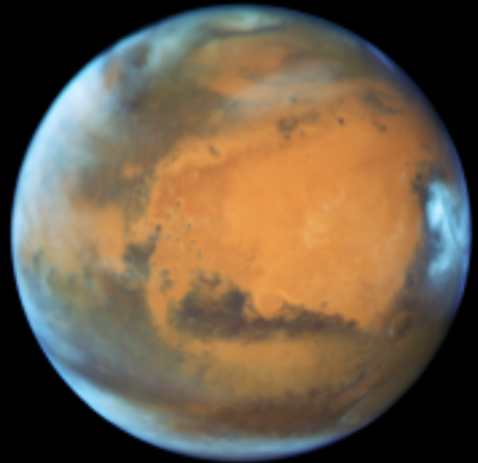
Large-scale baroclinic and barotropic instabilities in planetary atmospheres

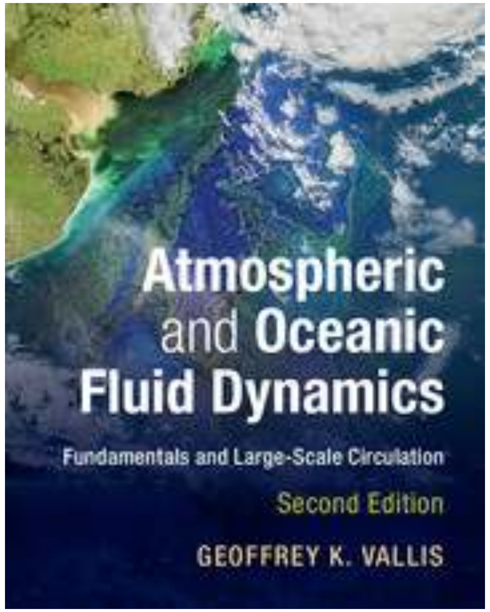
Peter Read

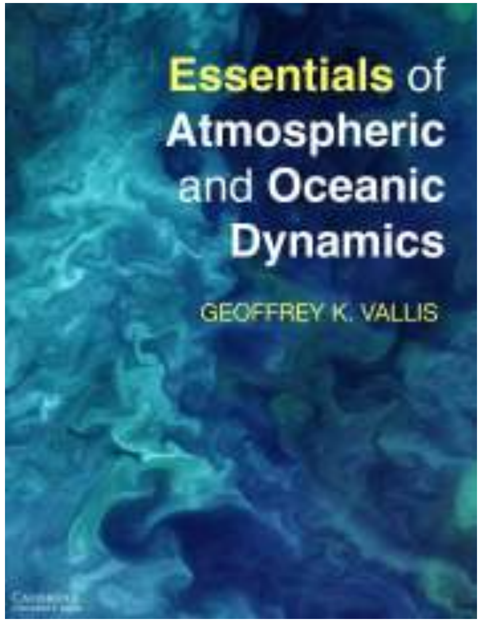
University of Oxford, UK



Aims and objectives

- Major processes in large-scale atmosphere and ocean circulations on Earth and other planets
  - Energetically dominant eddy-generating processes in rotating, stably-stratified flows
- Basic theory is well documented – e.g. see books by Geoff Vallis (2017 and 2019 - CUP)
  - Theoretical treatment here will follow Vallis, focusing on basic principles (not too many details!)
  - Illustrate basic ideas on stability conditions and nonlinear equilibration using laboratory experiments and simple GCM simulations
  - Applications to atmospheres of Earth, Mars and gas giant planets (Jupiter and Saturn)





QG instabilities (baroclinic and/or barotropic)

- QG potential vorticity equation [cf lectures by Jerome Noir & Keith Julien]

$$\left.\begin{aligned}&\frac{\partial q}{\partial t} + \boldsymbol{u}_g . \nabla q = 0; \qquad 0 < z < H \\ &\text{where } q = \nabla^2 \psi + \beta y + \frac{\partial}{\partial z}\left(F \frac{\partial \psi}{\partial z}\right); \qquad F = \frac{f_0^2}{N^2};\end{aligned}\right\} \quad (1)$$

- where $q = \nabla^2 \psi + \beta y + \frac{\partial}{\partial z}\left(F \frac{\partial \psi}{\partial z}\right)$; $F = \frac{f_0^2}{N^2}$;

  and $f = f_0 + \beta y$ where $f_0 = 2\Omega \sin \varphi_0$
- With boundary conditions $w = 0$ at $z = 0, H$
- i.e. $\frac{\partial b}{\partial t} + \boldsymbol{u}_g . \nabla b = 0$ at $z = 0, H$ where $b = f_0 \frac{\partial \psi}{\partial z}$ (2)

QG instabilities (baroclinic and/or barotropic)

- Consider basic state $\boldsymbol{u}_g = (U(y,z), 0)$

  for which $q = Q = \beta y - \frac{\partial U}{\partial y} + \frac{\partial}{\partial z}\left(F \frac{\partial \Psi}{\partial z}\right)$ and $U = -\frac{\partial \Psi}{\partial y}$
- Linearise (1) about U to get perturbation PV equation

$$\left.\begin{array}{l} \frac{\partial q'}{\partial t} + U \frac{\partial q'}{\partial x} + v' \frac{\partial Q}{\partial y} = 0;\; 0 < z < H \\ \text{with boundary conditions} \\ \frac{\partial b'}{\partial t} + U \frac{\partial b'}{\partial x} + v' \frac{\partial B}{\partial y} = 0;\; z = 0, H \end{array}\right\} \quad (3)$$

- Where buoyancy $b' = f_0 \frac{\partial \psi'}{\partial z}, \frac{\partial B}{\partial y} = -f_0 \frac{\partial U}{\partial z}$

QG instabilities (baroclinic and/or barotropic)

- Now seek solutions of the form

$$\psi' = Re[\tilde{\psi}(y,z)e^{ik(x-ct)}]$$

So $\tilde{q} = \frac{\partial^2\tilde{\psi}}{\partial y^2} + \frac{\partial}{\partial z}\left(F\frac{\partial\tilde{\psi}}{\partial z}\right) - k^2\tilde{\psi}$

- Substitute into (3) to obtain

$$\left.\begin{aligned} (U-c)\left(\frac{\partial^2\tilde{\psi}}{\partial y^2} + \frac{\partial}{\partial z}\left(F\frac{\partial\tilde{\psi}}{\partial z}\right) - k^2\tilde{\psi}\right) + \frac{\partial Q}{\partial y}\tilde{\psi} = 0;\ \ 0 < z < H \\ (U-c)\frac{\partial\tilde{\psi}}{\partial z} - \frac{\partial U}{\partial z}\tilde{\psi} = 0;\ \ z = 0, H \end{aligned}\right\} \quad (4)$$

- **Instability requires $Im[c] \neq 0$**

QG instabilities (baroclinic and/or barotropic)

- x(4) by $\tilde{\psi}^*$ and integrate (by parts) over domain $y_1 < y < y_2; 0 < z < H$ (with suitable boundary conditions at y_1 and y_2)
- So

$$\int_0^H \int_{y_1}^{y_2} \left[\left| \frac{\partial \tilde{\psi}}{\partial y} \right|^2 + F \left| \frac{\partial \tilde{\psi}}{\partial z} \right|^2 + k^2 |\tilde{\psi}|^2 \right] dy dz - \int_{y_1}^{y_2} \left\{ \int_0^H \frac{\partial Q/\partial y}{U - c} |\tilde{\psi}|^2 dz + \left[\frac{F \partial U/\partial z}{U - c} |\tilde{\psi}|^2 \right]_0^H \right\} dy = 0$$

- Note that first group of terms is **real and positive** but second group may be complex with imaginary part that satisfies

$$-c_i \int_{y_1}^{y_2} \left\{ \int_0^H \frac{\partial Q/\partial y}{|U - c|^2} |\tilde{\psi}|^2 dz + \left[\frac{F \partial U/\partial z}{|U - c|^2} |\tilde{\psi}|^2 \right]_0^H \right\} dy = 0 \qquad (5)$$

QG instabilities

$$-c_i \int_{y_1}^{y_2} \left\{ \int_0^H \frac{\partial Q/\partial y}{|U-c|^2} \left|\tilde{\psi}\right|^2 dz + \left[\frac{F \partial U/\partial z}{|U-c|^2} \left|\tilde{\psi}\right|^2 \right]_0^H \right\} dy = 0$$

- Thus, for $c_i \neq 0$ we require one of the following:

i. $\partial Q/\partial y$ changes sign in the interior domain – OR

ii. Interior $\partial Q/\partial y$ has the opposite sign to $\partial U/\partial z$ at $z = H$ – OR

iii. Interior $\partial Q/\partial y$ has the same sign as $\partial U/\partial z$ at $z = 0$ – OR if

iv. $\frac{\partial Q}{\partial y} = 0$ in the interior, $\partial U/\partial z$ has the same sign at $z = 0$ AND $z = H$

- **The Charney-Stern-Pedlosky generalization of the Rayleigh-Kuo stability criterion** [NB necessary but not sufficient]
- Example: Earth's mid-latitude troposphere
  - $\partial Q/\partial y$ dominated by $\beta > 0$ in the free atmosphere and $\partial U/\partial z > 0$ almost everywhere
  - Instability typically satisfied by criterion *iii.*

Barotropic instability

- Consider a basic state flow for which $U = U(y)$ only
- Now $\frac{\partial Q}{\partial y} = \beta - \frac{\partial^2 U}{\partial y^2}$ and (5) becomes

$$c_i \int_{y_1}^{y_2} \frac{\beta - \partial^2 U/\partial y^2}{|U - c|^2} |\tilde{\psi}|^2 dy = 0$$

