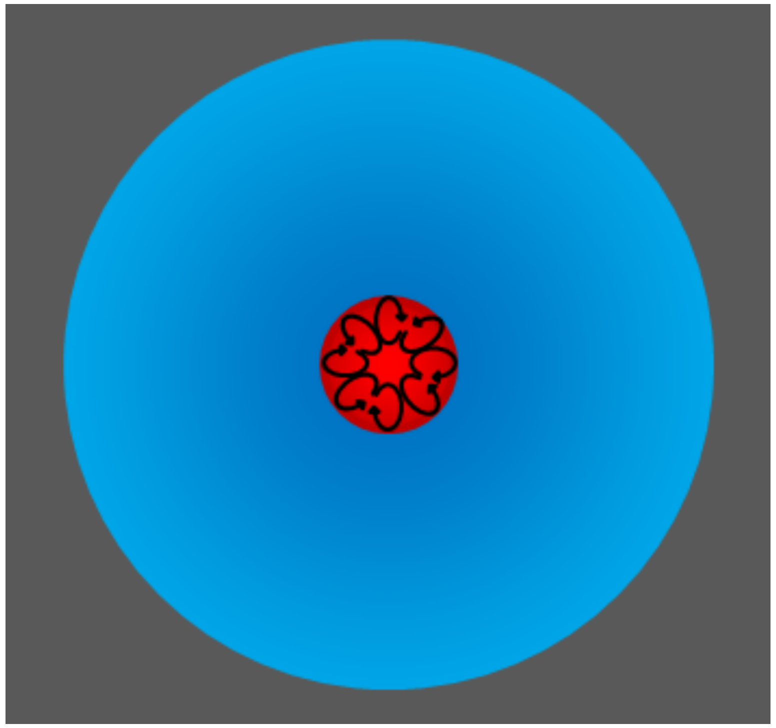
L4: Internal Gravity Waves

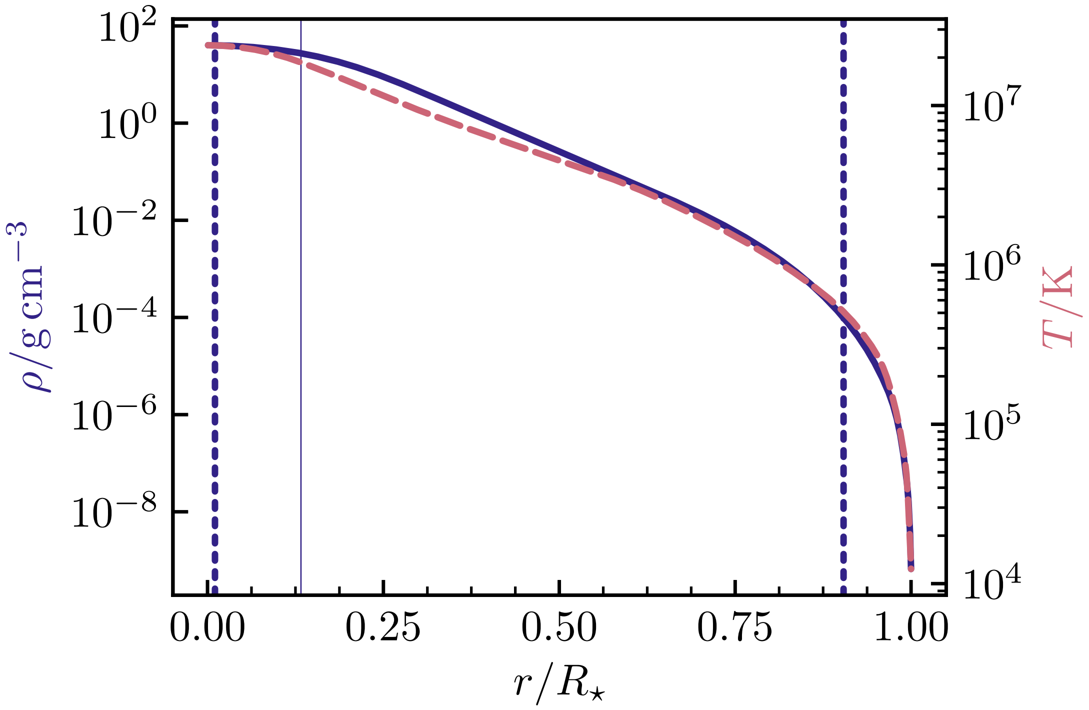
T.M. Rogers
Waves, Instabilities and Turbulence in GAFD, 8-14 July 2019 Cargese

Stellar Team at Newcastle

Imogen Cresswell (UG)
Rathish Ratnasingham (PhD)
Philipp Edelmann (Postdoc)

So what happens in Stars (Massive)





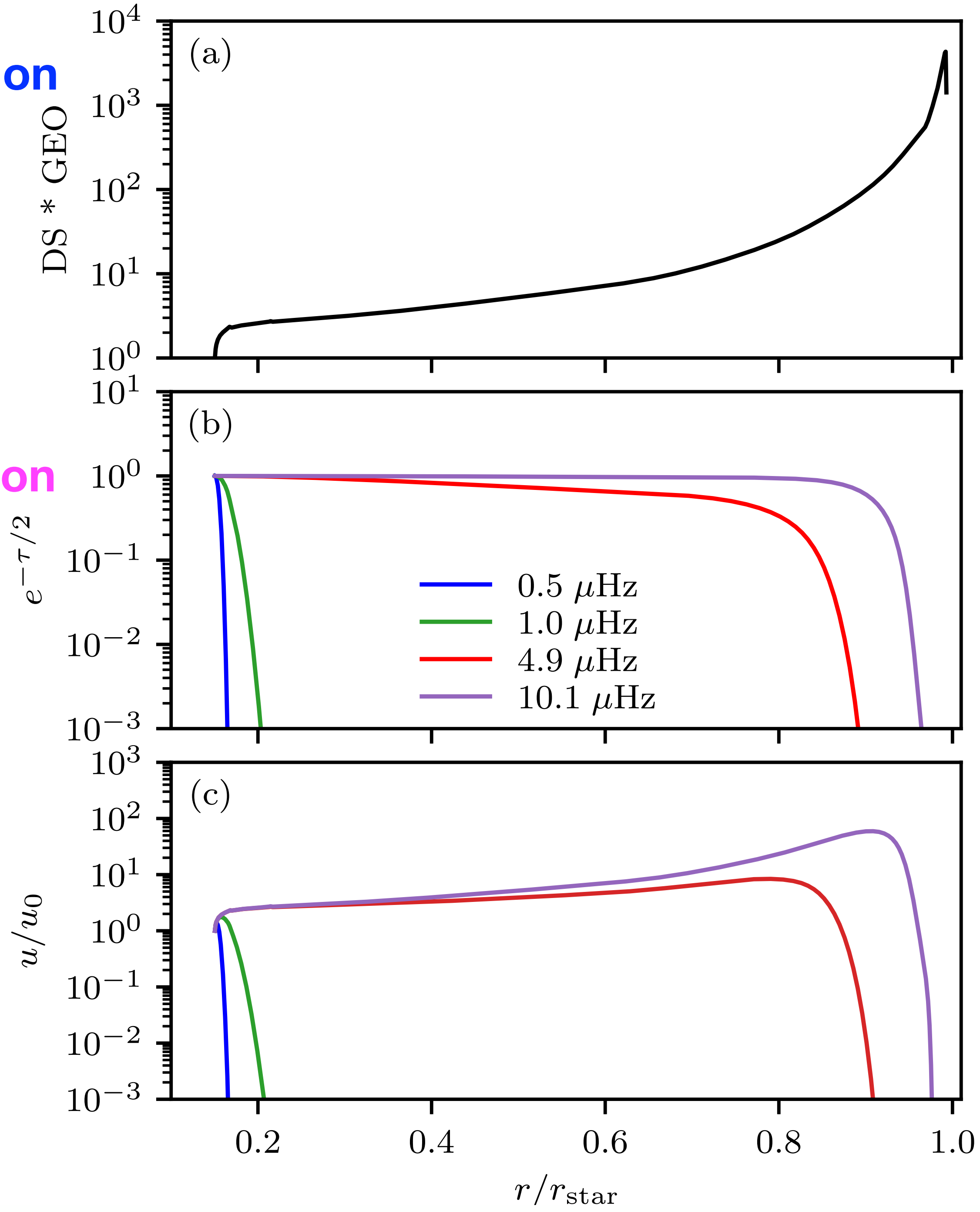
Density Stratification & Geometry

$$\rho^{-1/2}r^{-3/2}$$

Radiative Diffusion

$$e^{-\tau/2}$$

Combined



Ratnasingham et al. 2018

Numerical Model

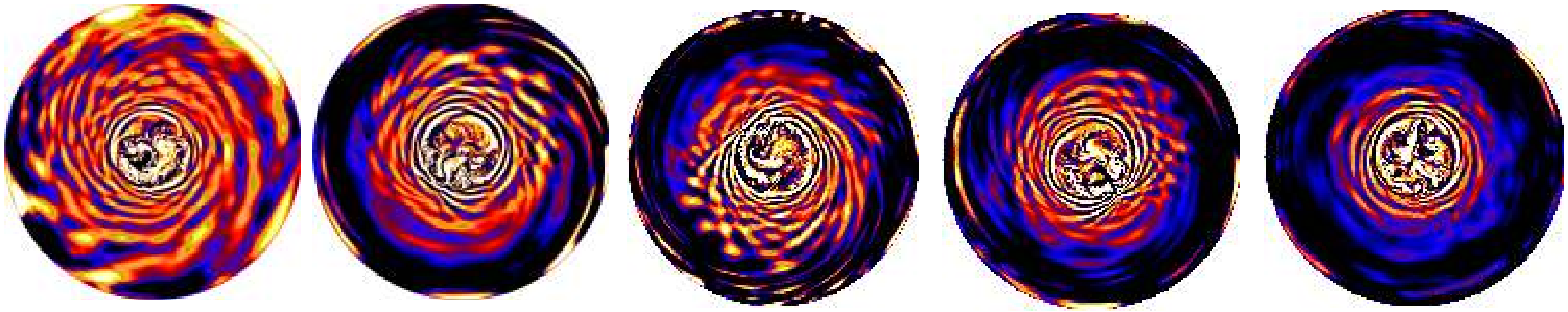
- We solve the full set of hydrodynamic equations in the anelastic approximation in 2D representing an equatorial slice of the star (and 3D sphere)
- Our reference state model is taken from the Cambridge STARS 1D stellar evolution code for a 3M_{sun} star: ~inner 15% convection+radiative envelope
- **The Basic Physical Picture**
- Turbulent convection generates IGW at the convective-radiative interface: get ***spectrum*** of waves
- As waves propagate outward toward the surface, their amplitudes are affected by two main effects:
    - wave amplitude increases because of decreasing density
    - wave amplitude decreases because of radiative dissipation
- *Some* waves make it to the surface with sufficient amplitude to break
- Wave breaking is accompanied by angular momentum transport
- Angular momentum transport causes surface rotation to change
- Leads to enhanced mixing

CAVEATS

- Since we have a realistic temperature gradient and our diffusion coefficients are too large: **we force the convection harder - higher luminosity than 3M_{sun} star in some models**
- **(In some models)** our convective velocities are ~10 x larger than mixing-length theory would predict, however surface amplitude not larger

Convectively driven waves in massive stars (sometimes) cause surface to spin ***retrograde***

Vorticity is shown:
White-prograde flow
Black-retrograde flow



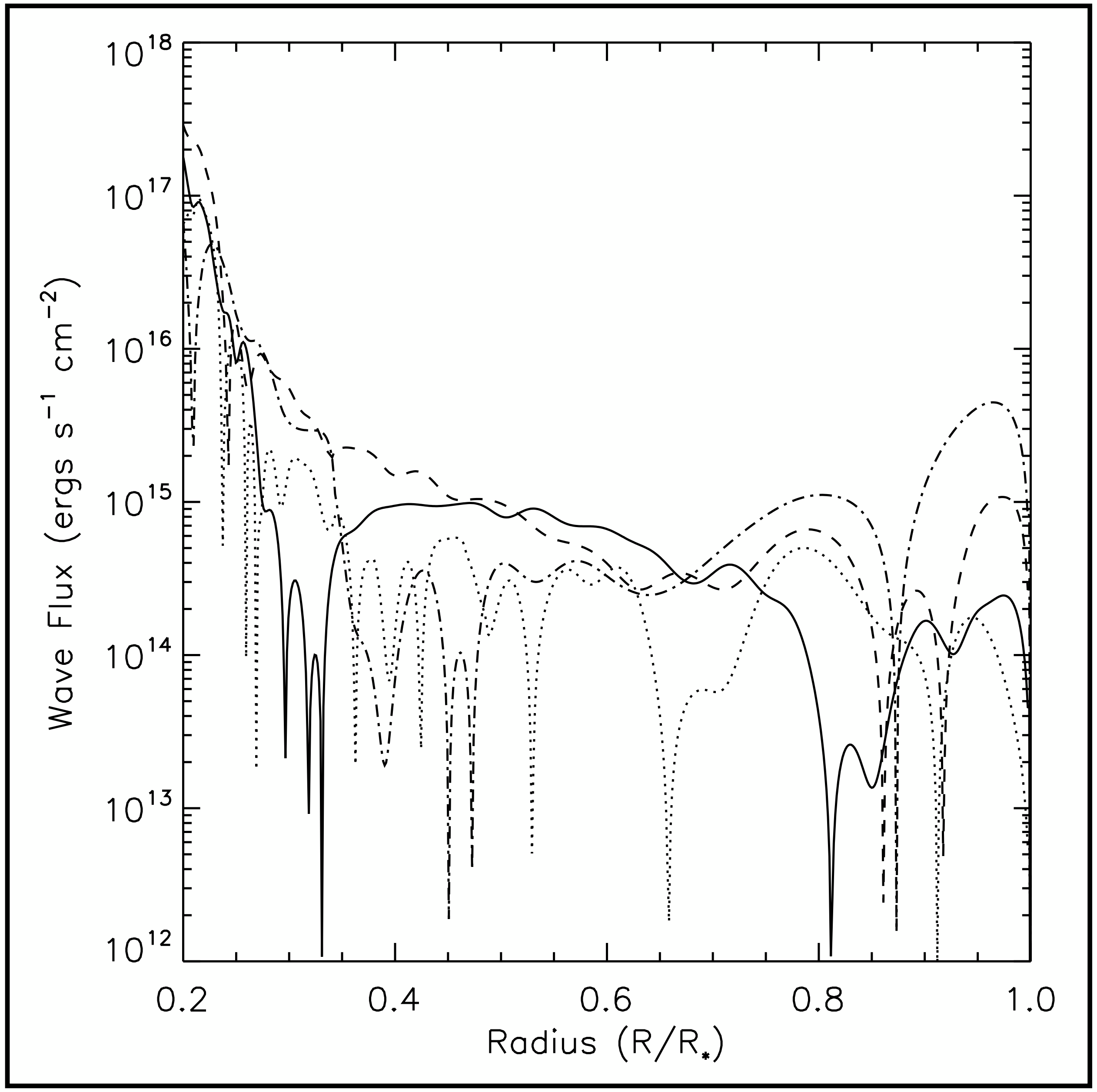
Rogers et al (2013)

Wave Generation

Press 1981 - “ Most important is to obtain a reliable estimate of the amplitude and spectrum of internal waves … obtaining this estimate must rely primarily on future 2D and 3D nonlinear, numerical hydrodynamic calculations”