- A necessary condition for instability ($c_i \neq 0$) is therefore that $\beta - \partial^2 U/\partial y^2$ change sign somewhere in the domain (criterion *i.*)

Example: barotropic edge waves

Figures from Vallis (2017)

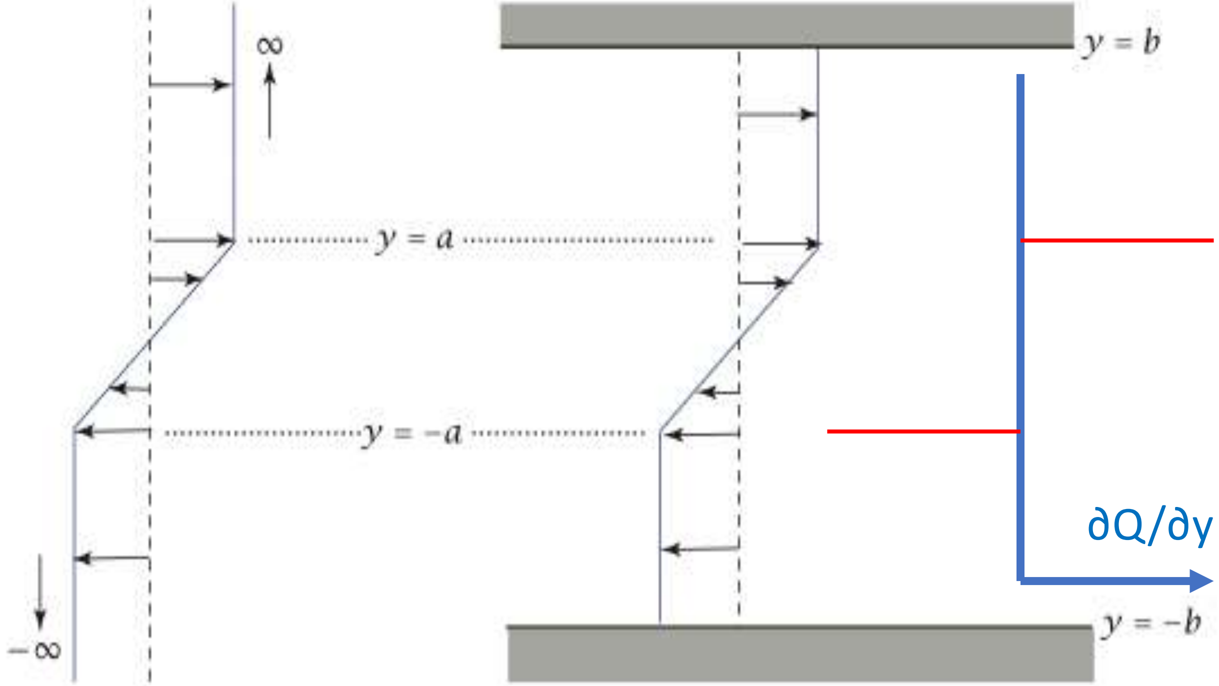


Fig. 9.4

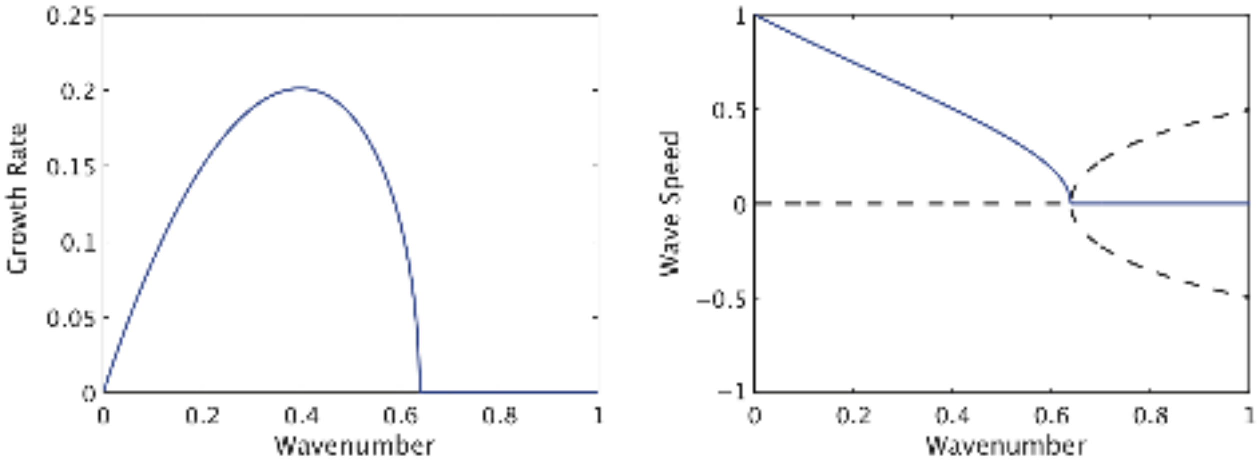


Fig. 9.5

- Edge waves propagate on PV gradient at *y* = ±*a*
- Dispersion relations. $c_{+a} = U_0 - \frac{\frac{U_0}{a}}{2k}; c_{-a} = -U_0 + \frac{\frac{U_0}{a}}{2k}$
- For both waves to interact, phase speeds must be equal $\Longrightarrow c = 0, k = 1/2a$

Example: barotropic edge waves

Figures from Vallis (2017)

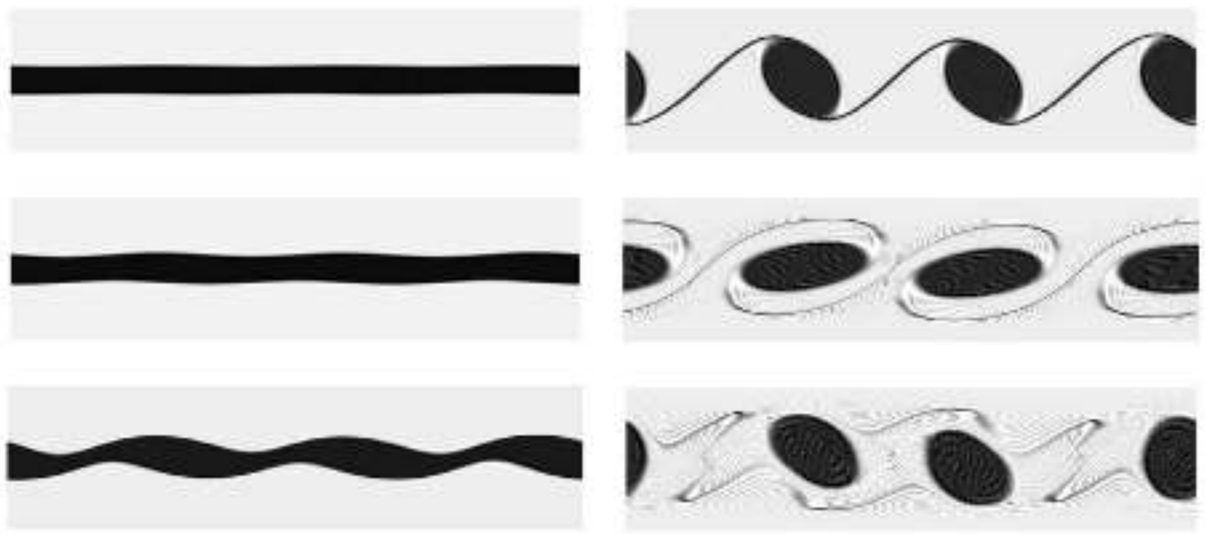
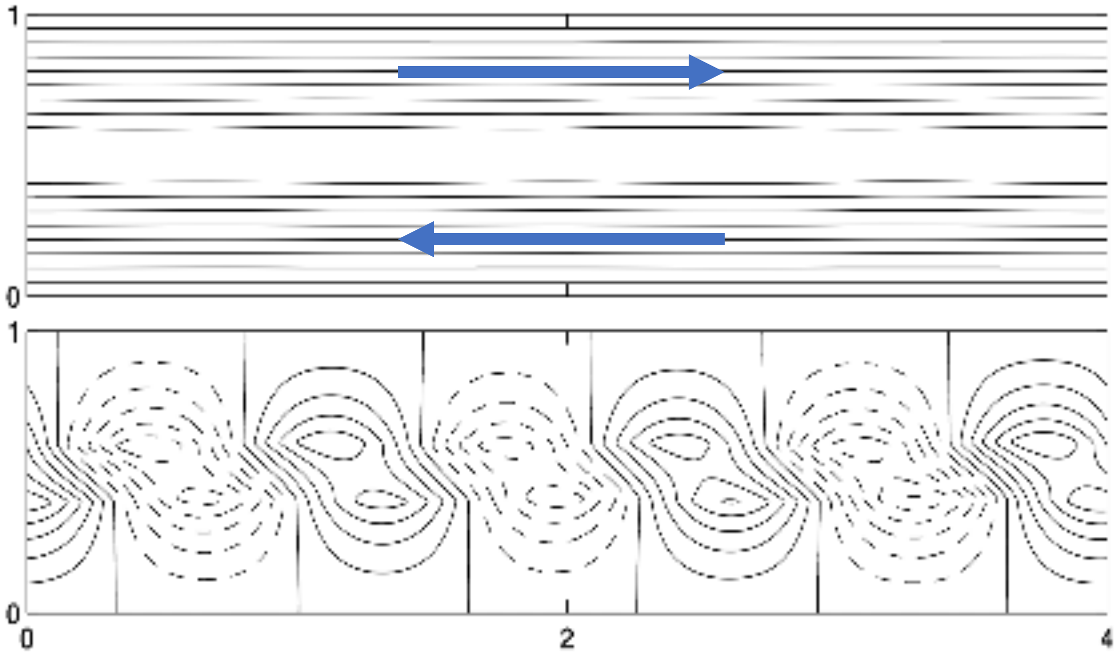


Fig. 9.6



9.7

Vorticity pattern during development of barotropic instability

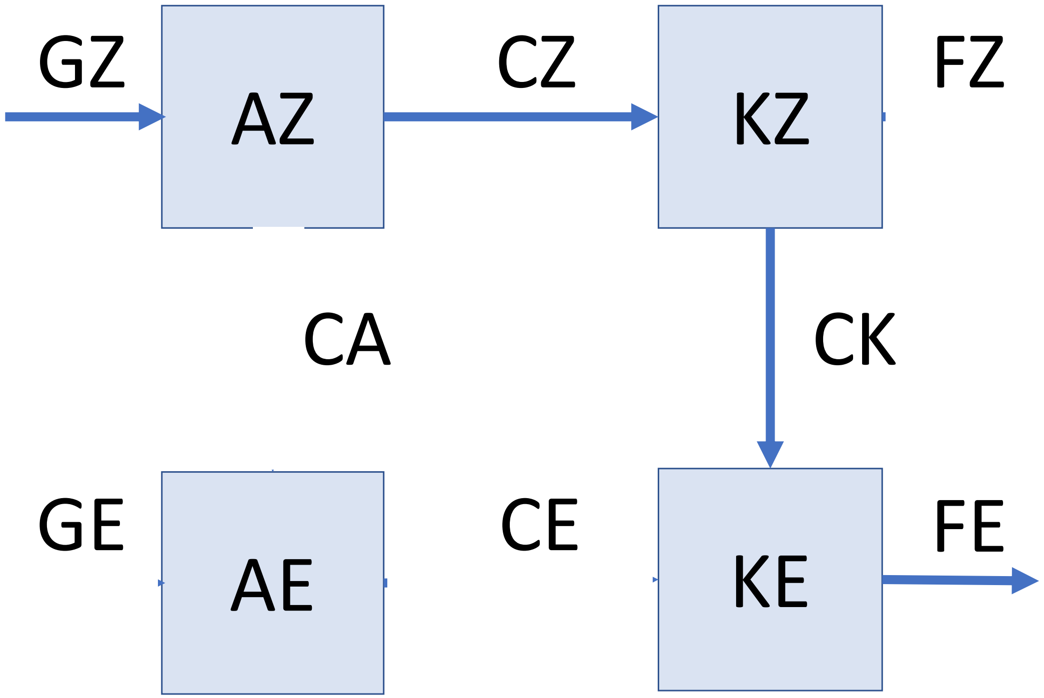
- Varicose instability

Total (upper) and perturbation (lower) streamfunction

- Note how perturbation leans into the shear to extract EKE from the background flow

Example: barotropic edge waves

Lorenz energy cycle



Conversion route for barotropic instability

Figure from Vallis (2017)

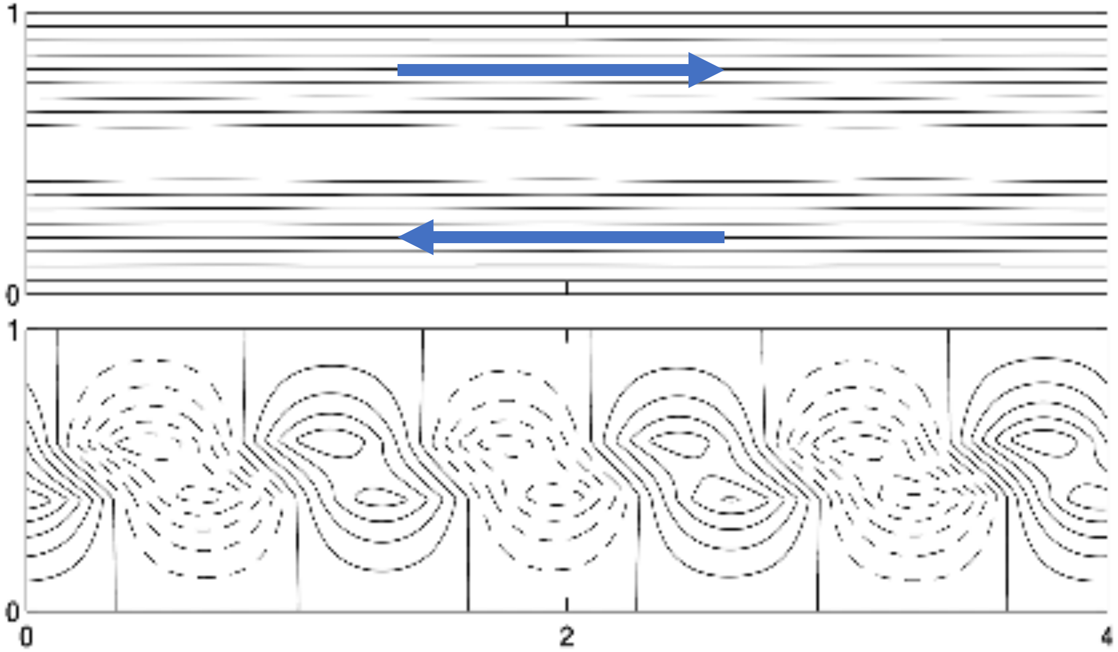
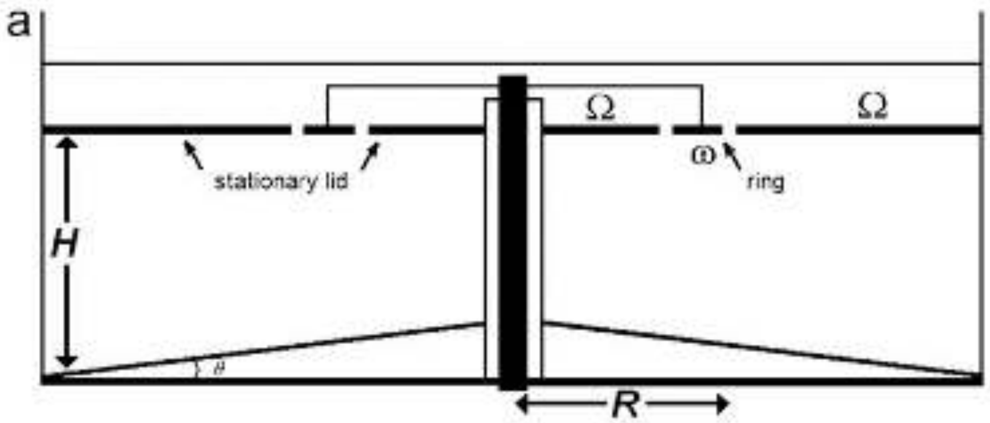


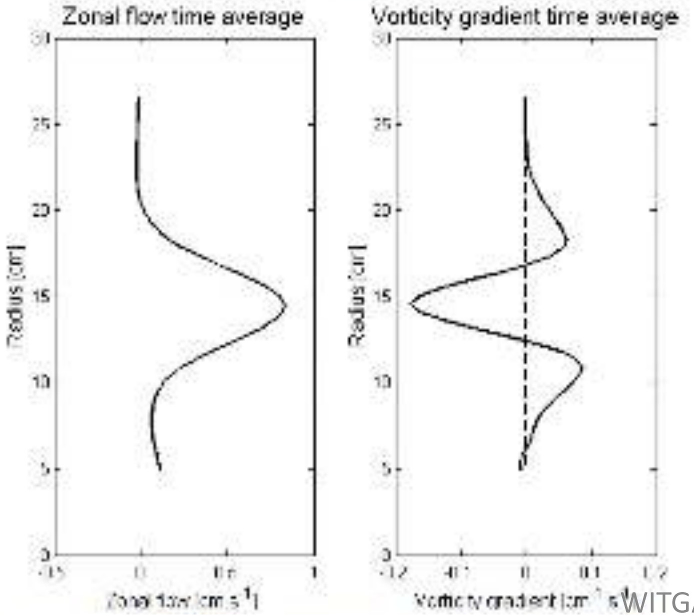
Fig. 9.7 Total (upper) and perturbation (lower) streamfunction