Wave Flux

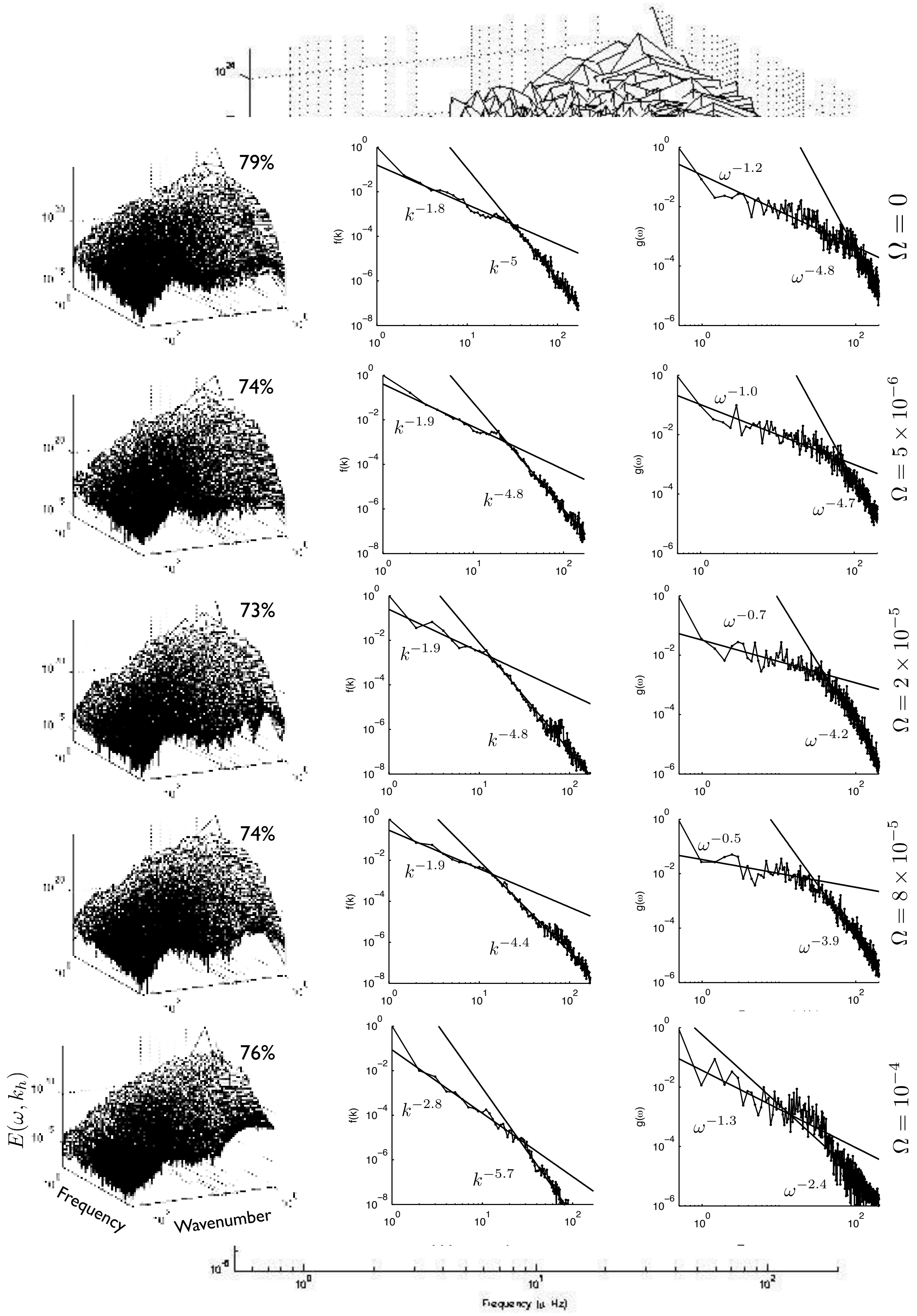
- Theoretical Prediction (Kumar et al. 1999, Lecoanet & Quataert 2012): $F_w \sim MF_c$
- Convective Flux ~10^{19} , Wave Flux ~10^{17}, Convective Mach number ~10^{-3}-10^{-2}
- *Analytic theory matches fairly well with numerical simulations for total wave flux*



Wave Spectrum

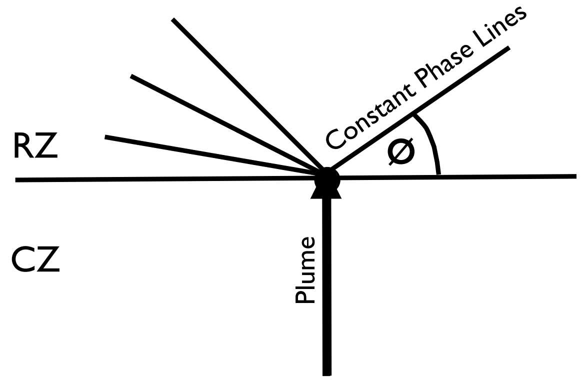
- Spectra is consistently combined power laws in both frequency and wavenumber
- Combined power laws is robust feature of all models and all times (slopes vary somewhat)
- The structure of the spectra likely due to the combined action of plumes and eddies driving waves of different scales and frequencies

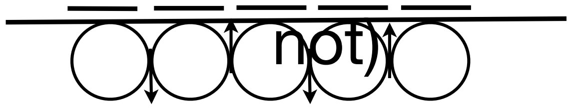
These spectra are not in agreement with theoretical predictions which generally have a steeper function of frequency and a power law/exponential for wavenumber



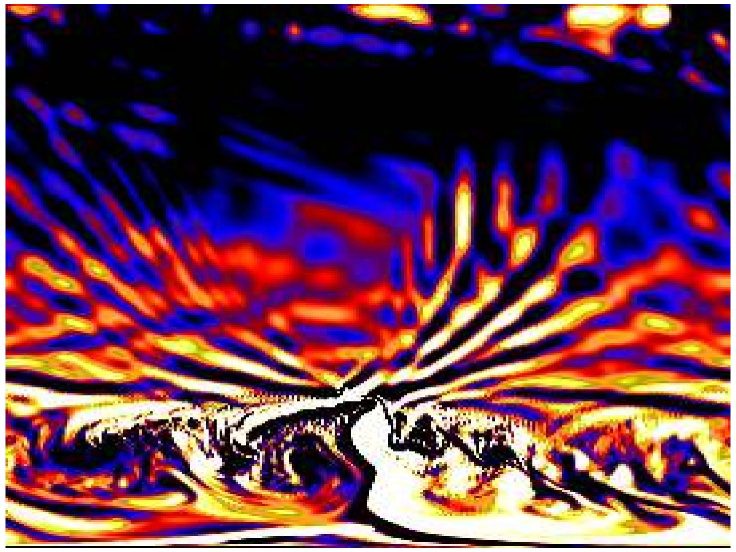
Wave Excitation

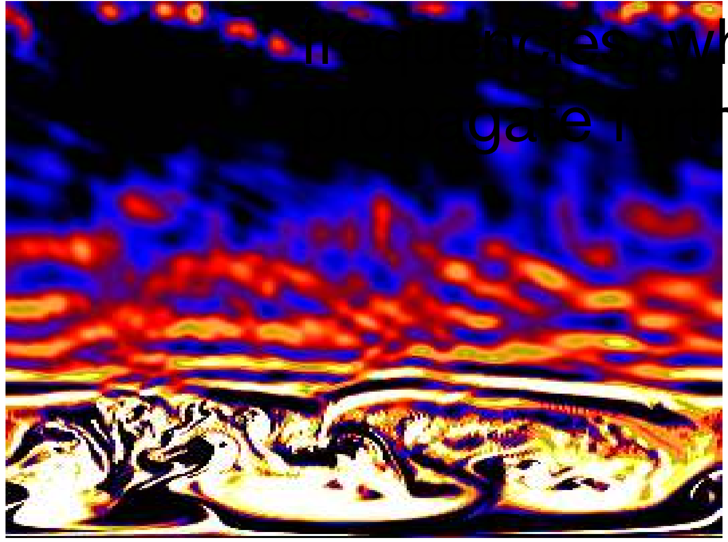
$$\omega = \pm N sin\phi$$


Plume Excitation


Eddie excitation

- Simulations resolve both eddy and plume excitation of waves (theoretical spectra generally do not)
- This is important because the plumes directly force higher frequencies, which are able to propagate further before dissipating

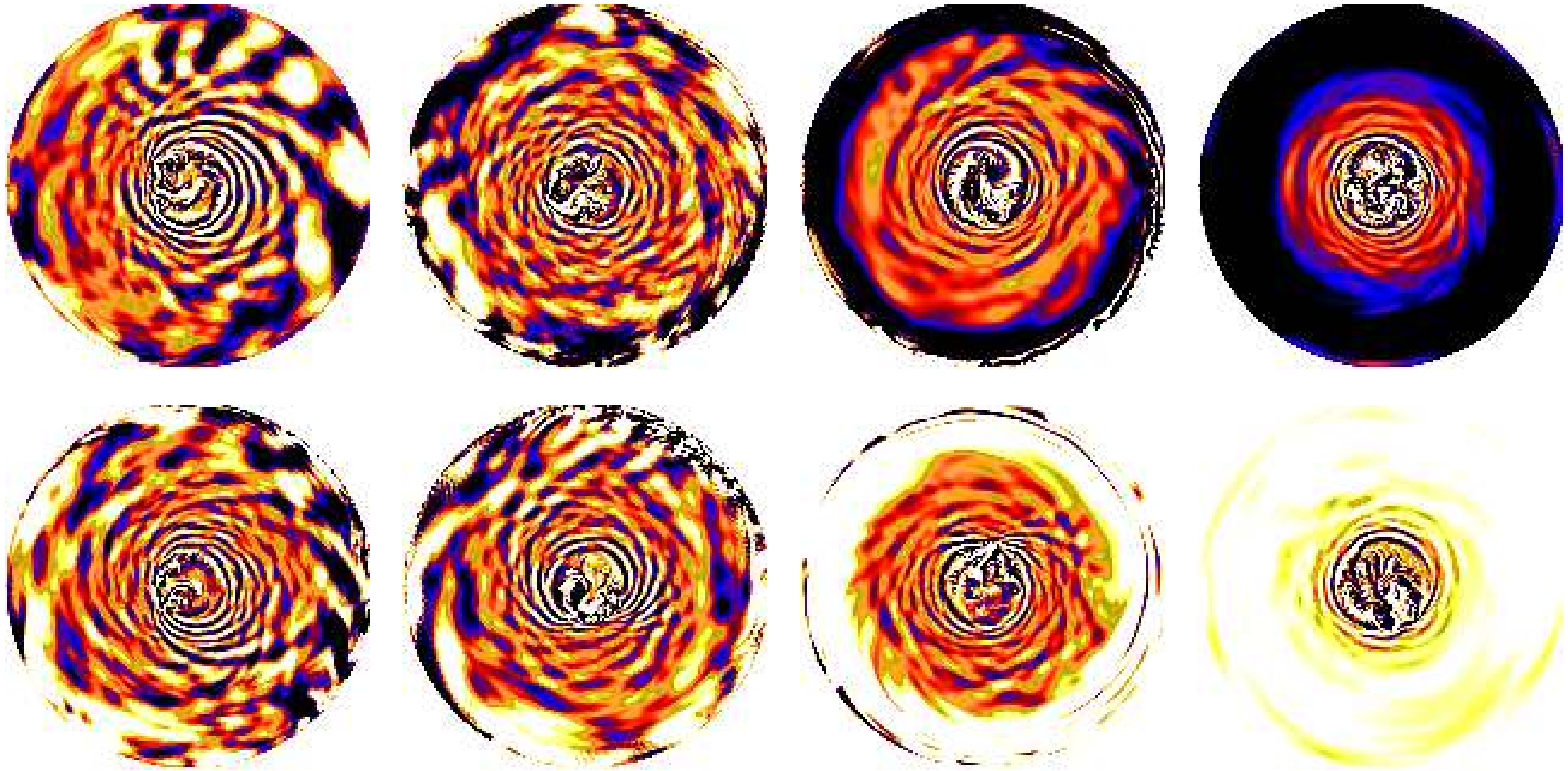




Plumes directly force both low and high frequency waves (shallow and steep angles) of large horizontal scale

Eddies drive low frequency waves of small and large horizontal scale

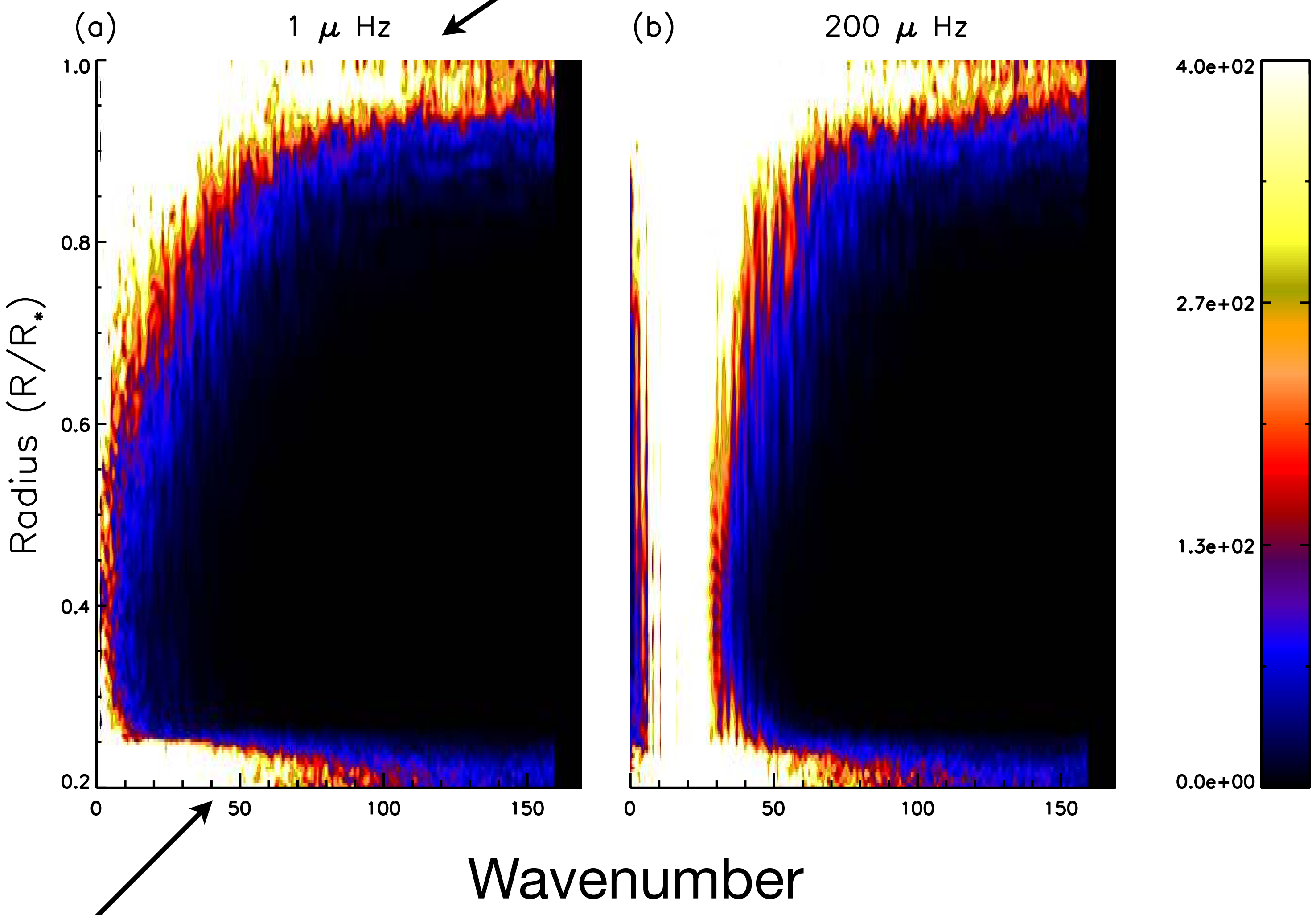
Results:
Angular Momentum Transport by IGW



Wave Amplification and Breaking at Surface

- Large scale waves generated at the convective-radiative interface
- Only the largest scale survive through the bulk of the interior (depending on frequency)
- At surface, smaller scales are generated-> indicative of wave breaking