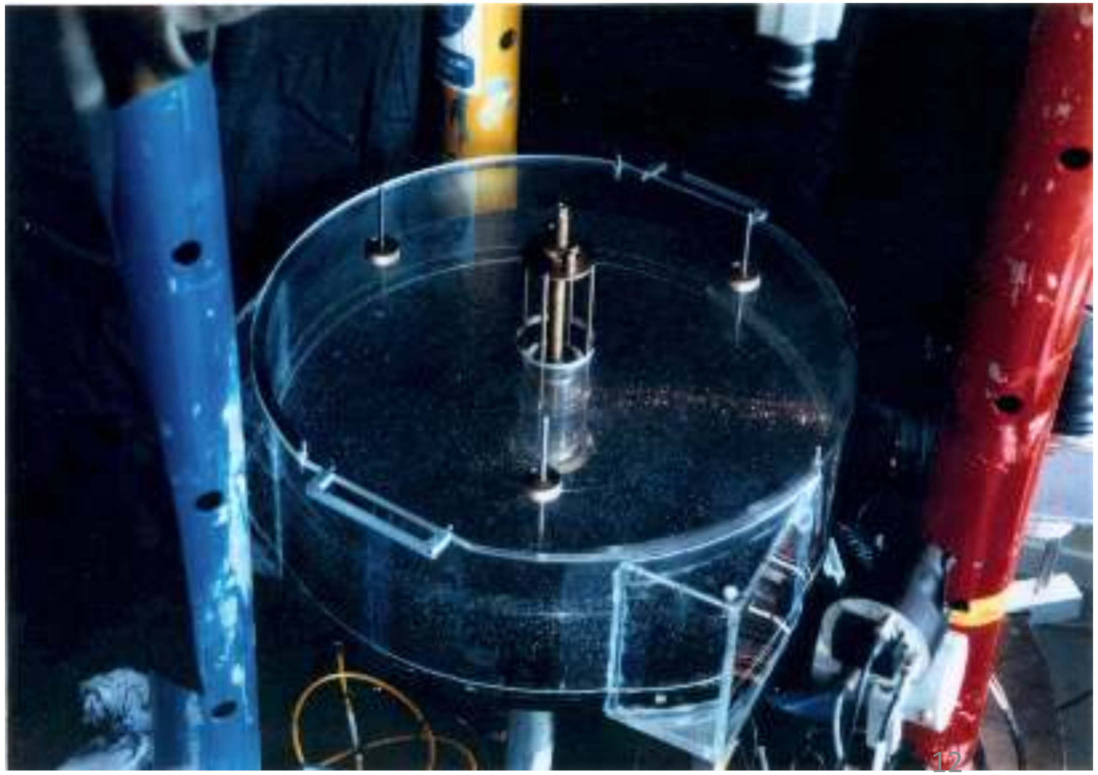
- Note how perturbation leans into the shear to extract EKE from the background flow

Barotropic jet experiment

- Experiment to drive a barotropically unstable jet in a rotating fluid
- Cylindrical tank filled with homogeneous fluid (water)
- Differentially rotating ring (or disk) creates a jet (or shear layer) of width ~ $E^{1/4}$
- Topographic β-plane

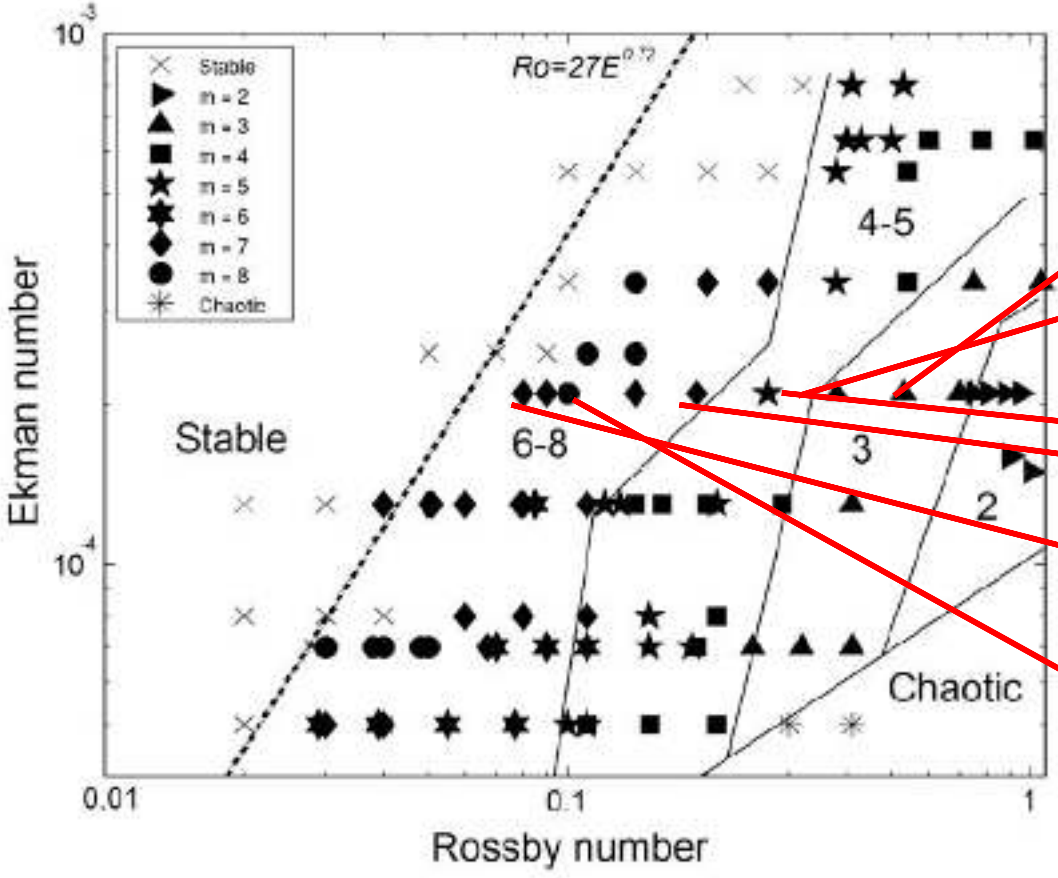




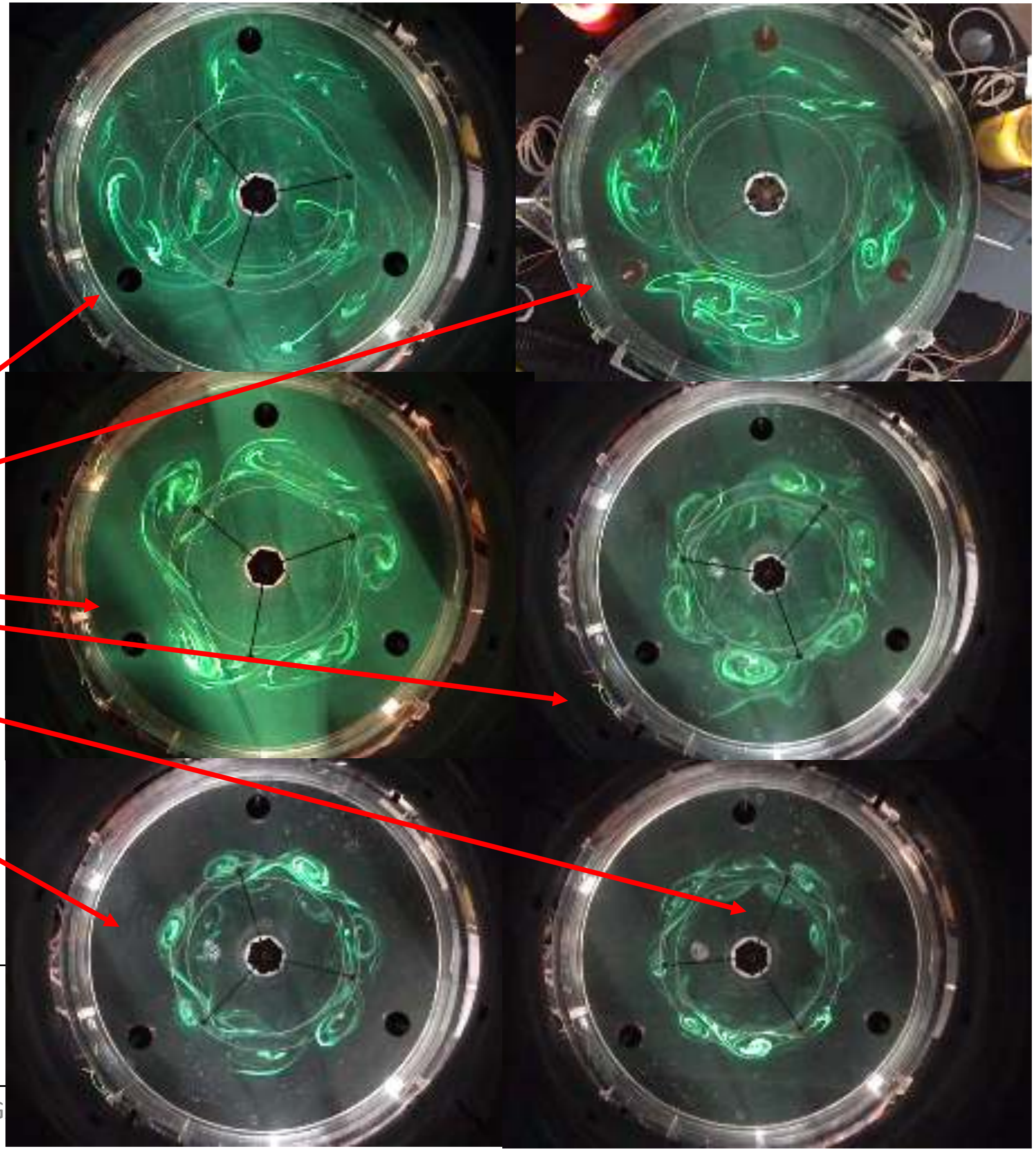


CIRCULATION REGIMES

Barotropic jet experiment



Regime Diagram



Classical example of linearized baroclinic instability: the Eady problem (1949)

- Highly simplified "minimal" model
  I. Motion is on an f-plane ($\beta = 0$)
  II. Uniformly stratified ($N^2 = constant$)
  III. Basic state has uniform vertical shear and no lateral shear: $U(z) = \Lambda z = Uz/H$
  IV. Flow contained between two rigid, flat, horizontal boundaries at $z = 0, H$
- $\frac{\partial Q}{\partial y} = 0$ in interior so potential for instability only via criterion *iv.*
- Trial separable solution $\psi' = Re\left[\Phi(z) \sin ly\, e^{ik(x-ct)}\right]; l = n\pi/L$
- Unstable for $\mu = L_d[k^2 + l^2]^{1/2} < \mu_c \approx 2.399 \ldots$
  - where $L_d = NH/f_0$ - Rossby radius of deformation
  - Wavelength of maximum growth $\lambda_m \approx 3.9L_d$
  - Maximum growth rate $\sigma_{max} = [kc_i]_{max} \approx \frac{0.31U}{L_d} = \frac{0.31\Lambda f_0}{N} \approx \frac{f_0}{\sqrt{Ri}}$

Eady model of baroclinic instability

Growth rate/phase speed + wave structures (Vallis 2017)

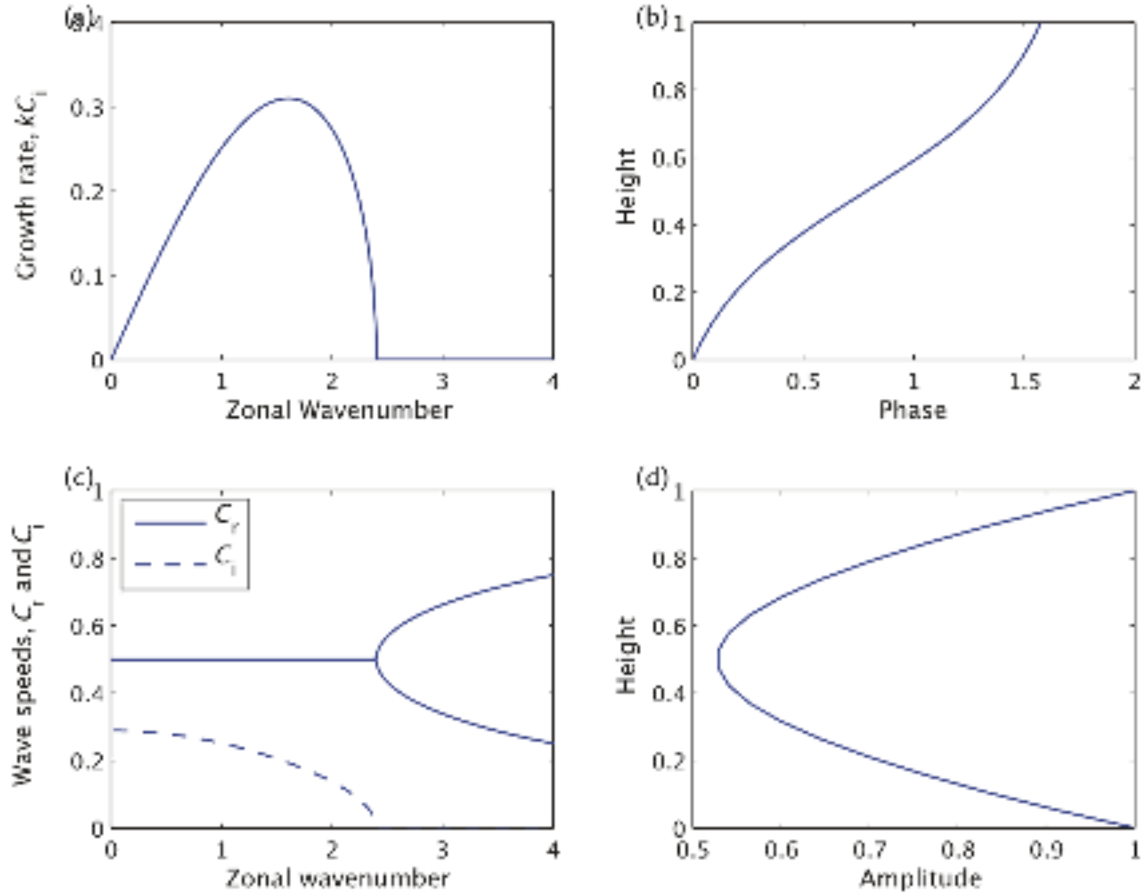


Fig. 9.10

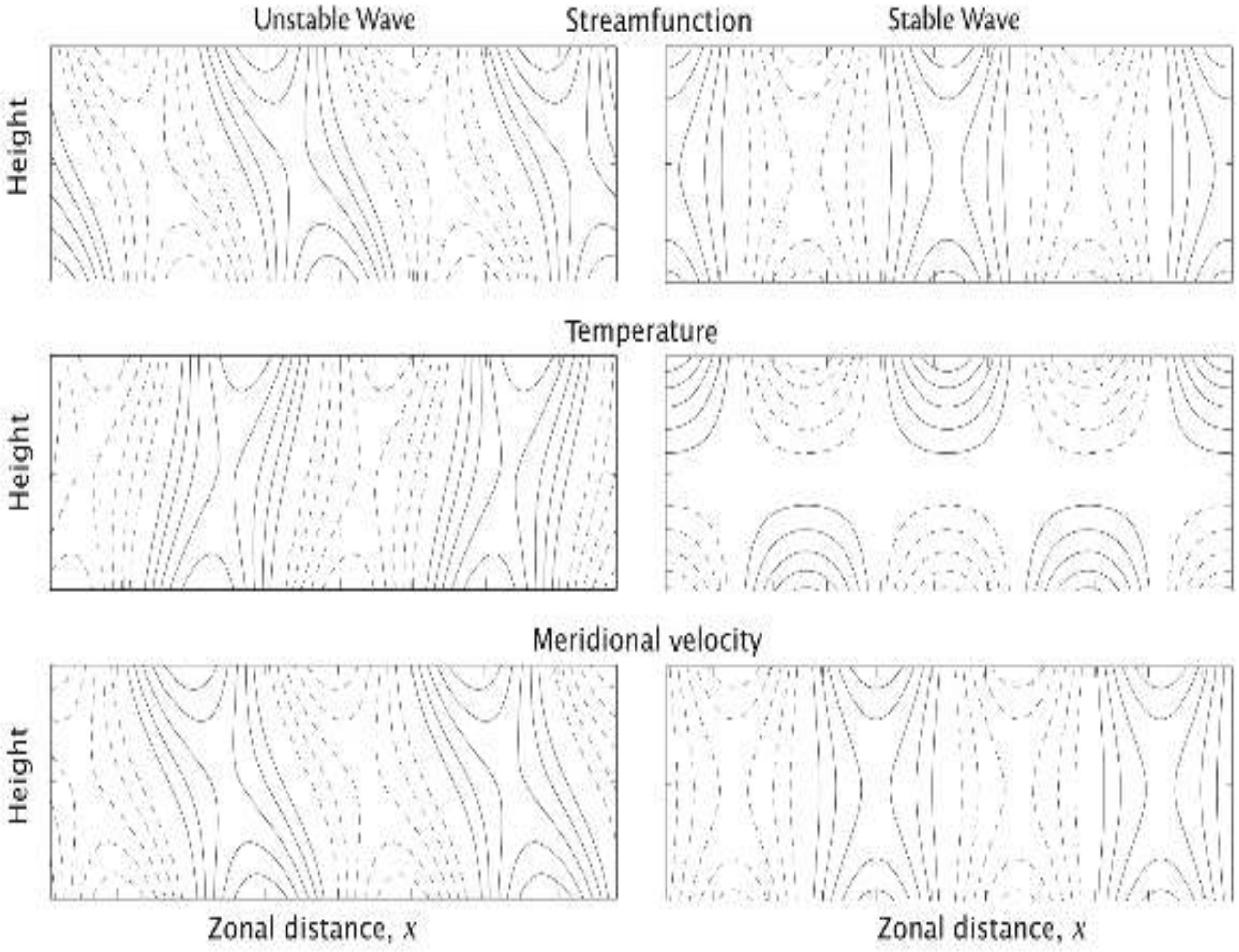
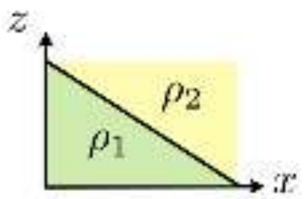


Fig. 9.12

Wave structures (from Vallis 2017)

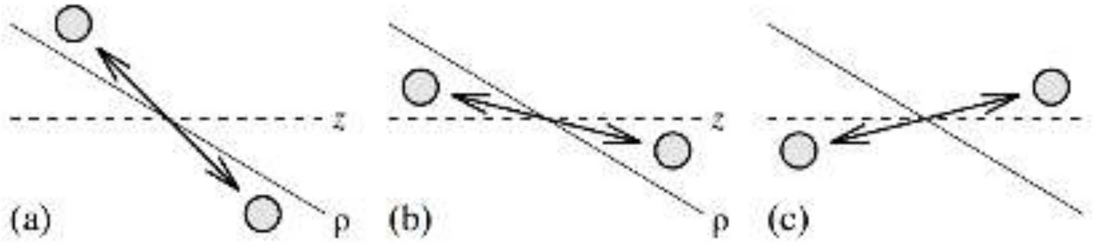
Physics & energetics of baroclinic instability



$\rho_1 > \rho_2$,

$\Delta P = -gV(\rho_2 - \rho_1)(z_2 - z_1)$.