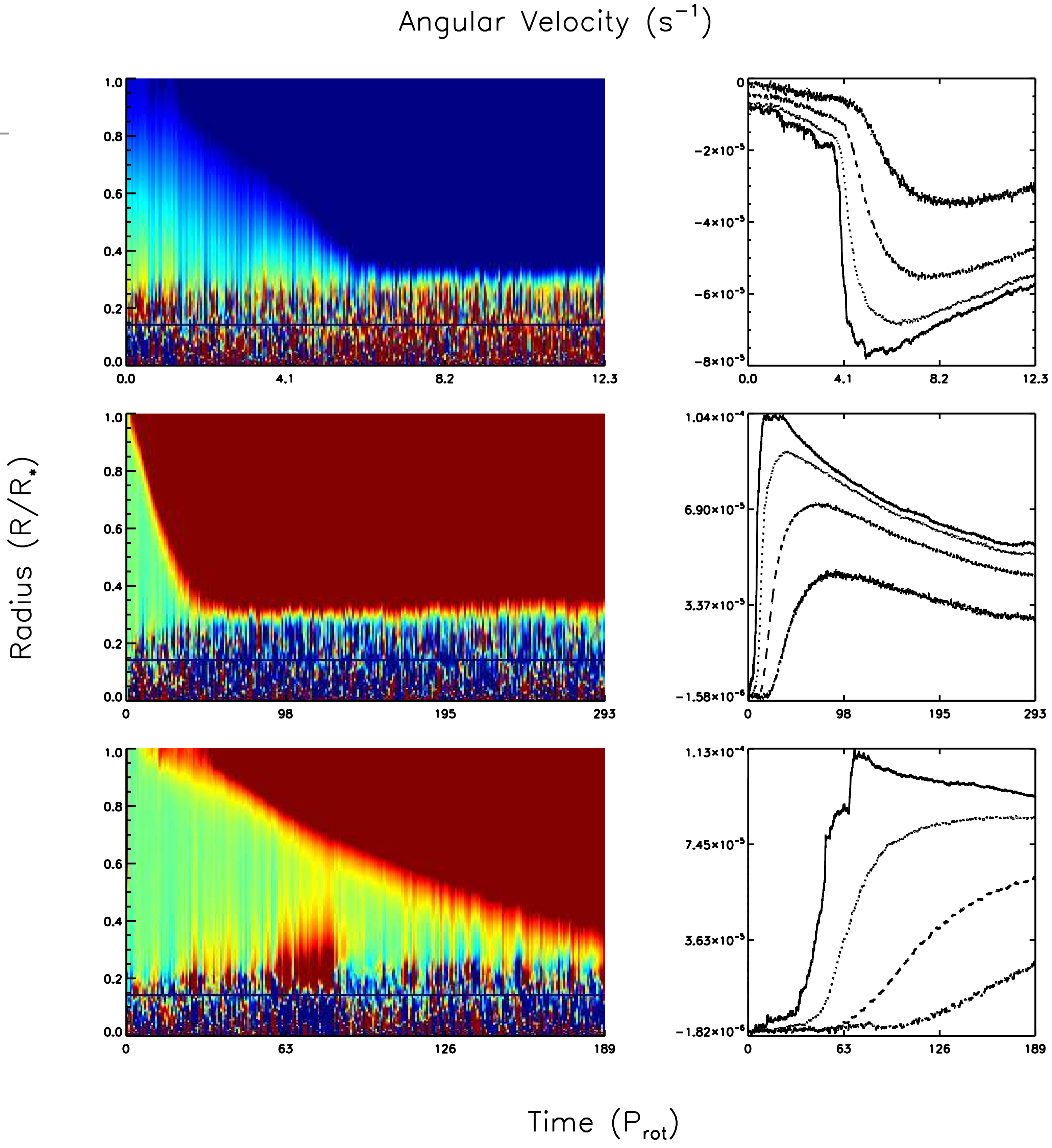
Waves generated at surface by breaking



Waves generated at convective-radiative interface

Angular Velocity Evolution

- Shear layer develops first at the surface then migrates toward the source in time
  - Initial development is due to wave breaking, followed by critical layer formation and absorption
- Convection zone starts to spin (predominantly) with the opposite sense as radiative envelope (to conserve AM)
- Rapid AV variation in short time, conservative extrapolation ~10^3-10^4 rotation period
- However, it is unclear whether this will reverse as in QBO



Rogers et al. 2013

Results:
Direct Comparisons to Observations

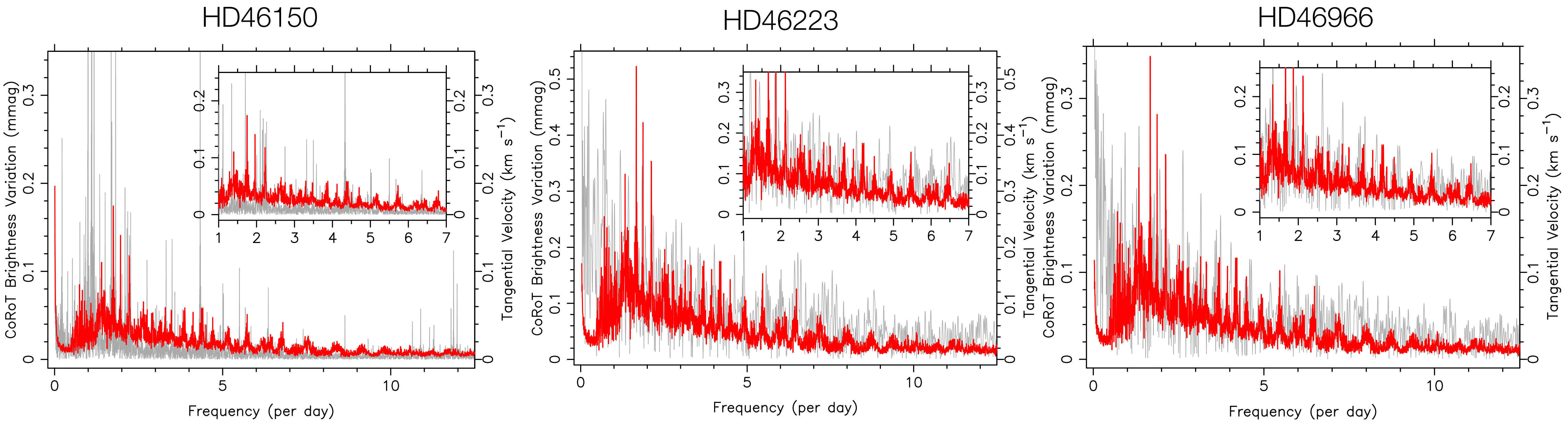
Although standing wave *modes* are readily observed in stars we have very few observational constraints on propagating (and dissipating) waves, i.e. the ones responsible for angular momentum transport and mixing. This is changing with recent asteroseismic detections.

Brightness Variations in O-stars

🟥 Simulations

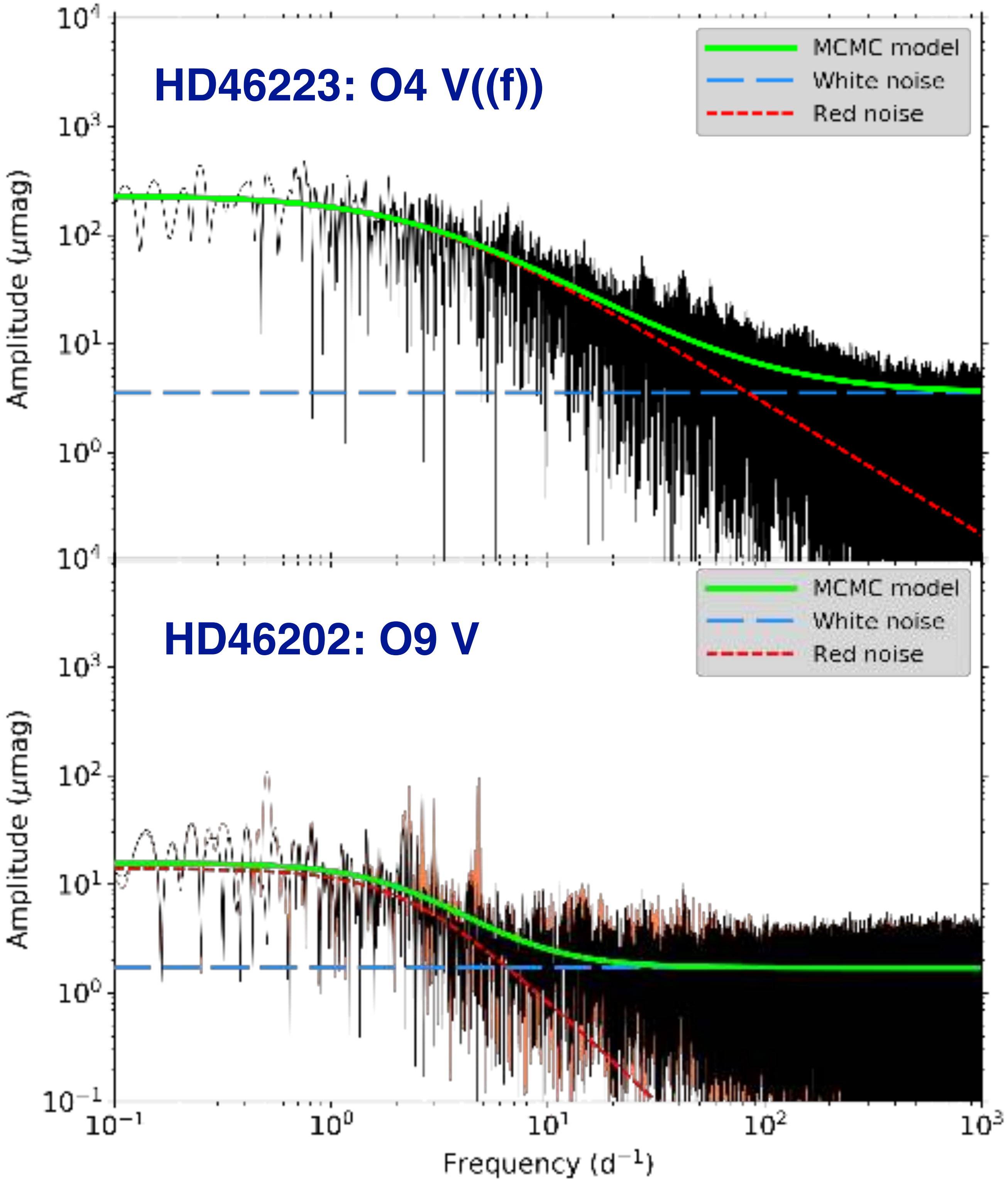
▭ Observations

Aerts & Rogers 2015

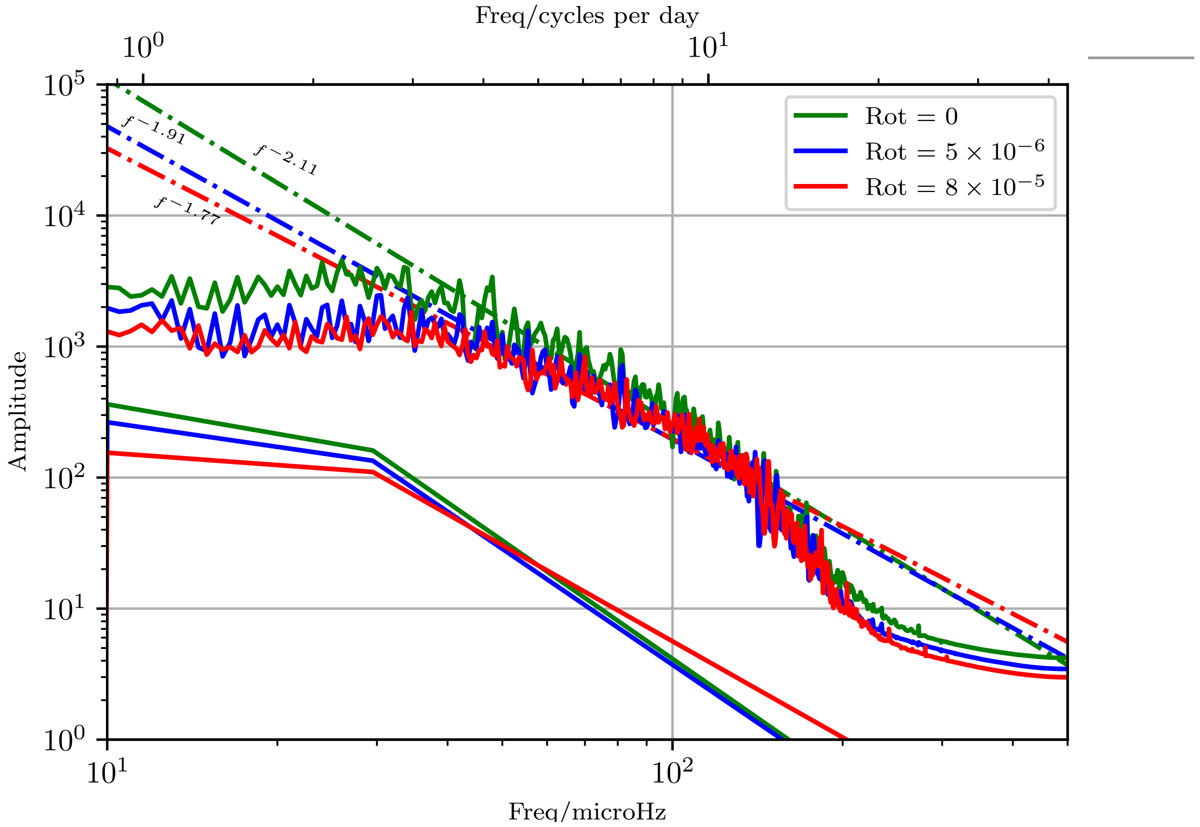


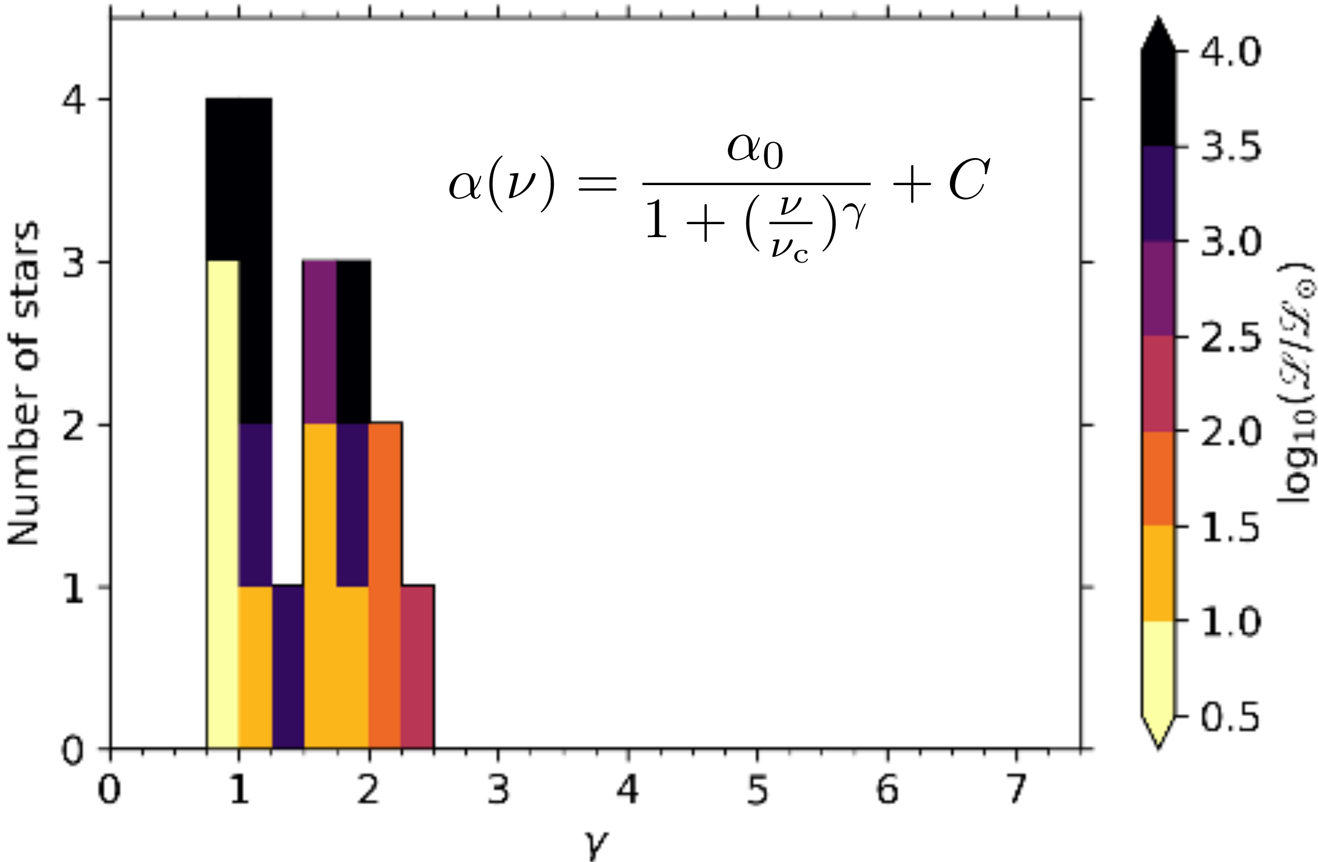
- O stars do not show heat driven g-modes, but show power excess at high frequencies
- Accounting for variation in mass and conversion between observed brightness fluctuations and velocity, spectra match well (except at lowest frequencies)
- We found that numerical models which were differentially rotating (core-envelope) matched observations better than uniformly rotating models
- In addition to these three stars, there are possible detections of stochastically excited waves in 3 other stars (Aerts et al. 2017, Aerts et al. 2018, Ramiaramanantsoa et al 2018)