3 cases of interest:

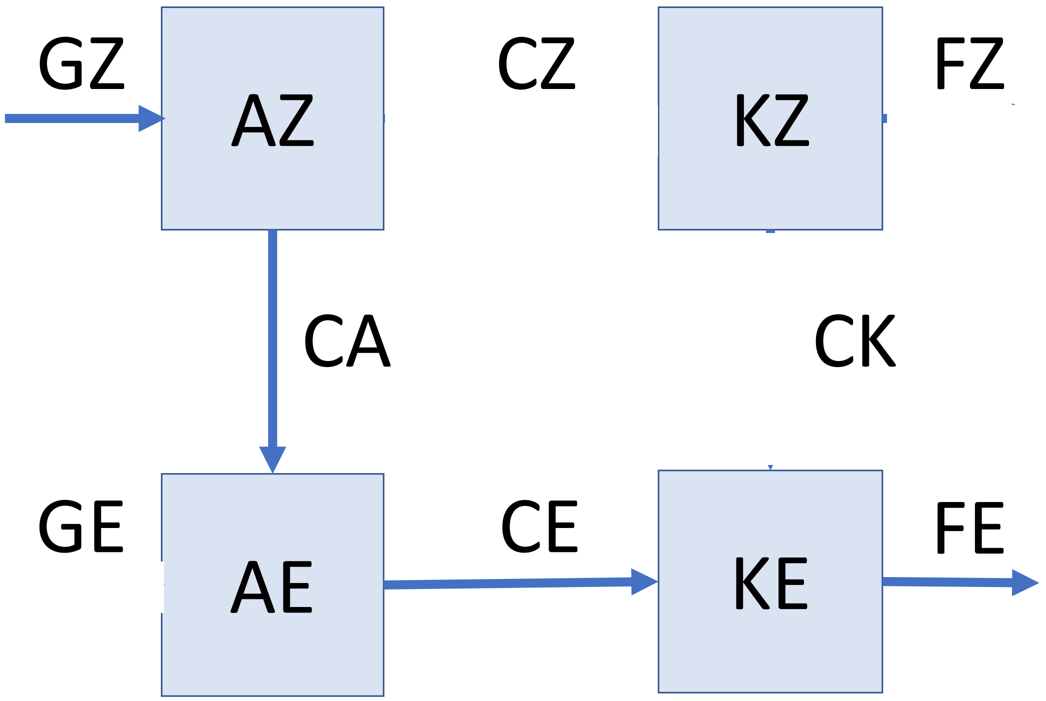


(a) $\Delta P > 0$,
need energy input,
⇒ stable.

(b) $\Delta P < 0$,
releases potential energy.
⇒ possibly unstable.

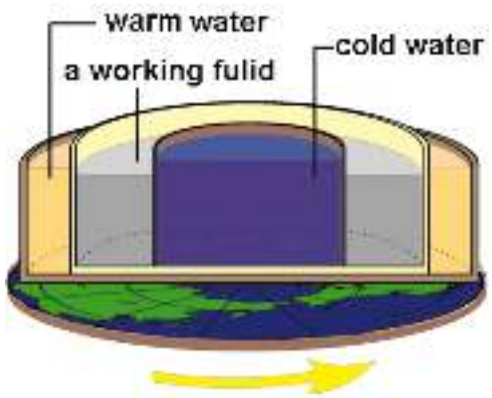
(c) $\Delta P > 0$,
need energy input,
⇒ stable.

Lorenz energy cycle



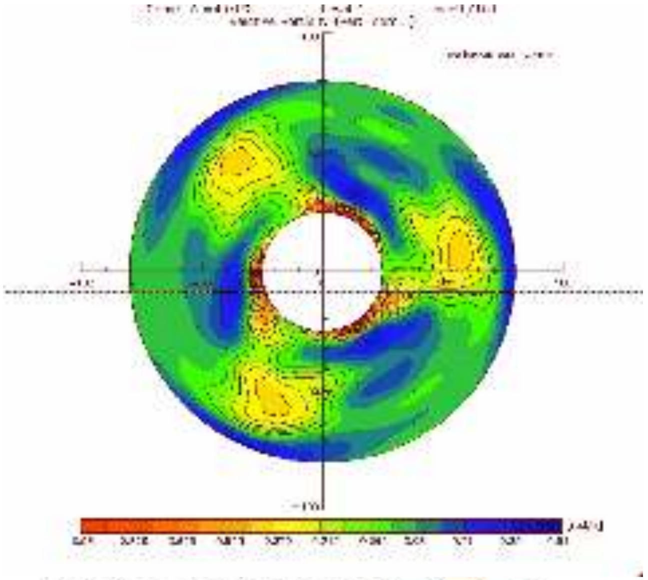
Conversion route for baroclinic instability

Baroclinic instability in the laboratory:
the rotating annulus experiment



- *Baroclinic instability*

- a potential energy releasing instability in the atmosphere and oceans

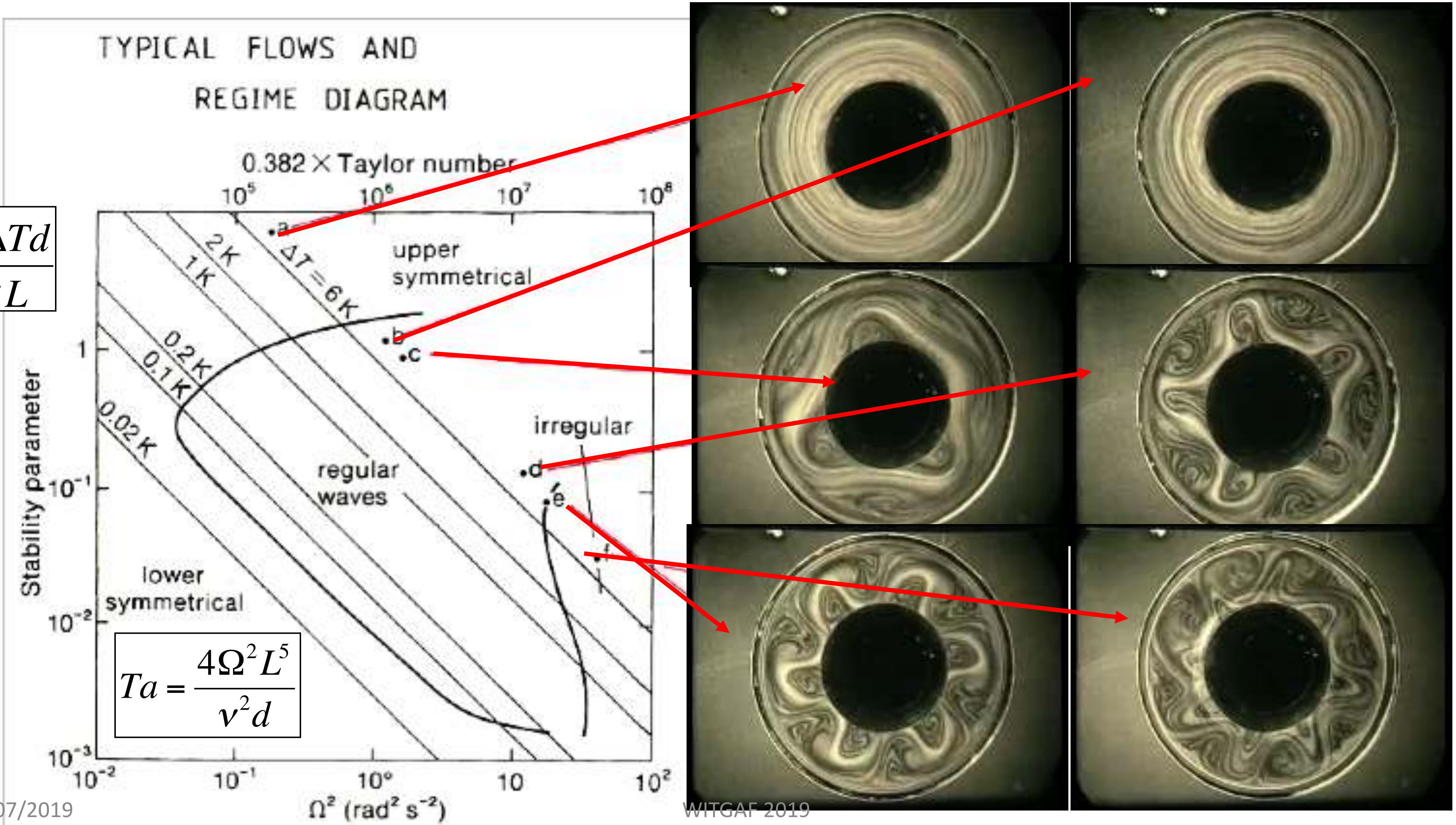


CIRCULATION REGIMES

The rotating annulus experiment

Flow patterns [Pfeffer et al. - FSU]

$$\Theta = \frac{g\alpha\Delta T d}{\Omega^2 L}$$

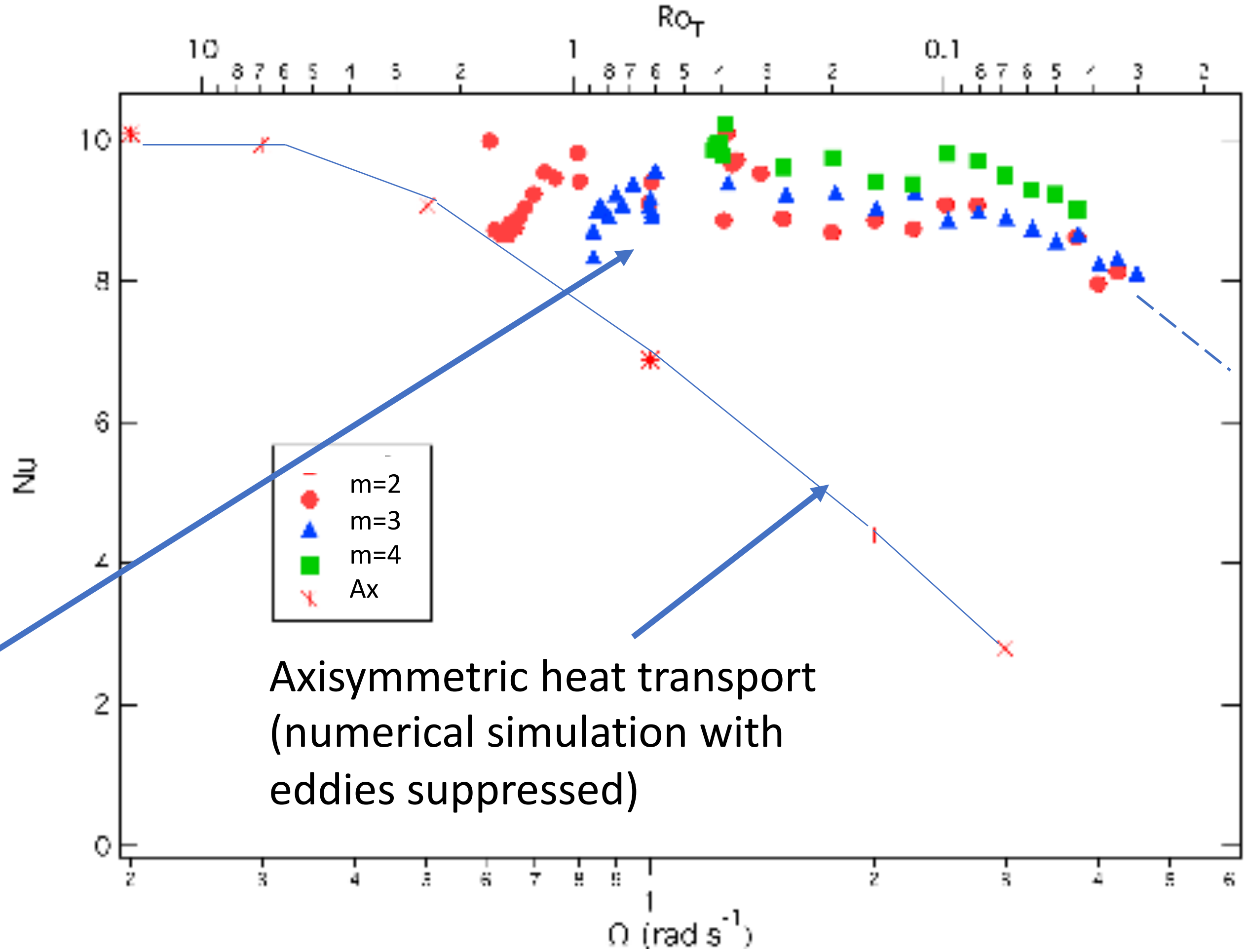


$$Ta = \frac{4\Omega^2 L^5}{\nu^2 d}$$

Nonlinear equilibration

- What happens as instability grows to finite amplitude?
- It modifies its basic state to reduce instability
  - Reduce isotherm slope (releases APE)
  - Mix PV to reduce ∂Q/∂y
  - Sharpen or reduce zonal jets?
- If $\tau_{Adv} = L/U \ll \tau_{Forcing}$, growing perturbations may hold basic state close to marginal instability
  - Baroclinic adjustment?
  - Heat transport by 3D flow ~independent of Ω in weakly supercritical flows….

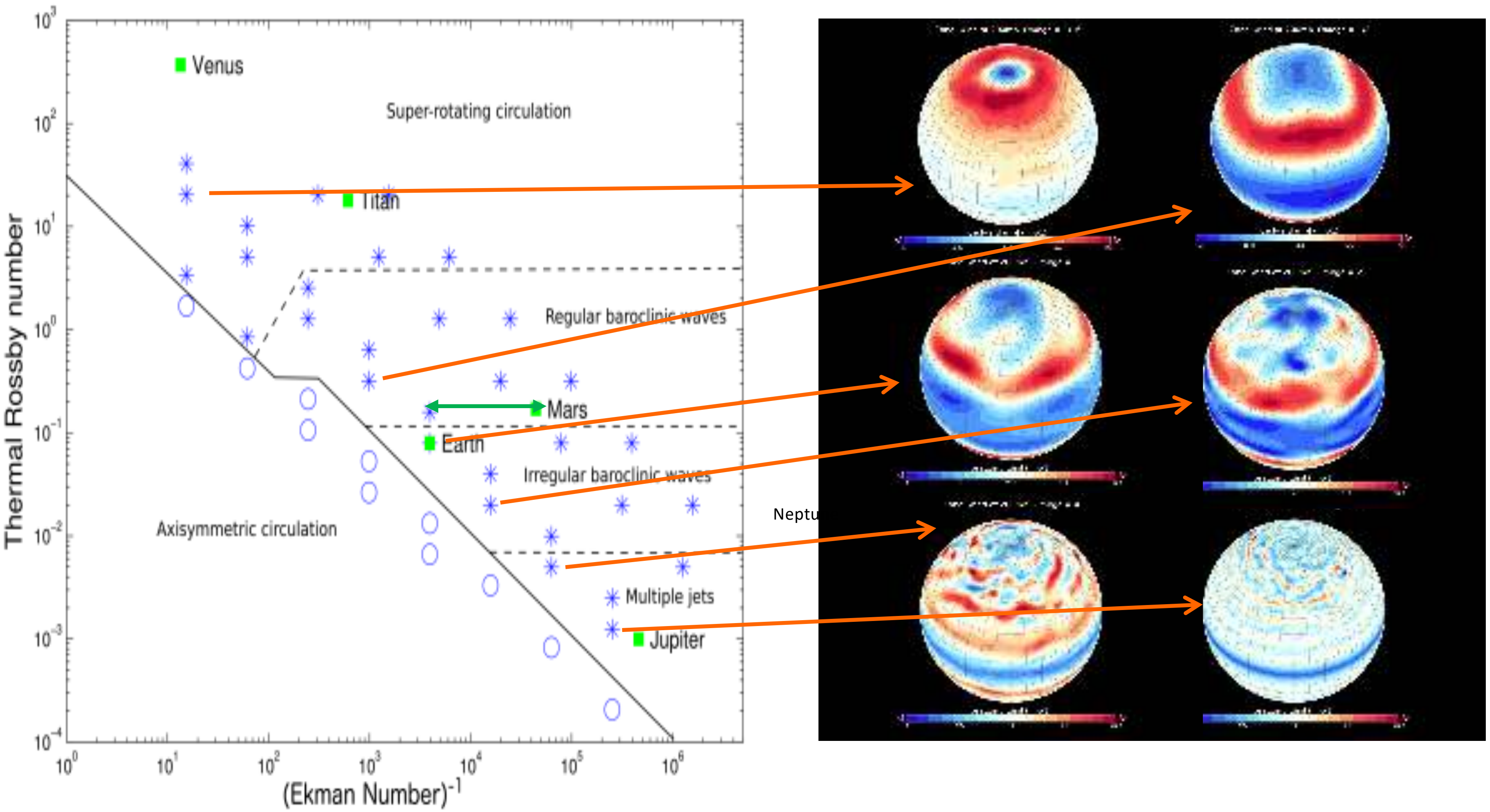
Heat transport (Read 2003 JFM)



Circulation regimes in simple global atmospheric models Wang+ (2018 QJRMS)

Shading is u at 200 hPa

- Simplified atmospheric numerical circulation model (Univ. Hamburg PUMA) of Earth-like planetary atmosphere
- Relaxation towards prescribed temperature *T(ϕ,z)*
- Simple linear surface friction
- No topography, moisture or oceans
- Vary planetary rotation rate (and other parameters)