More recent Observations



Bowman et al. 2019





$$\alpha(\nu) = \frac{\alpha_0}{1 + (\frac{\nu}{\nu_c})^\gamma} + C$$

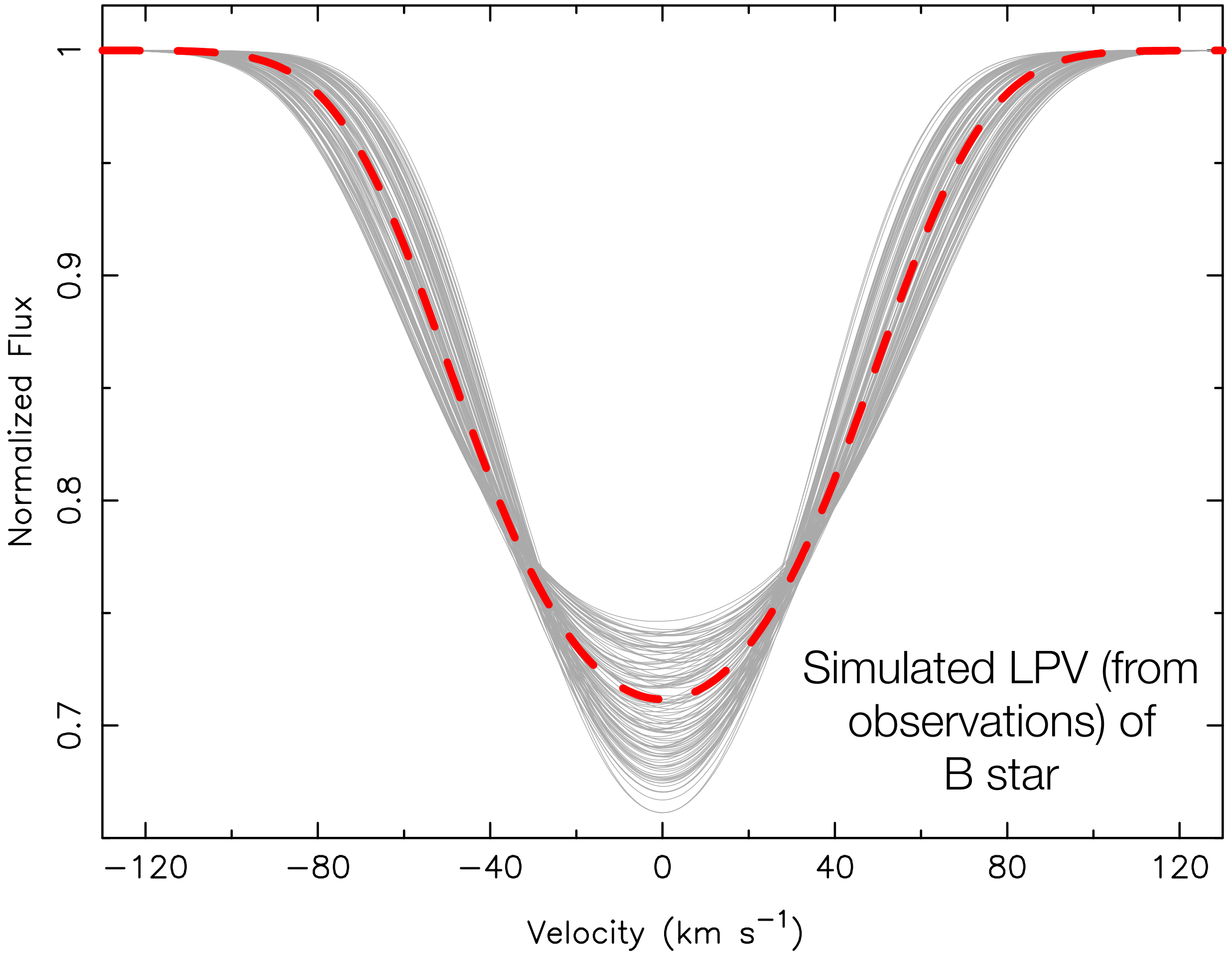
Macroturbulence

- Upper main sequence shows evidence of time dependent, non-doppler line broadening (LPV)

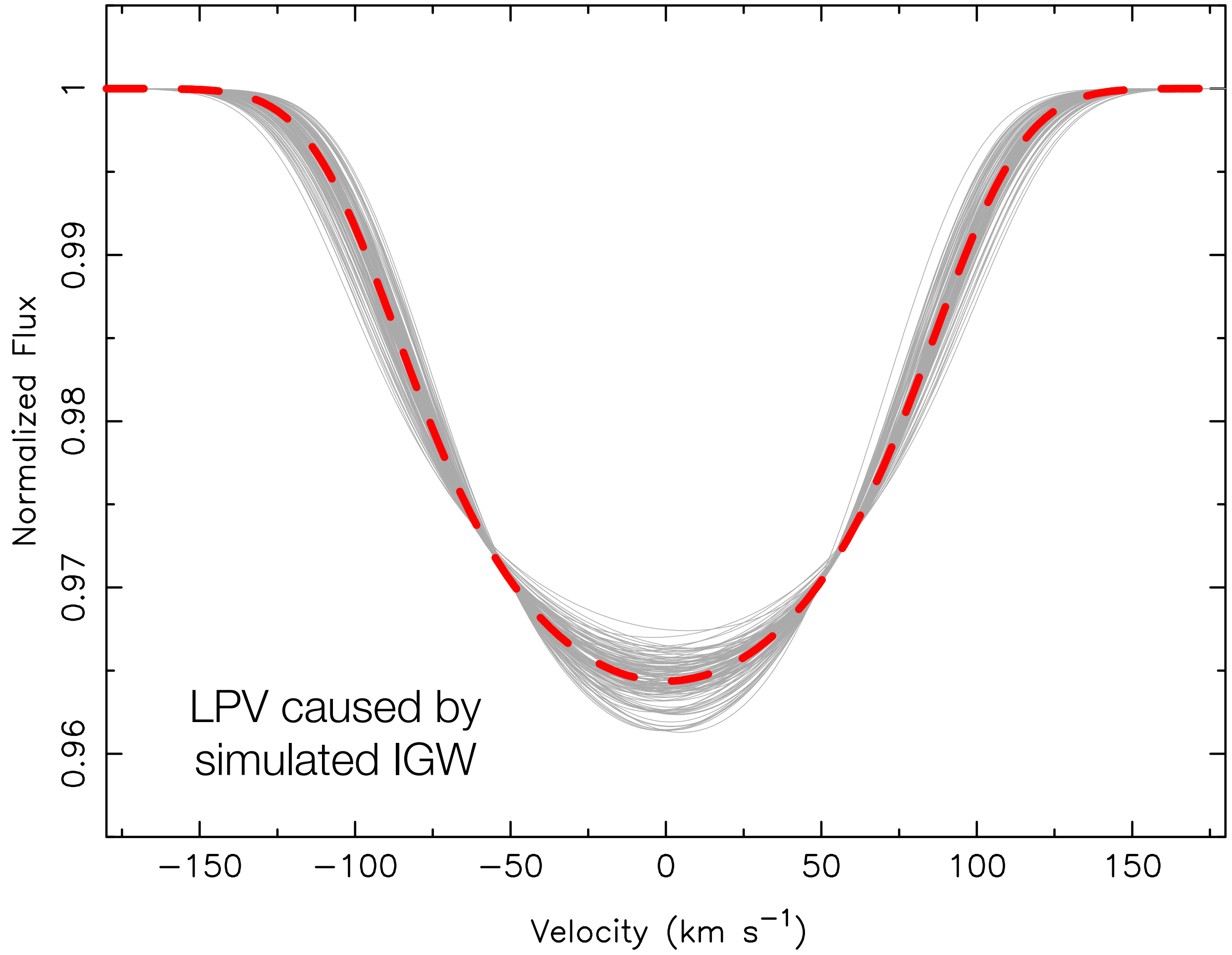
- Broadening has been referred to as “macroturbulence”. Expected in low mass stars, but hard to reconcile with expected quiescent envelope of higher mass stars. Could be surface convection zone (Cantiello), heat driven g-modes (Aerts)

- The same IGW that explain spectra, also show LPV similar to what is expected in O stars

Aerts & Rogers 2015



Simulated LPV (from observations) of B star



LPV caused by simulated IGW

Differential Rotation in Massive Main Sequence Stars

Using multiplets of g-modes which probe convective-radiative boundary and multiplets of p-modes which probe surface conditions, can get a measure of core-envelope differential rotation

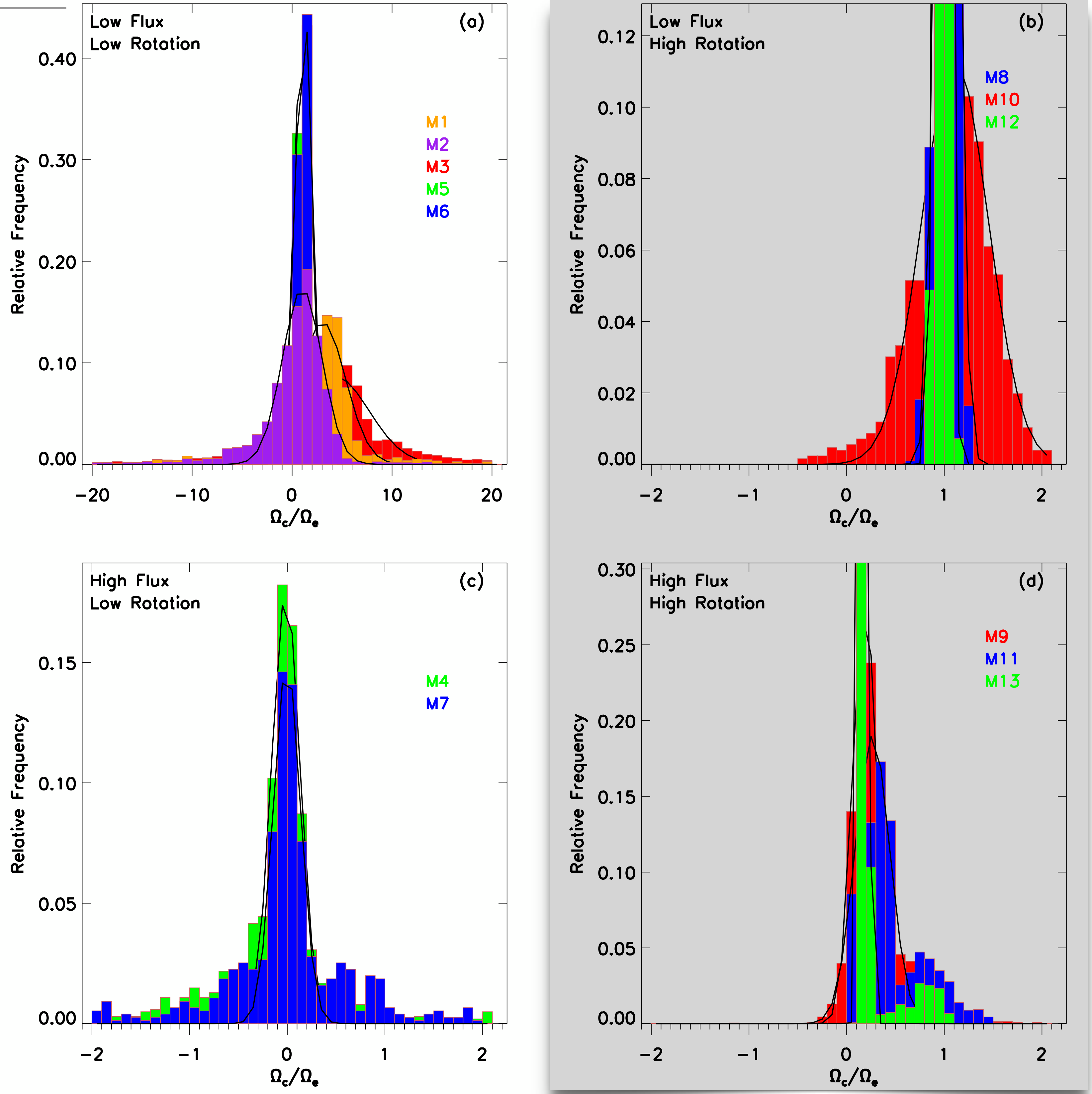
Star	Ω_c/Ω_e
HD 129929, B (Aerts et al. 2003)	3.6
HD 29248, B (Ausseloos et al. 2004)	5.0
HD 157056, B (Briquet et al. 2007)	~1
KIC 9244992, F (Saio et al. 2015)	0.97
KIC 11145123, A/F (Kurtz et al. 2014)	1.03
KIC 10080943a, F (Schmid et al. 2015)	tentative but slightly larger than 1
KIC 10080943b, F (Schmid et al. 2015)	tentative but slightly less than 1
KIC 10526294, B (Triana et al. 2015)	-0.3

Note: these are all slow rotators

Observations of core-envelope differential rotation in Intermediate and Massive Main Sequence Stars

- Simulation suite which had a single fiducial (3M$_{sun}$) model, varying initial rotation rate and convective flux
- Low Flux/Low Rotation models give $\Omega_c/\Omega_e \approx 1-5$ similar most of the observations
- Low Flux/High Rotation models (not yet observed) have $\Omega_c/\Omega_e \approx 1$ but notably, not exactly 1
- High Flux/Low Rotation models show retrograde surface flows which are larger than core (KIC 10526294)
- High Flux/High Rotation models (not yet observed) show prograde surface flows which are larger than core

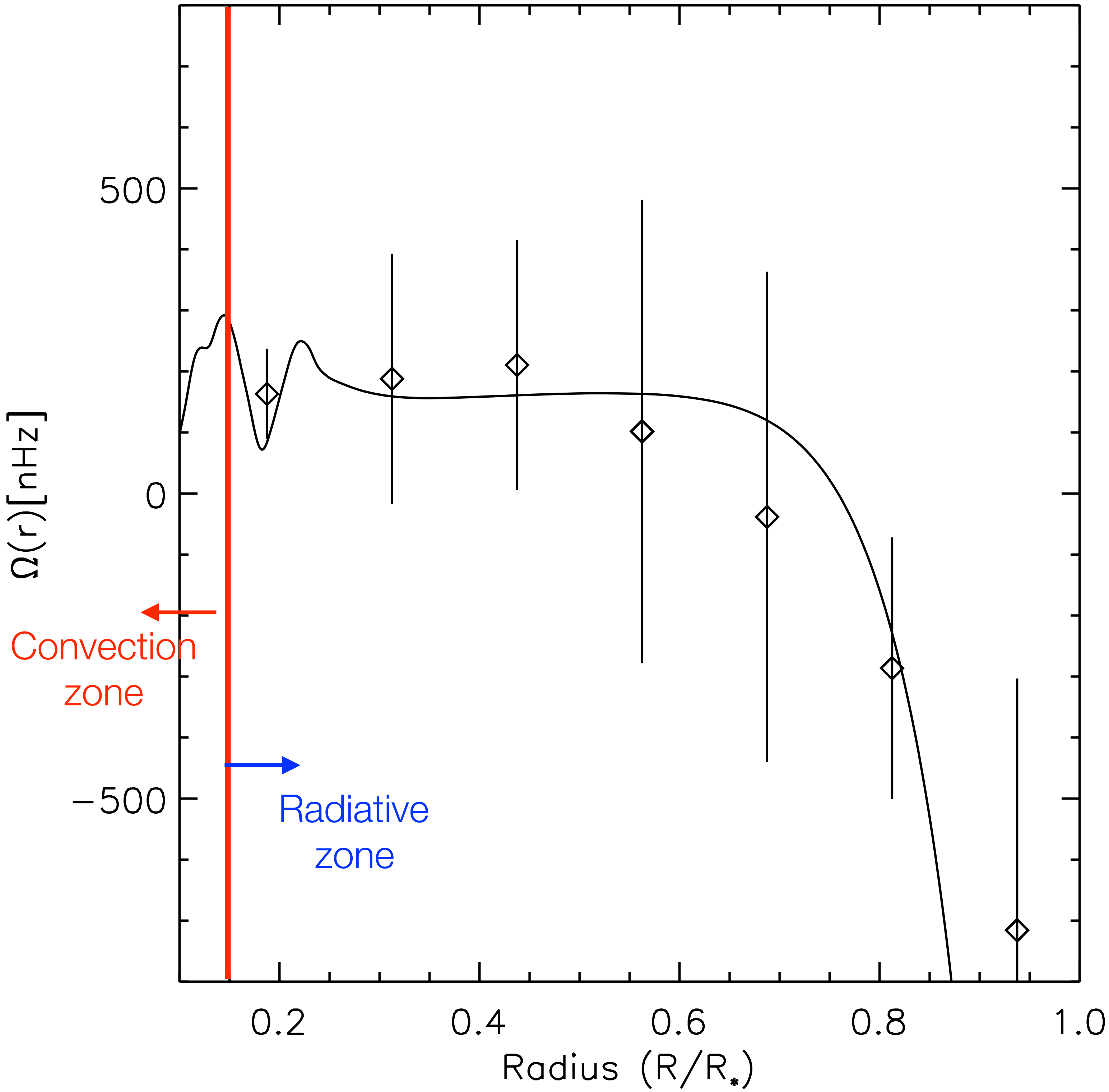
Rogers 2015



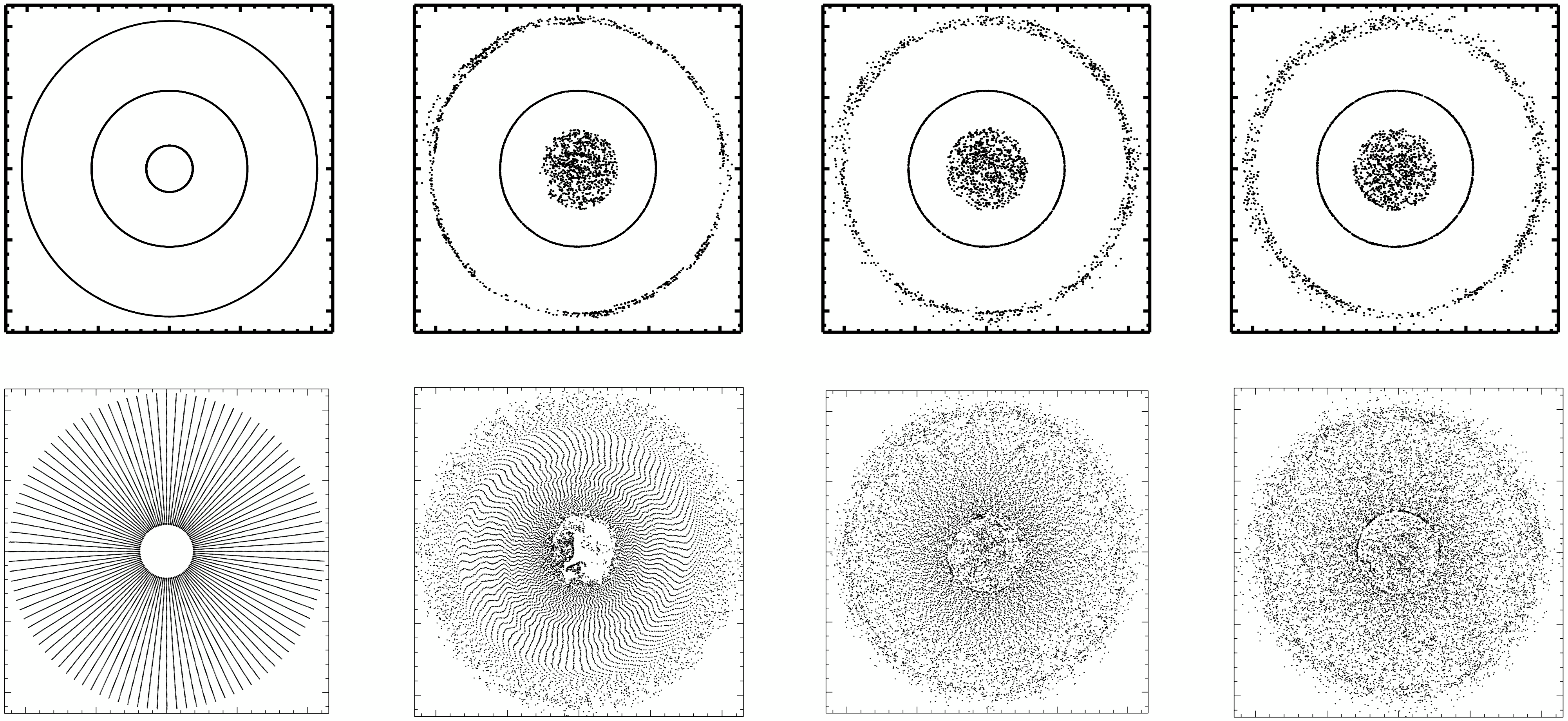
Differential Rotation in KIC 10526294

- Used 19 g-mode multiplets, 3.25 M_{sun} star did full inversion to recover differential rotation, ***first time done in a star other than the Sun***
- Found that the envelope is spinning faster than the core and *in the opposite direction*

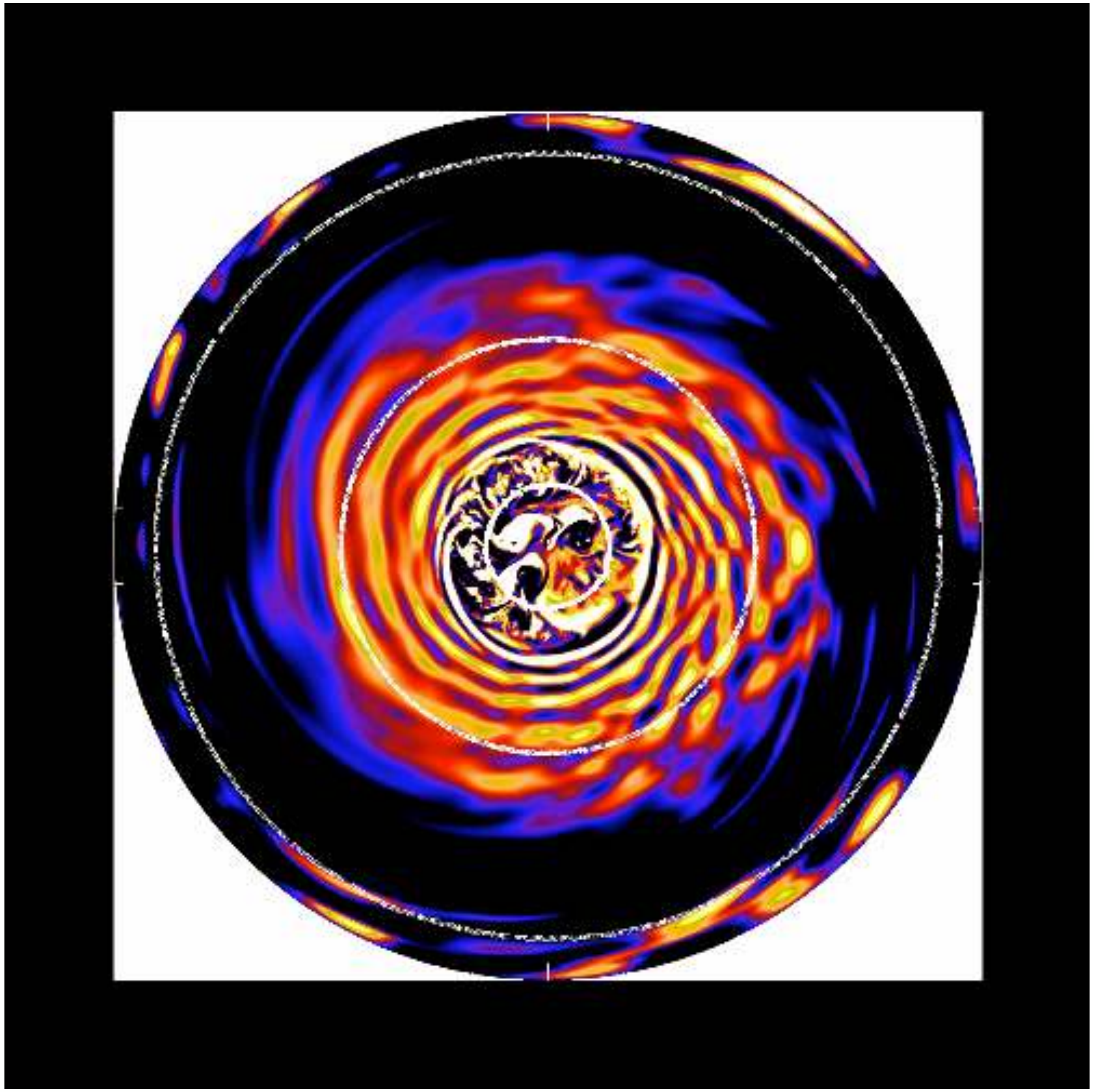
Triana et al. 2015

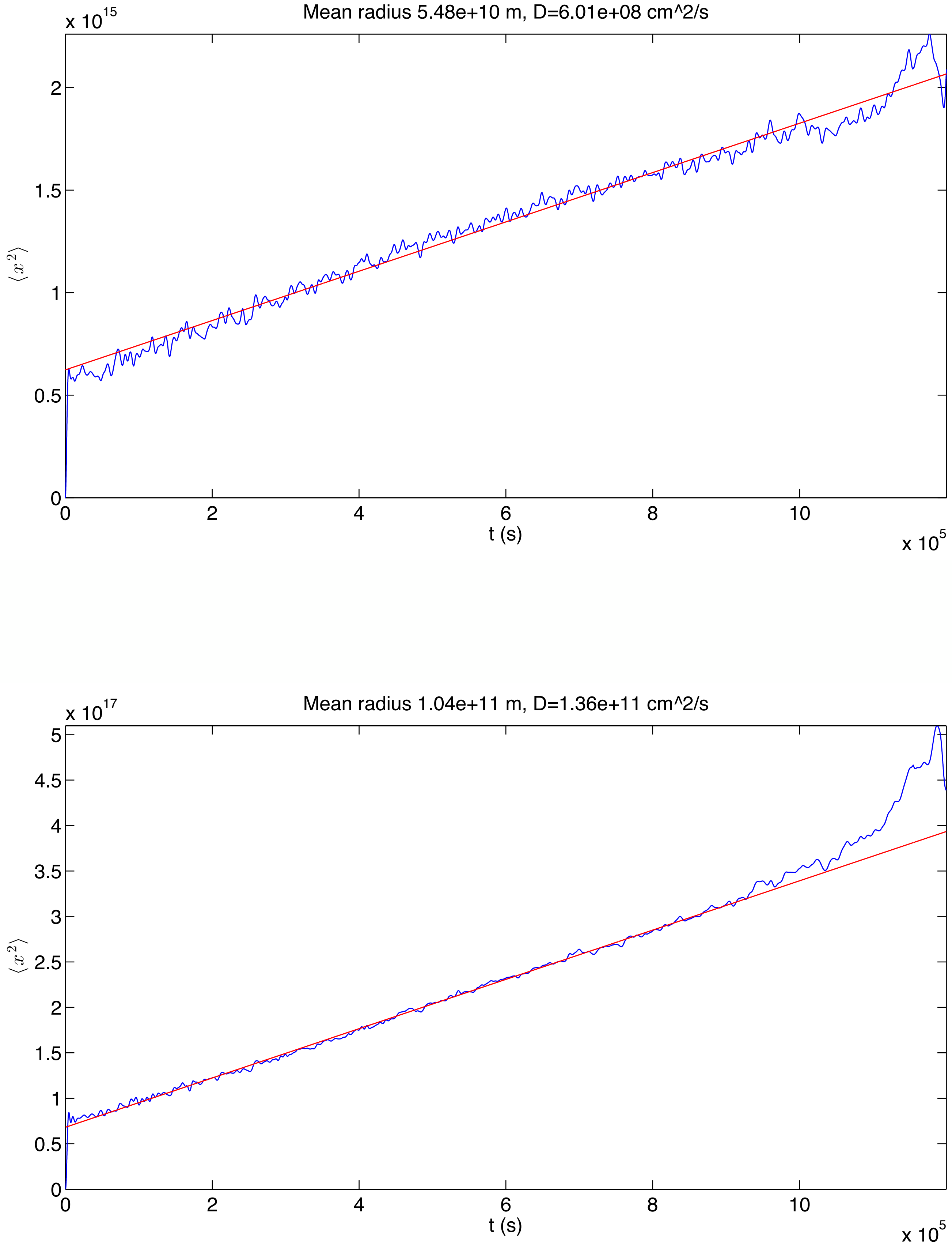


Results:
Chemical mixing by Waves

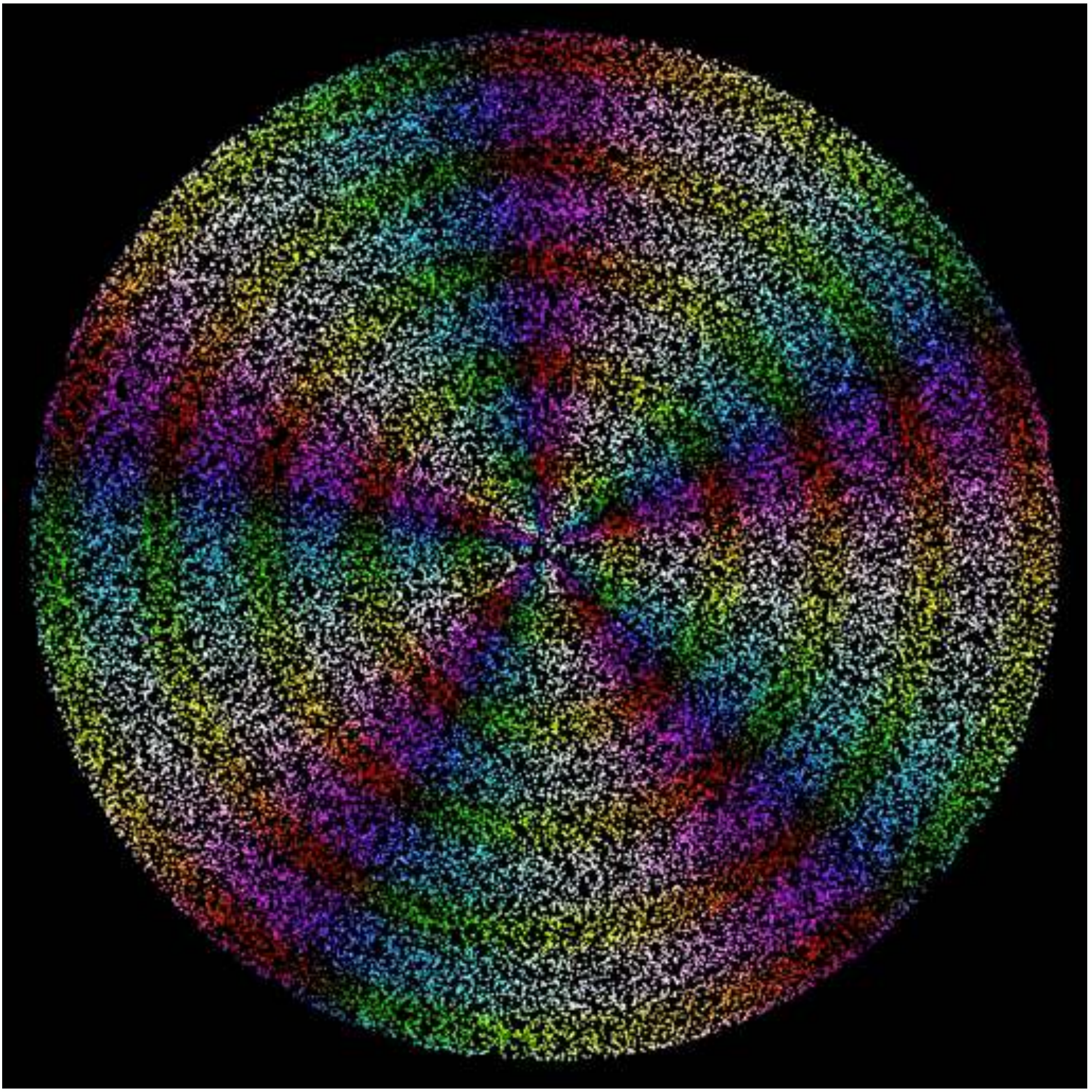


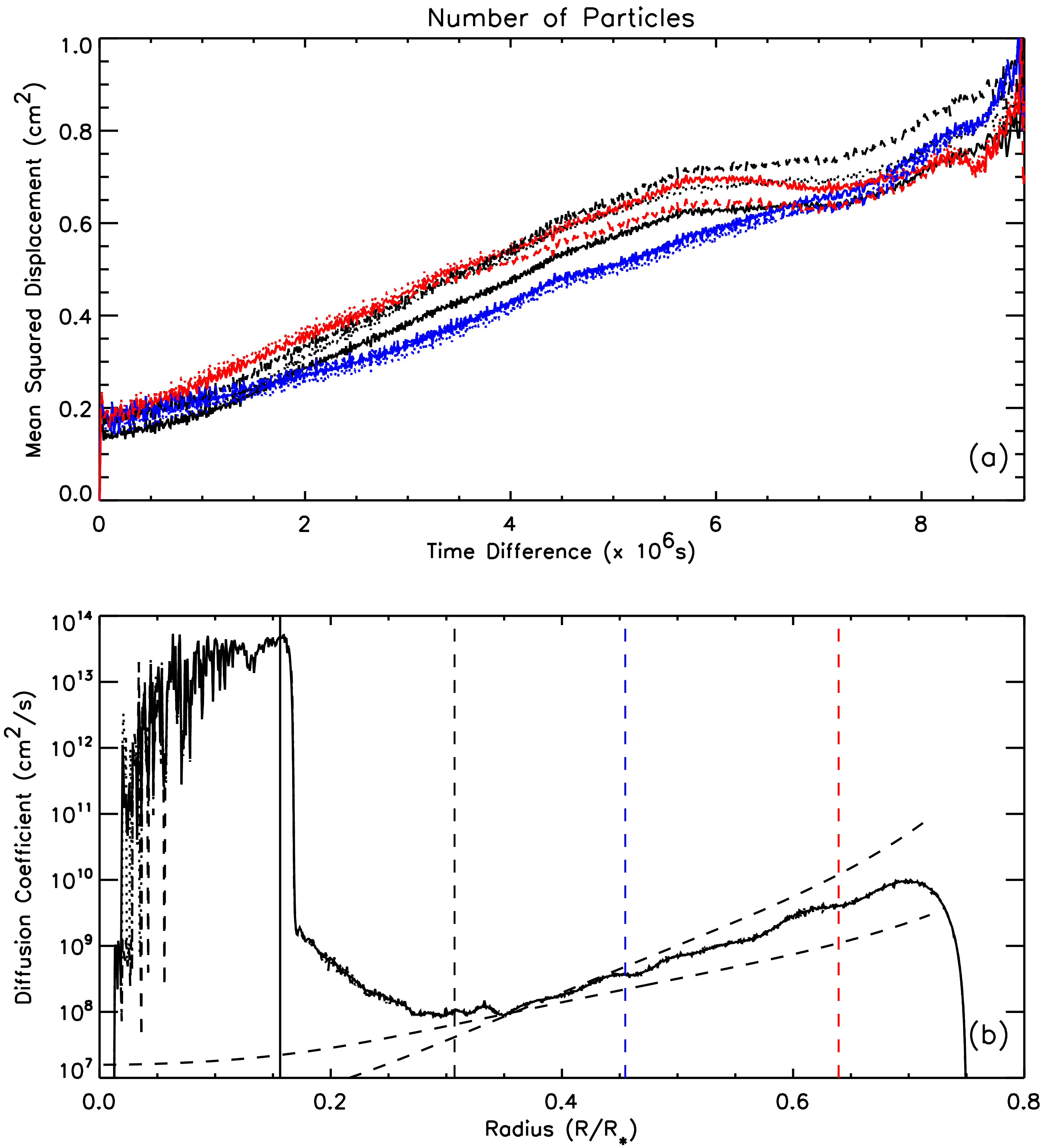
Chemical Mixing by IGW: Tracer Particles





Chemical Mixing by IGW: Tracer Particles

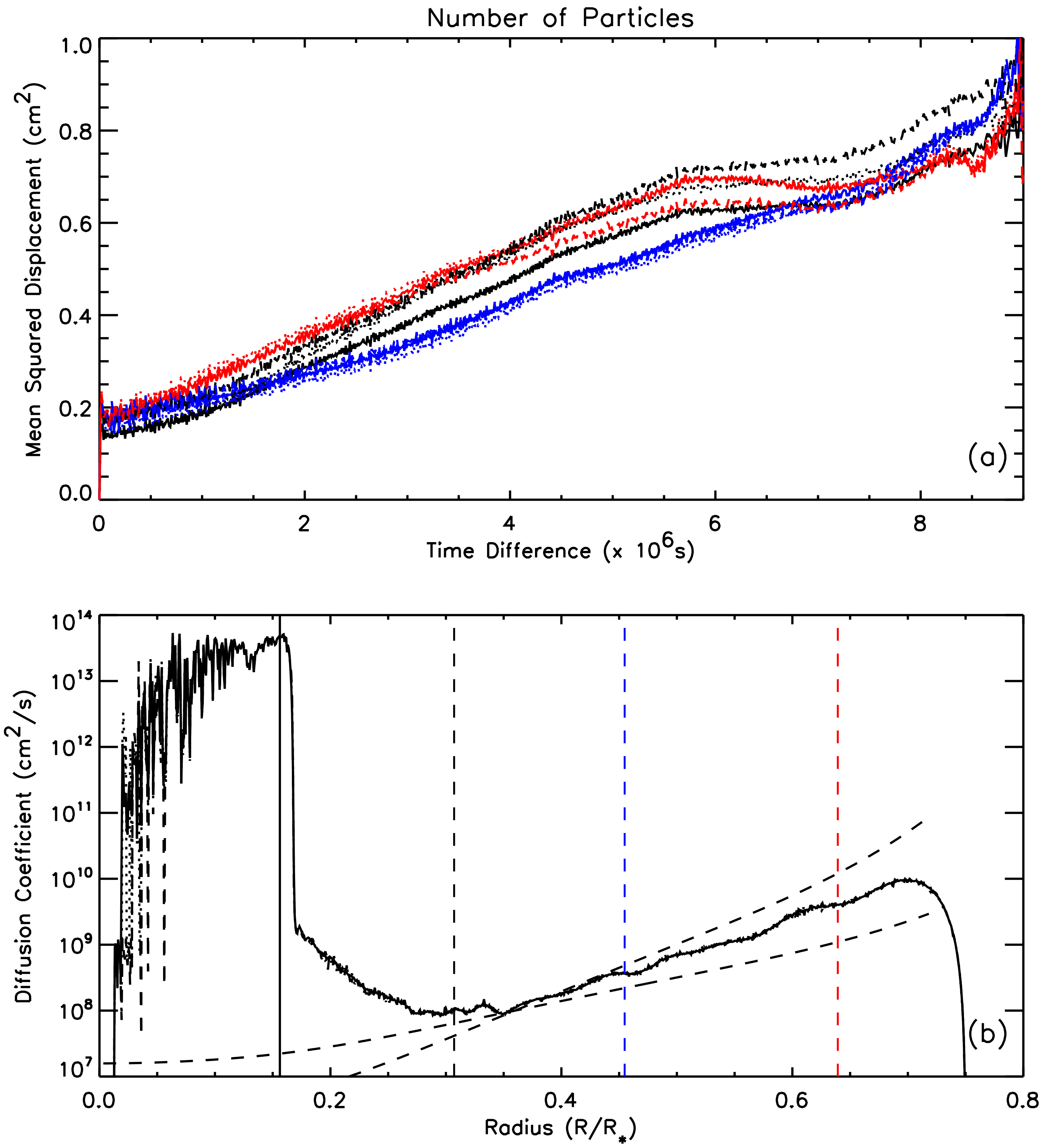




Asteroseismology can now place constraints on mixing within the radiation zone

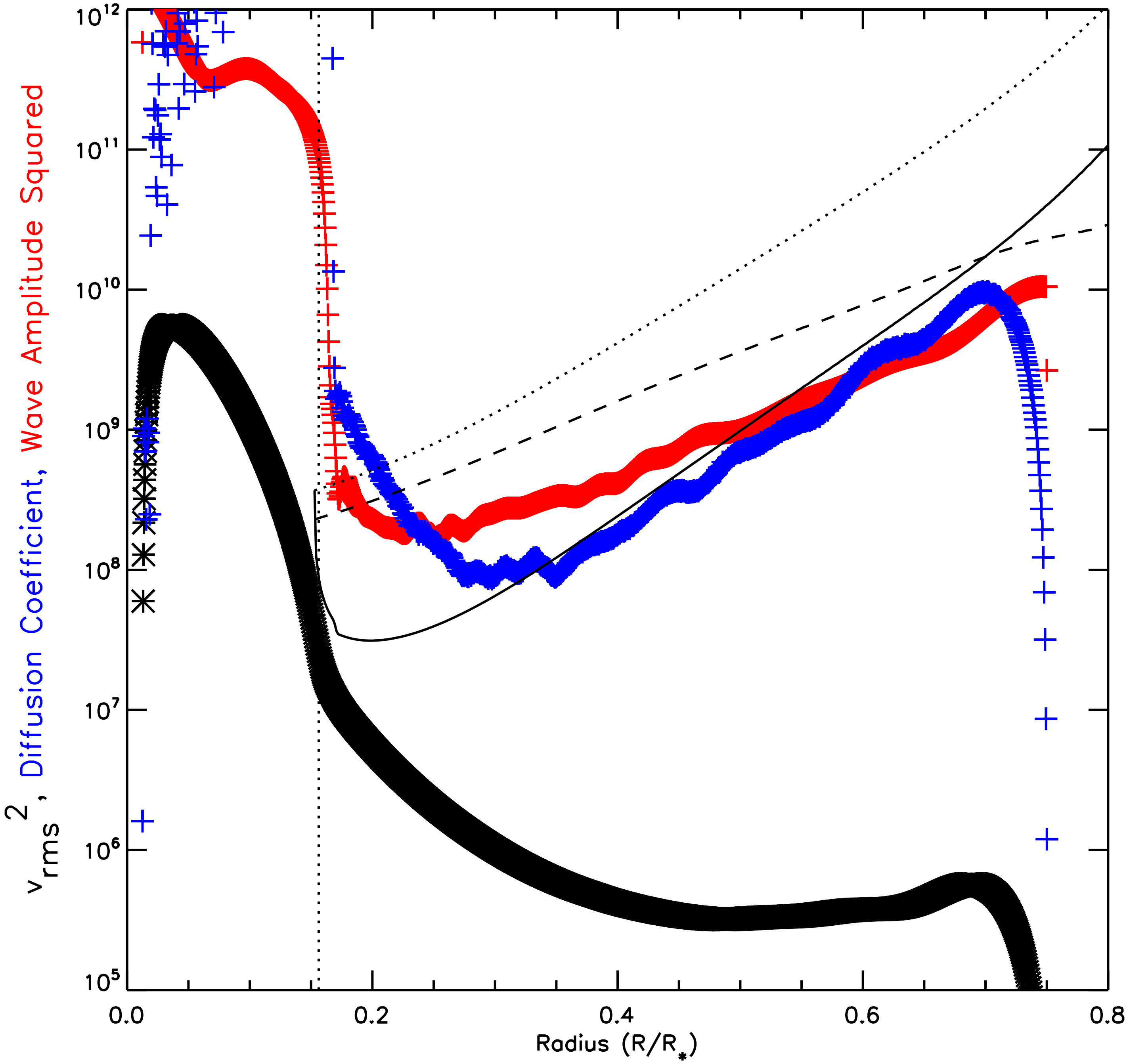
Dependencies

- Radial profile is robust to parameter variations
- Overall amplitude is set by convective velocities
- What sets the profile?



Parameterisation

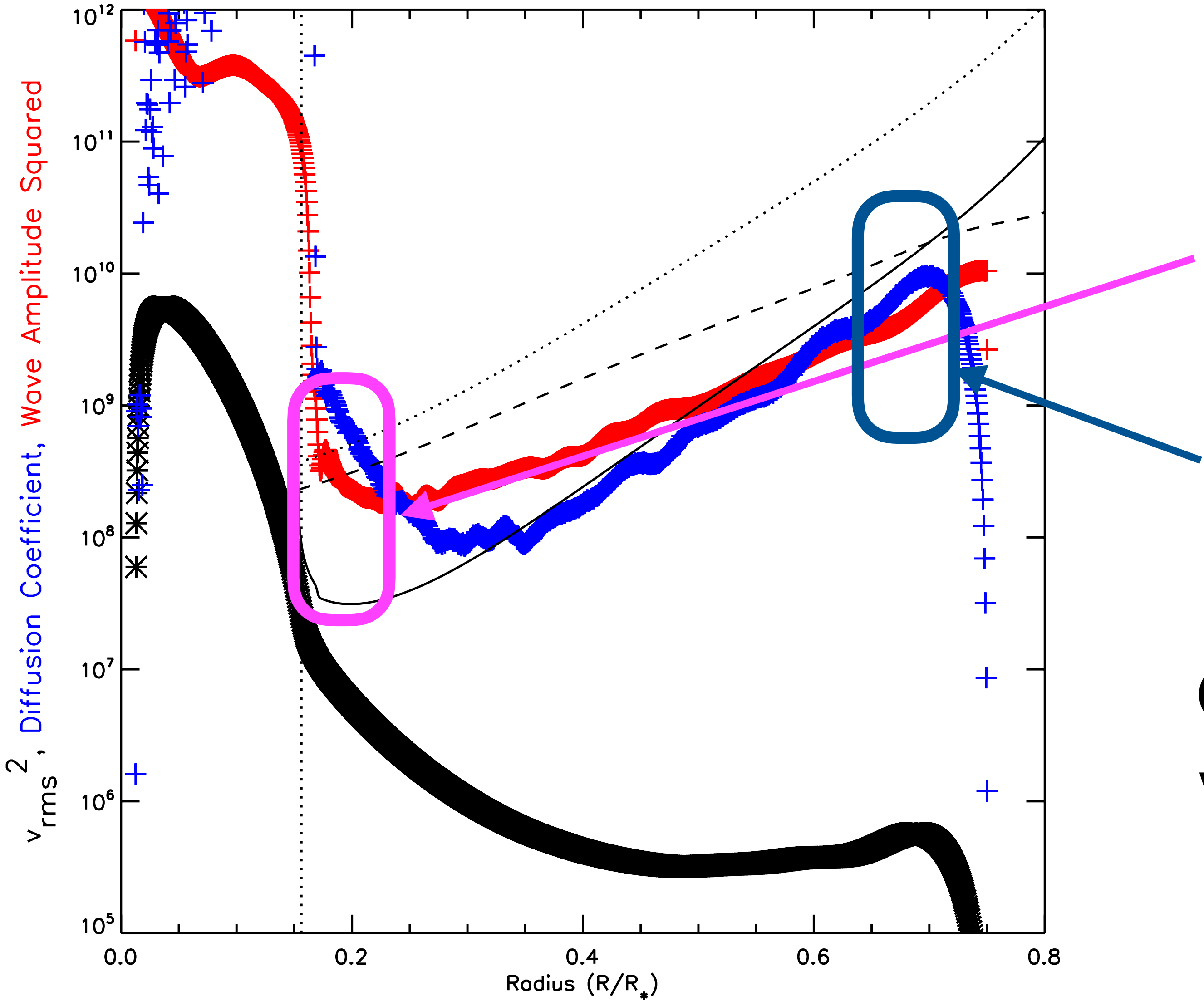
- Diffusion Coefficient does **NOT** depend on local fluid velocities
- Diffusion coefficient closely matches the wave amplitude squared- which depends predominantly on the density stratification and the wave generation spectrum
- This means we can implement a parameterisation of this into a stellar evolution code (Gade-Pederson et al. 2018)



$$D_{mix} = A v_{wave}^2$$

$$v_{wave} = v_{rms-cz}\left(\omega, k_h\right) \left(\frac{\rho(r)}{\rho_{tcz}}\right)^{-1/2} e^{-\tau(\omega,k_h,r)}$$

Parameterisation



The amplitude is not well constrained (diffusivities are too high)

Asteroseismology can constrain

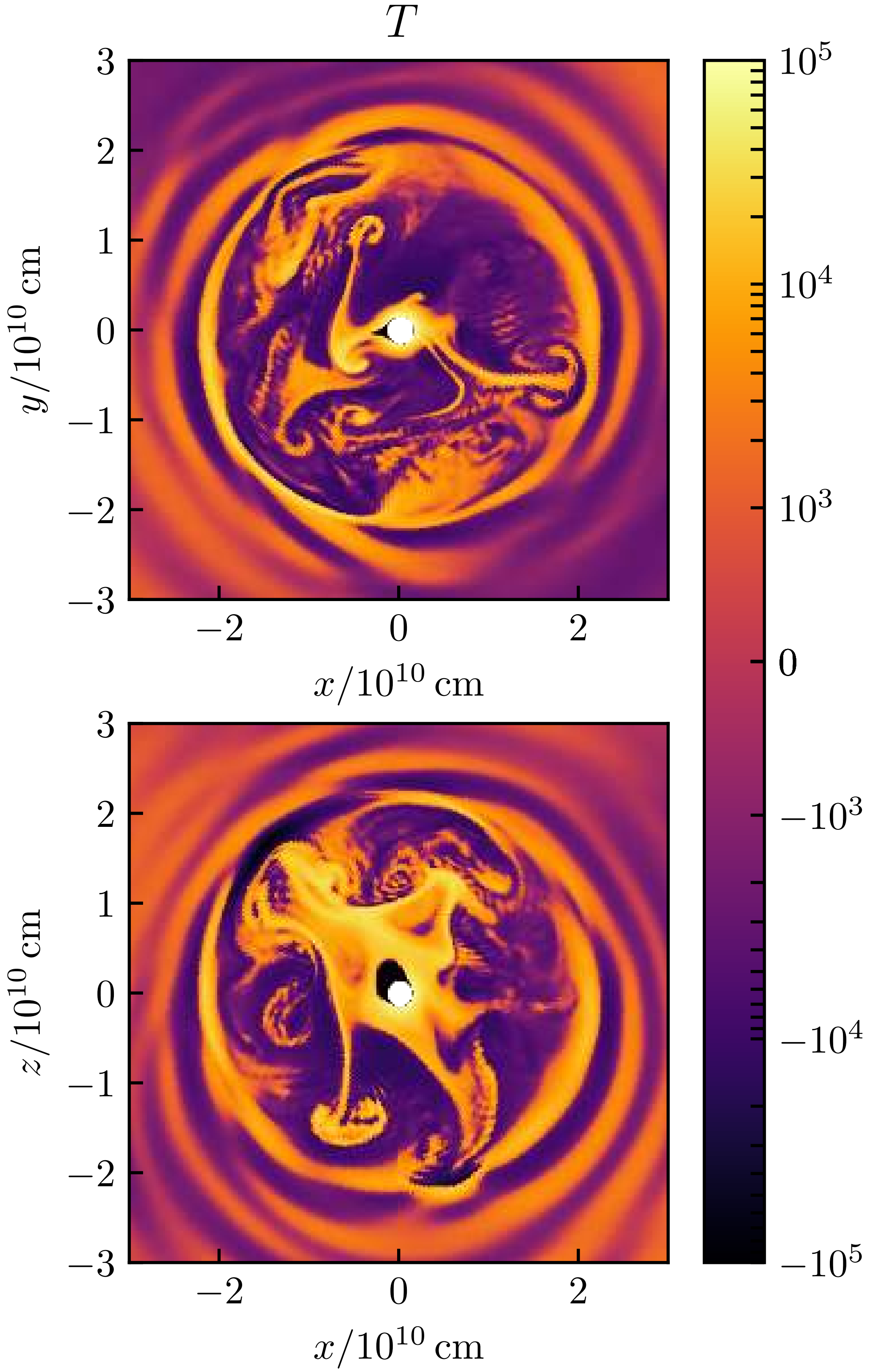
Spectroscopy can constrain

Currently don’t have stars where we have both observations **BUT more variable stars in Kepler to be analysed and many in TESS FOV**

3D Simulations

Pseudo-spectral, spherical harmonic decomposition in (r,theta), finite difference in vertical. Solves anelastic equations for ~90% stellar radius.

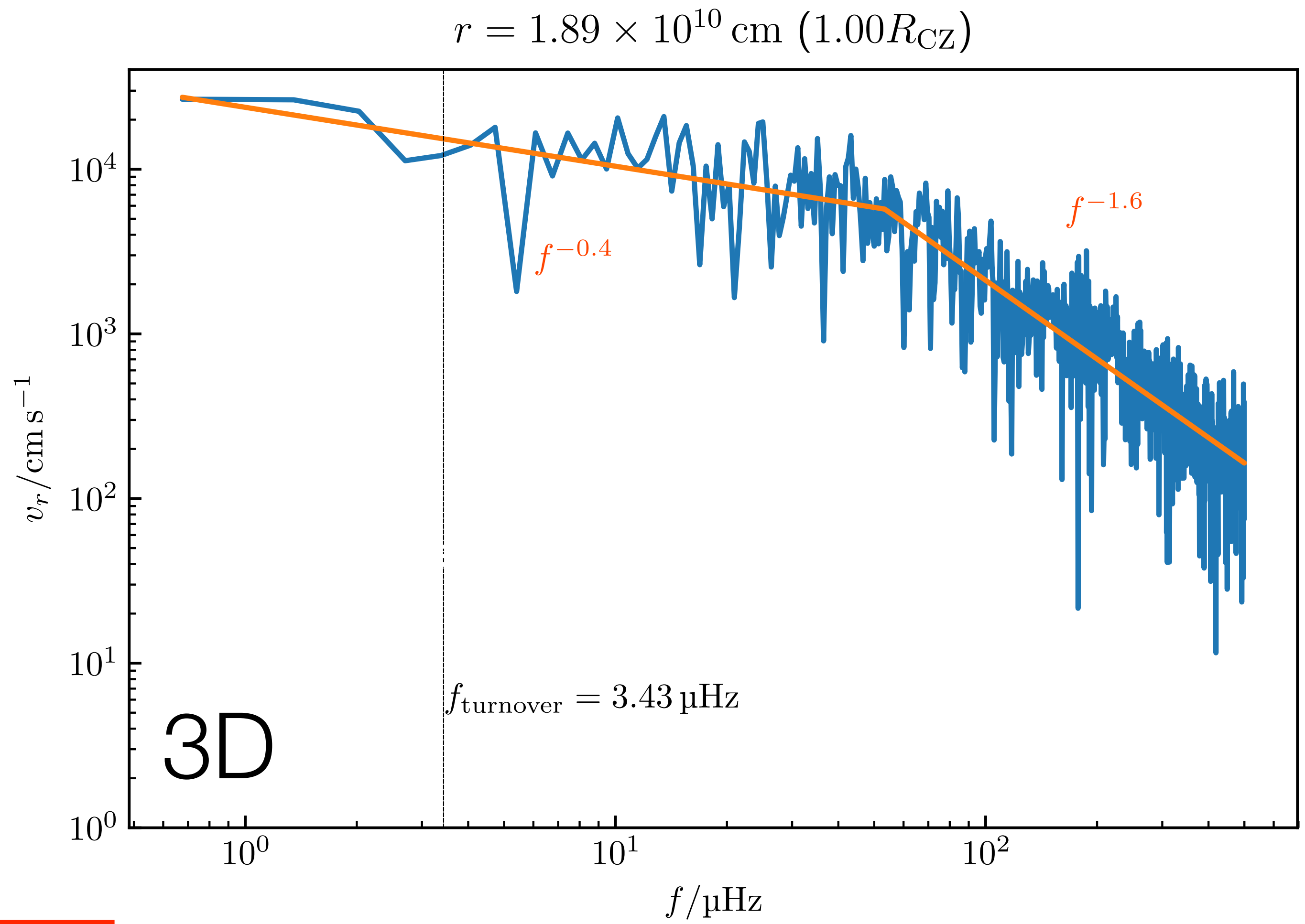
3D Simulations

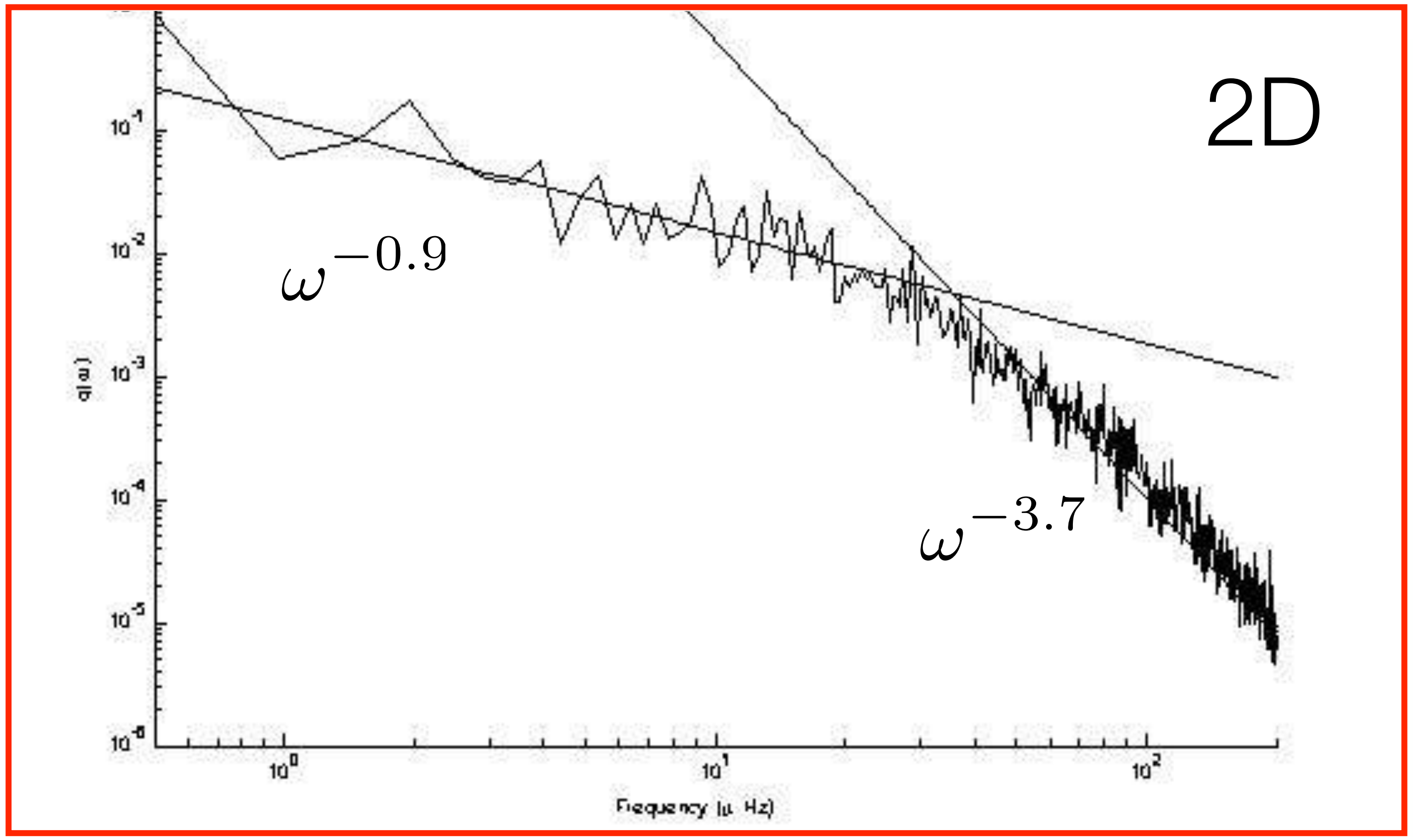


Convection dominated by plumes. Waves are over damped (numerical constraints) but we can still look at wave generation

Wave Spectrum

Remarkably the wave generation spectrum in 3D looks very similar to that in 2D - double power law, relatively flat at low frequencies





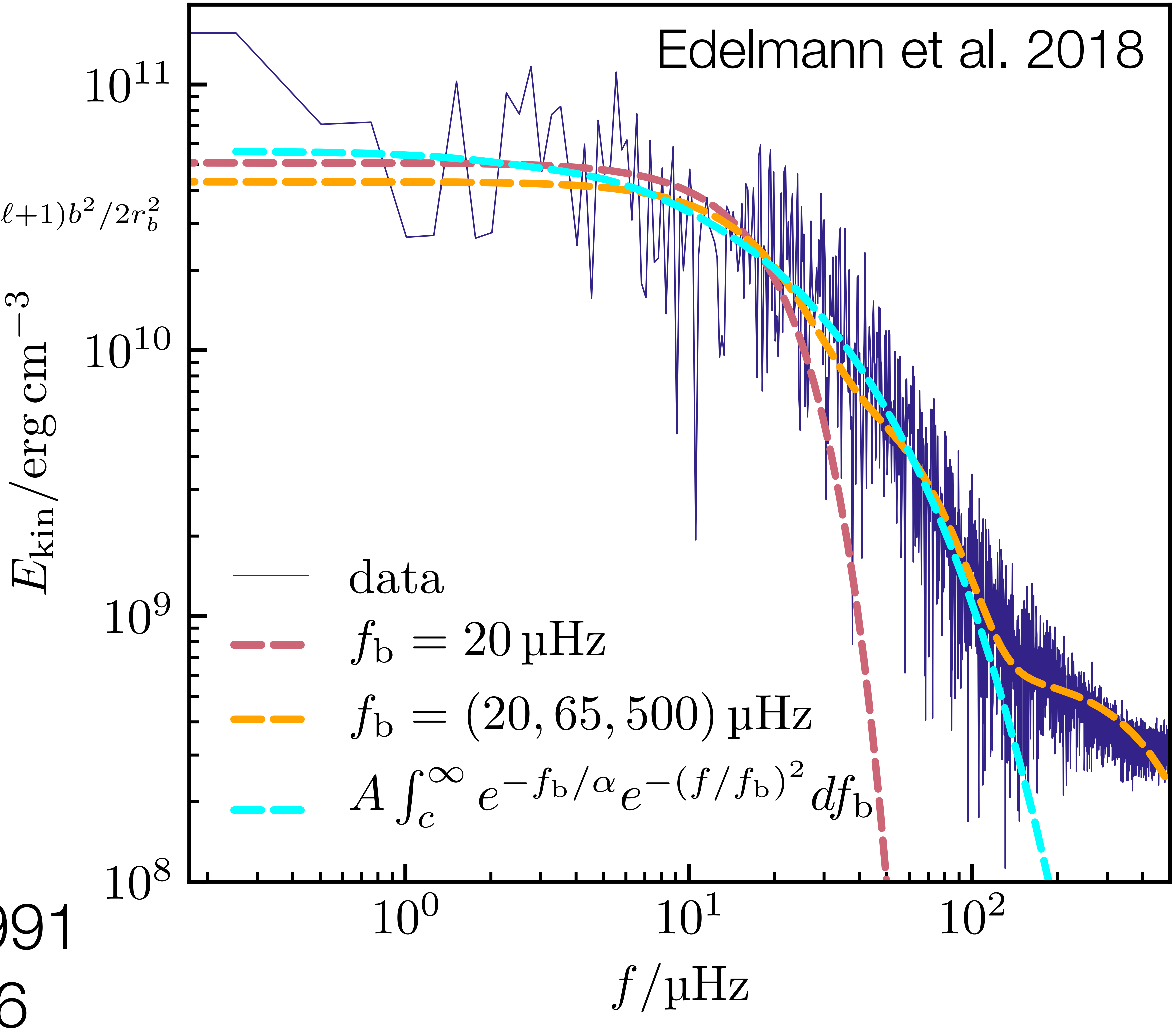
Note: this is radial velocity, energy would have:

$$E \propto \omega^{-0.8}$$

Plume model for wave generation

$$F_L\left(r,\omega,\ell,m\right) \sim$$

$$\frac{1}{4\pi r^2}\frac{AS_p\rho_b V_b^3}{4}F_R\frac{e^{-\omega^2/v_p^2}}{\nu_p}e^{-\ell(\ell+1)b^2/2r_b^2}$$

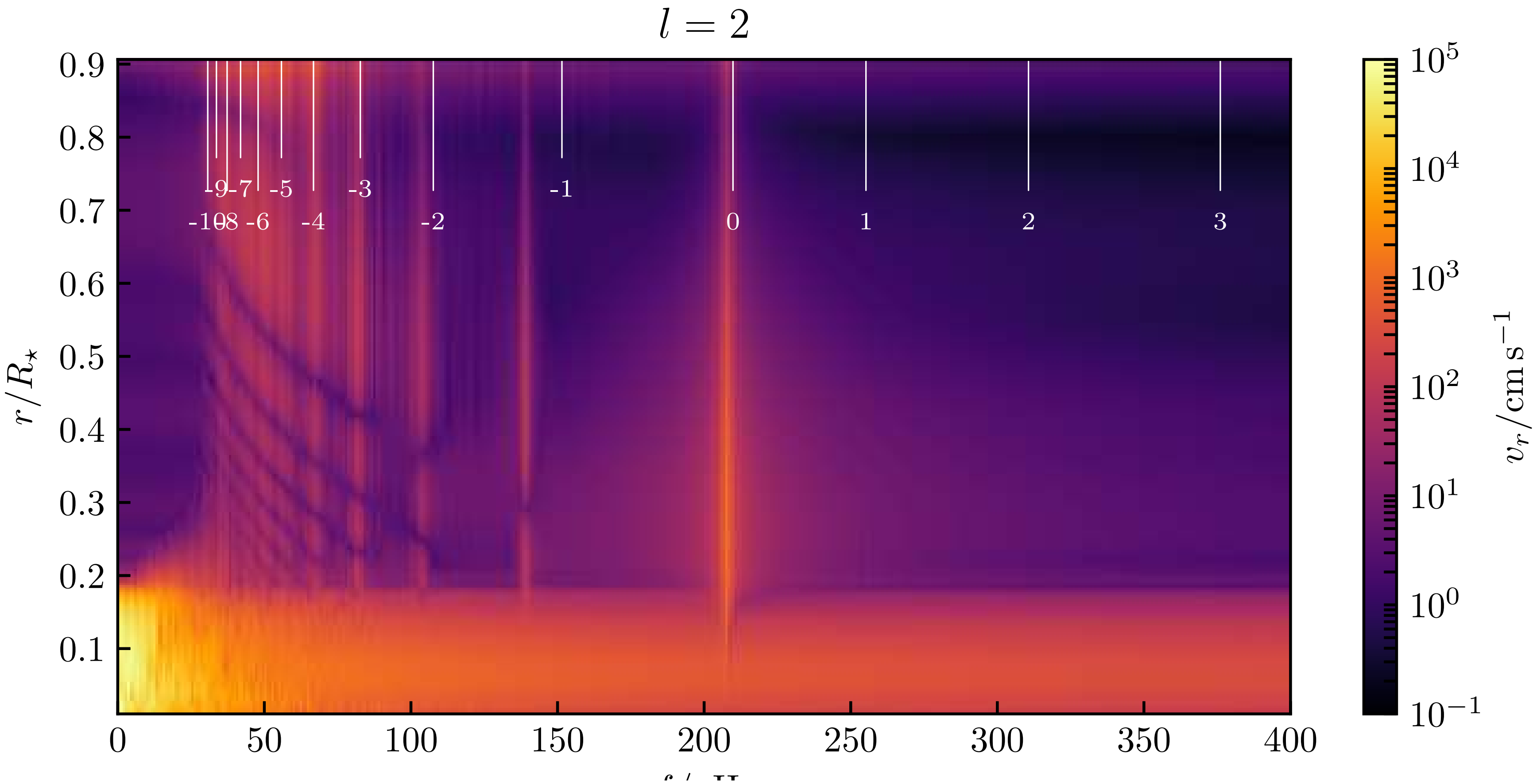


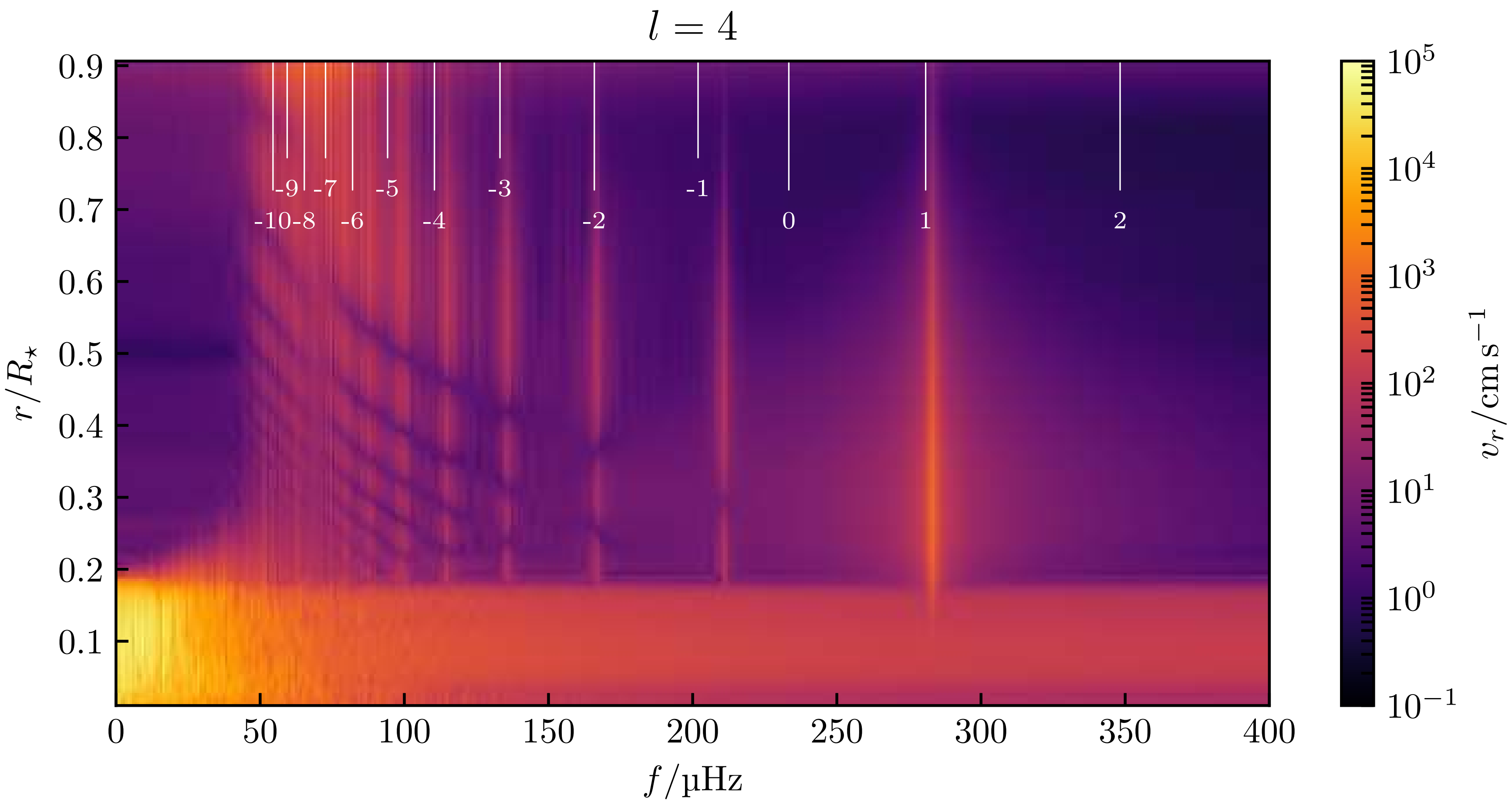
Rieutord & Zahn 1991
Pincon et al. 2016

Standing Modes

Simulations also reproduce standing mode pattern predicted with 1D oscillation code GYRE

Although there are frequency differences which get worst at higher l

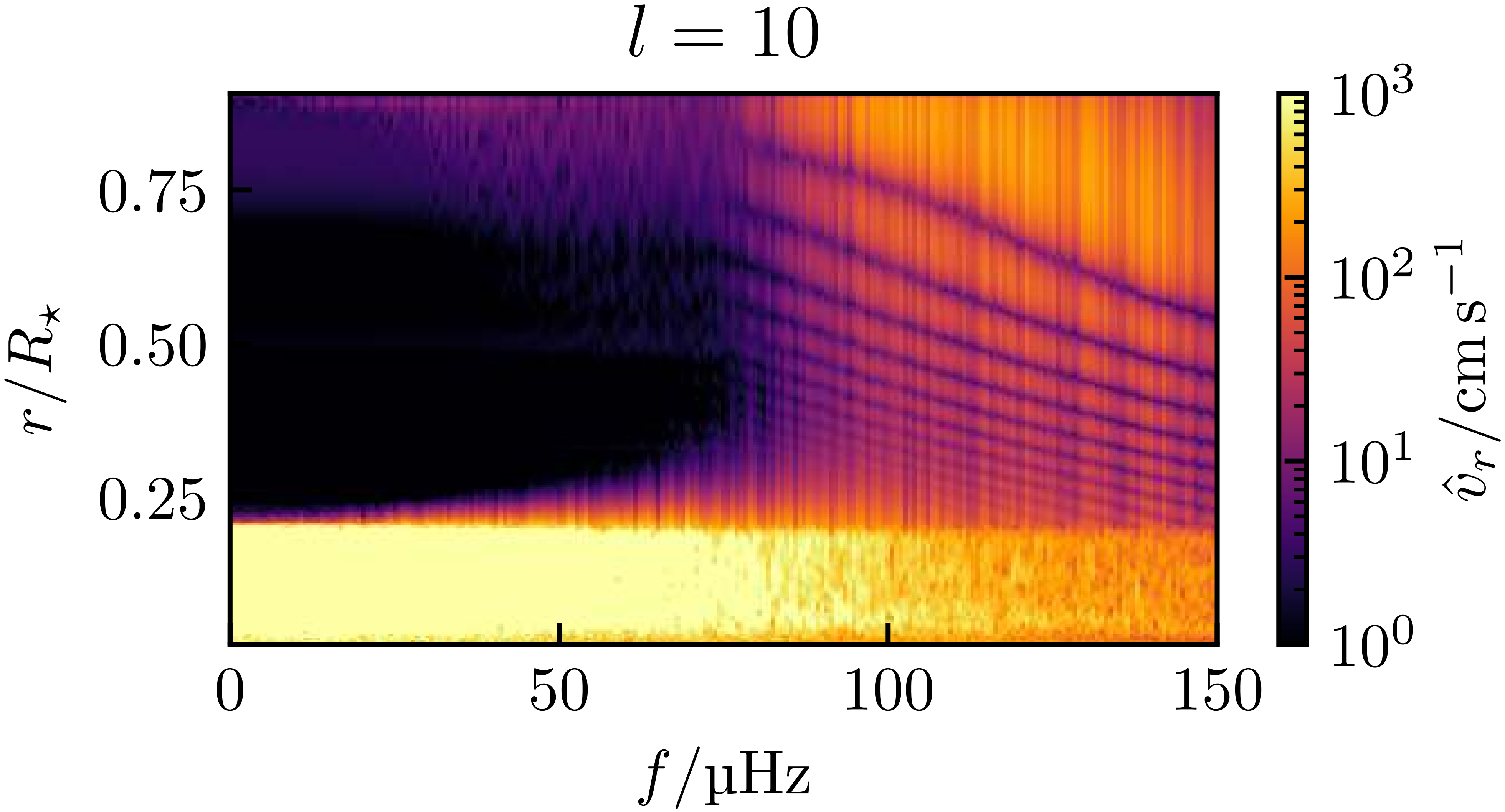




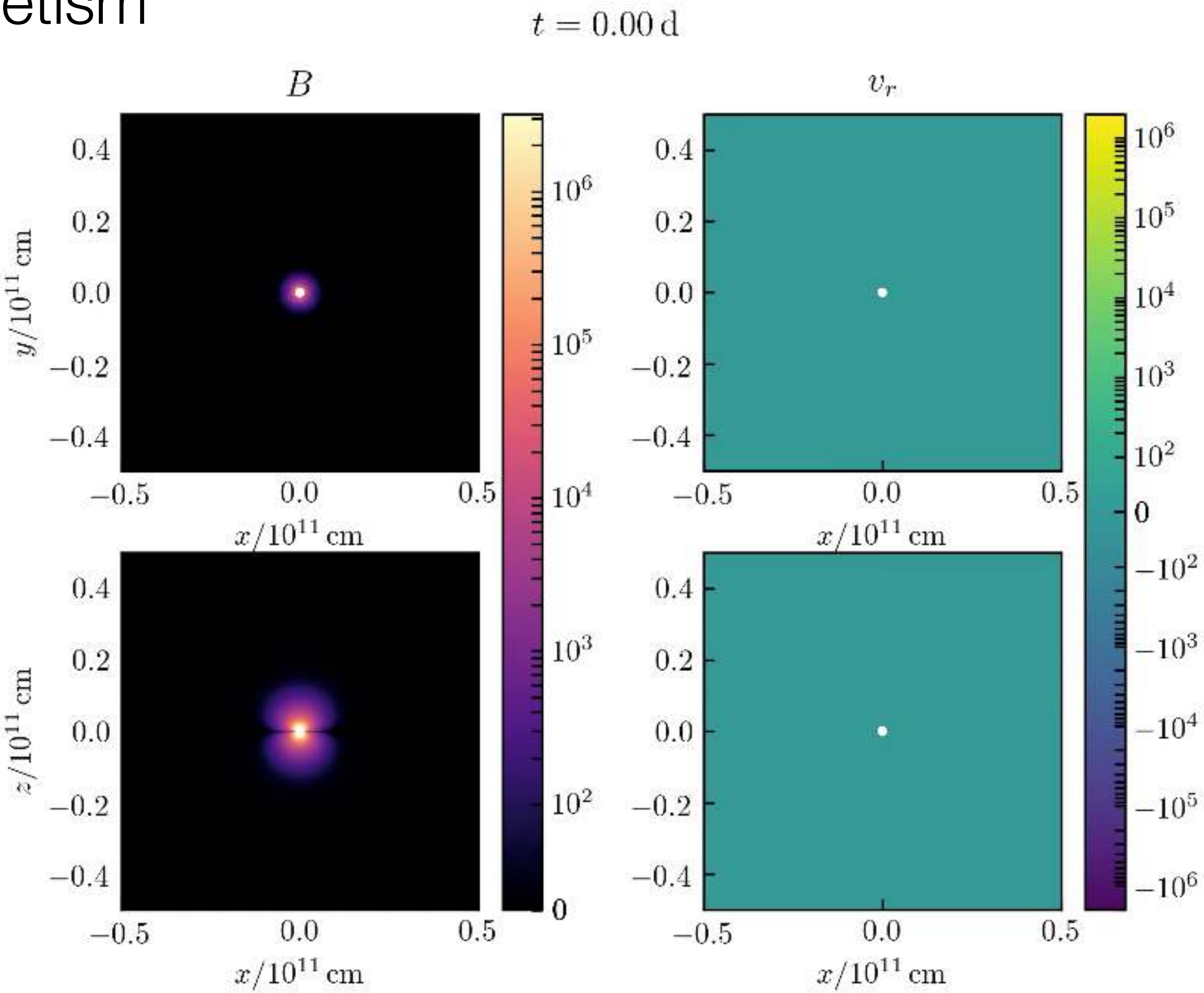
Surface spectra (as a function of frequency) are also similar to 2D

Wave Breaking

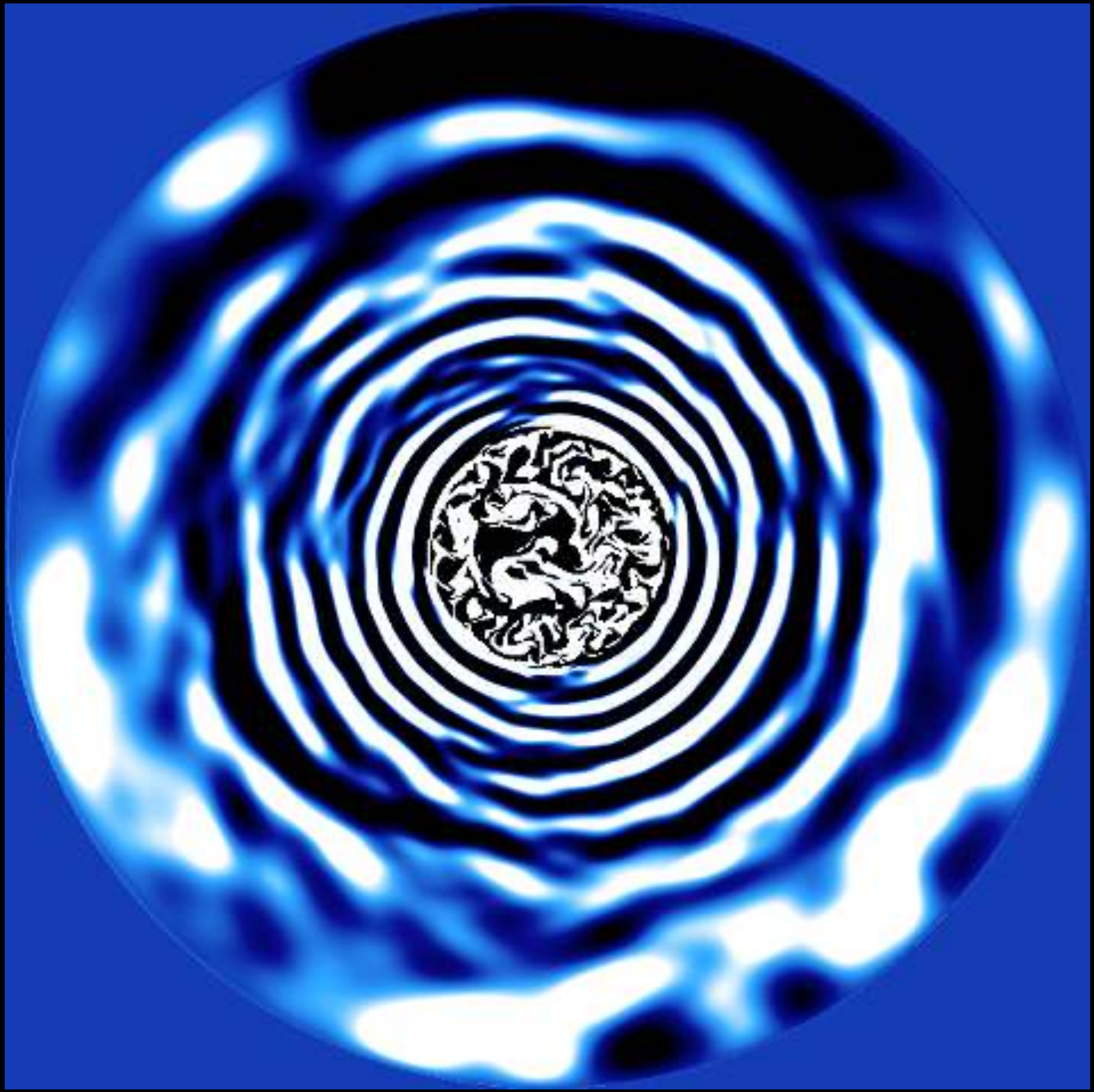
- Don't see much evidence for wave breaking (and subsequent transfer of AM) in the 3D simulations -> too diffusive
- Working on pushing down the diffusivities/increasing resolution



Magnetism


$t = 0.00\,\mathrm{d}$
B
v_r
$y/10^{11}\,\mathrm{cm}$
$z/10^{11}\,\mathrm{cm}$
$x/10^{11}\,\mathrm{cm}$
0.4
0.2
0.0
−0.2
−0.4
−0.5
0.5
10^6
10^5
10^4
10^3
10^2
0
-10^2
-10^3
-10^4
-10^5
-10^6

Conclusions



- Its clear, both observationally and numerically, that IGW could be important for mixing and angular momentum transport in Stars
- Its likely these waves are more important in massive stars than in low mass (solar type) stars
- Its possible that simplified models may not work (in massive stars) because of the possibility of wave breaking/critical layers and other nonlinear interactions: parameterisations will need to be carefully calibrated
- Calibration may come from observations (asteroseismic+spectroscopic)