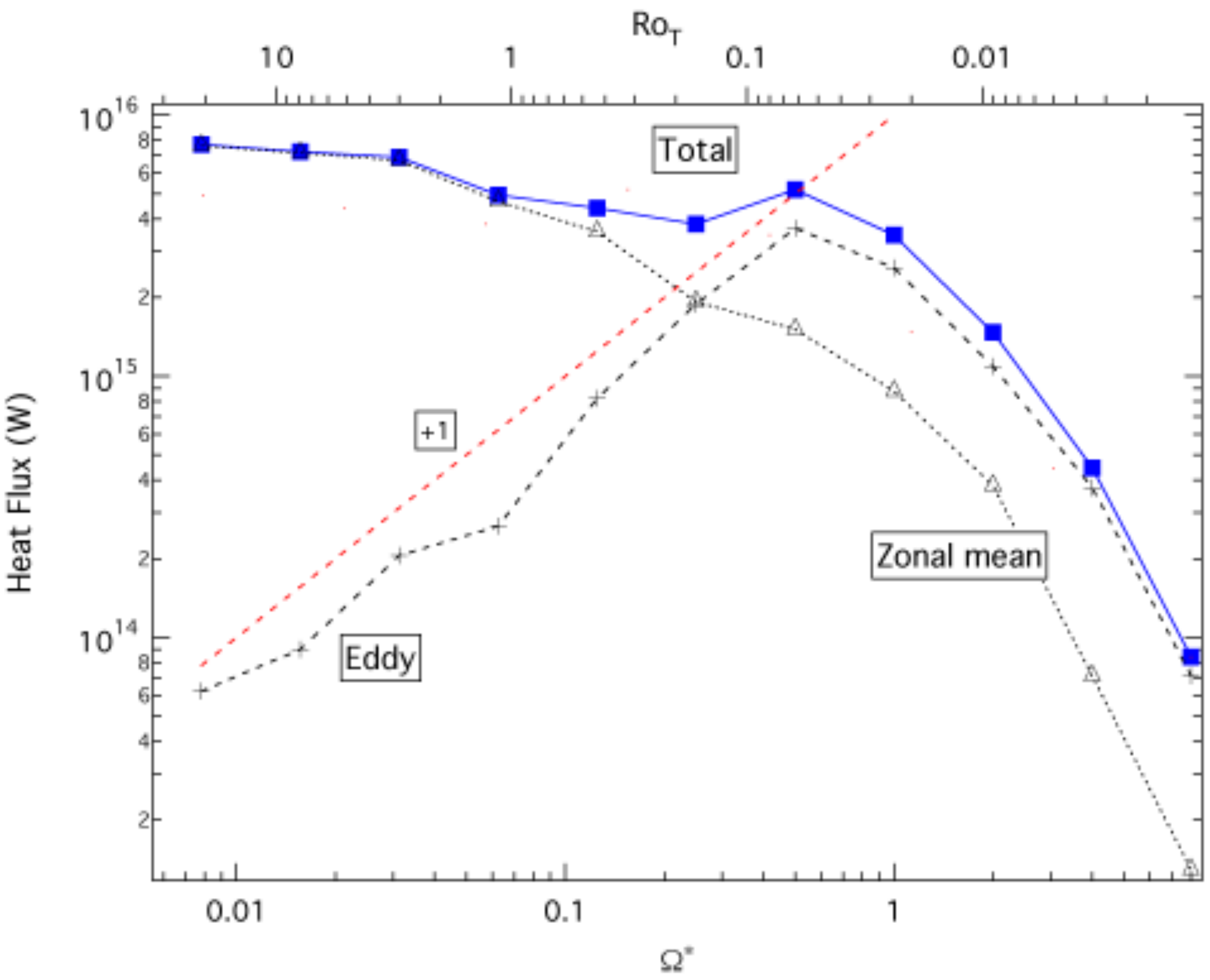
Planetary parameters

Ω/Ω^*	$\Theta = Ro_T$	N_J	$4\Omega^4\tau_R^4$
1/16	20	0.04	1.7×10^5
1/8	5	0.07	2.7×10^6
1/4	1.3	0.14	4.4×10^7
1/2	0.32	0.28	6.8×10^8
1	0.08	1.57	1.1×10^{10}
2	0.02	3.1	1.8×10^{11}
4	0.005	6.3	2.9×10^{12}
8	0.001	14.5	4.6×10^{13}

Cf
- Titan (1.7×10^5)
- Mars[?] (6.8×10^8)
- Earth (1.1×10^{10})
- Uranus & Neptune? (1.8×10^{11})
- Saturn & Jupiter (4.6×10^{13})

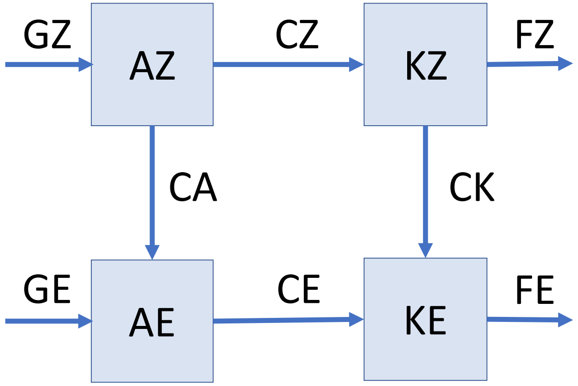
Heat transport & baroclinic adjustment?

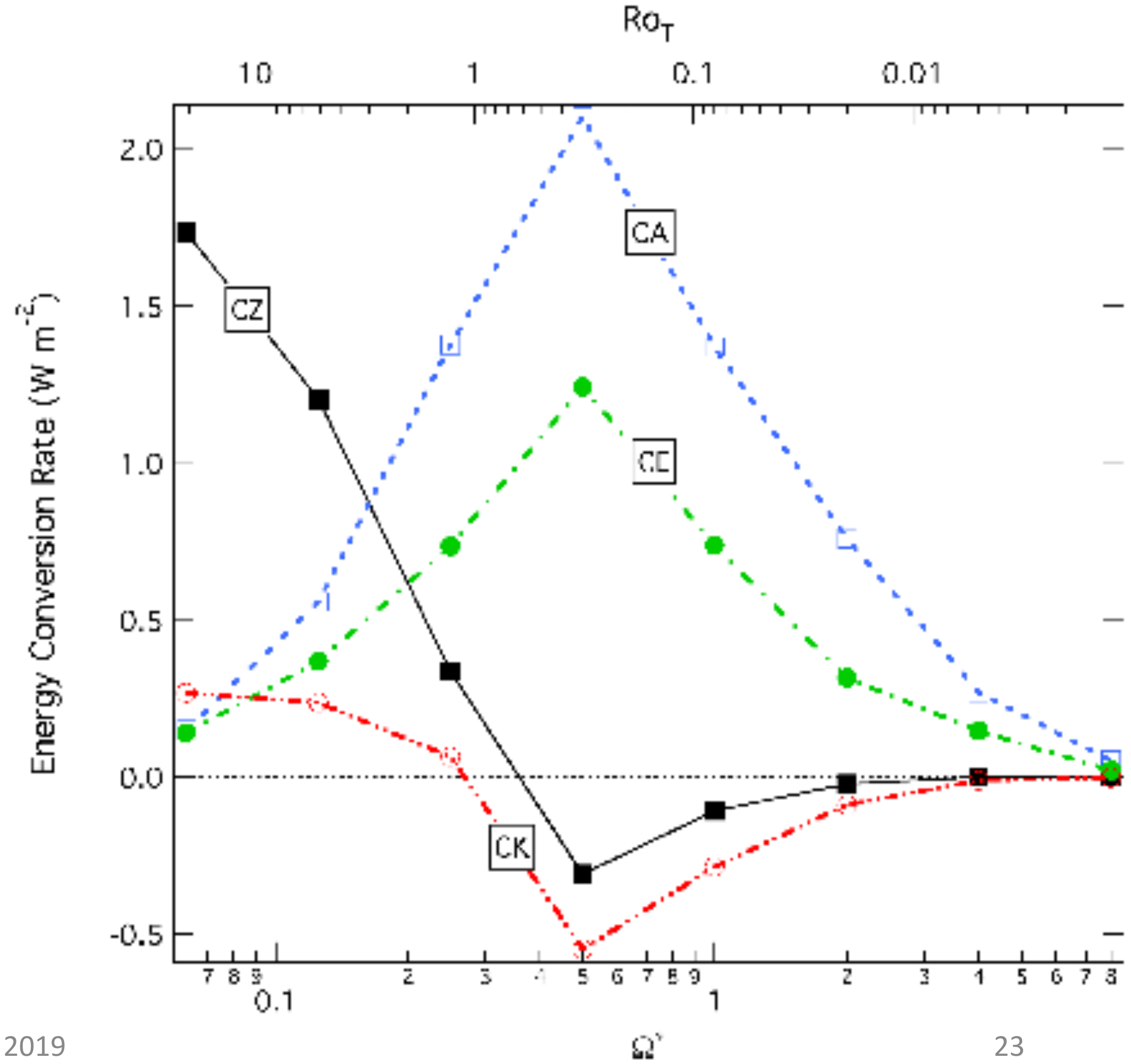
- Vary $\Omega/\Omega_{earth} = \Omega^*$ from 1/128 – 8
- Peak total meridional heat flux ~independent of Ω^* for $\Omega^* < 1$
    - Growth in eddy heat transport compensates for decrease in zonally symmetric transport with Ω
    - Eddy heat transport $\propto \Omega^*$ for $\Omega^* < ½$ ($Ro_T > 0.1$)
    - A form of baroclinic adjustment?
- NB eddies always present…?
    - Unlike in the lab….?
    - Baroclinic or barotropic?



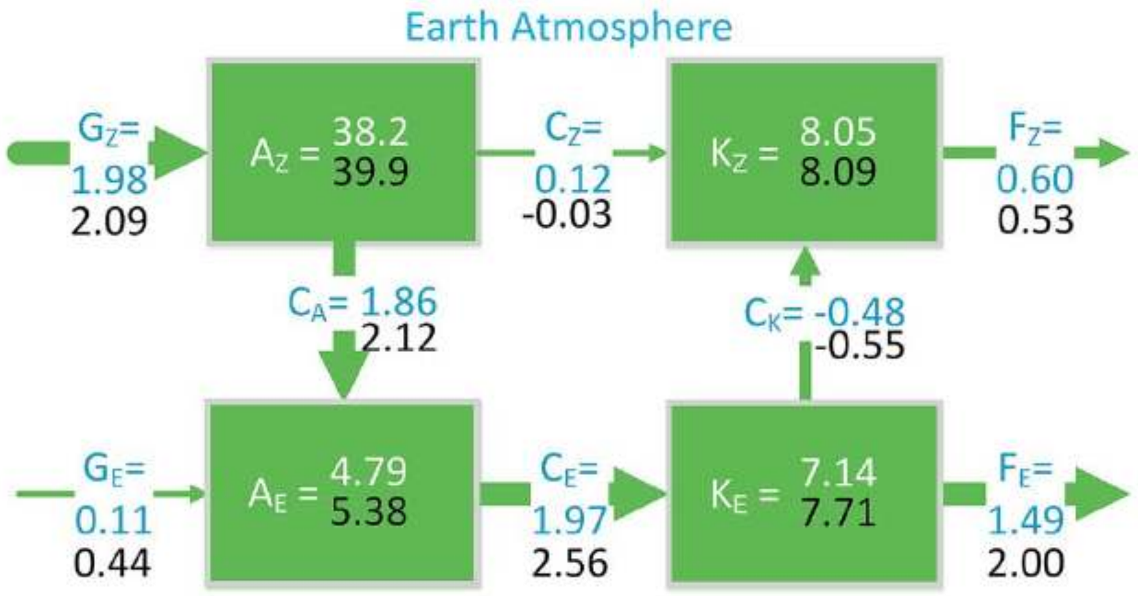
Eddies in global circulation: baroclinic or barotropic?

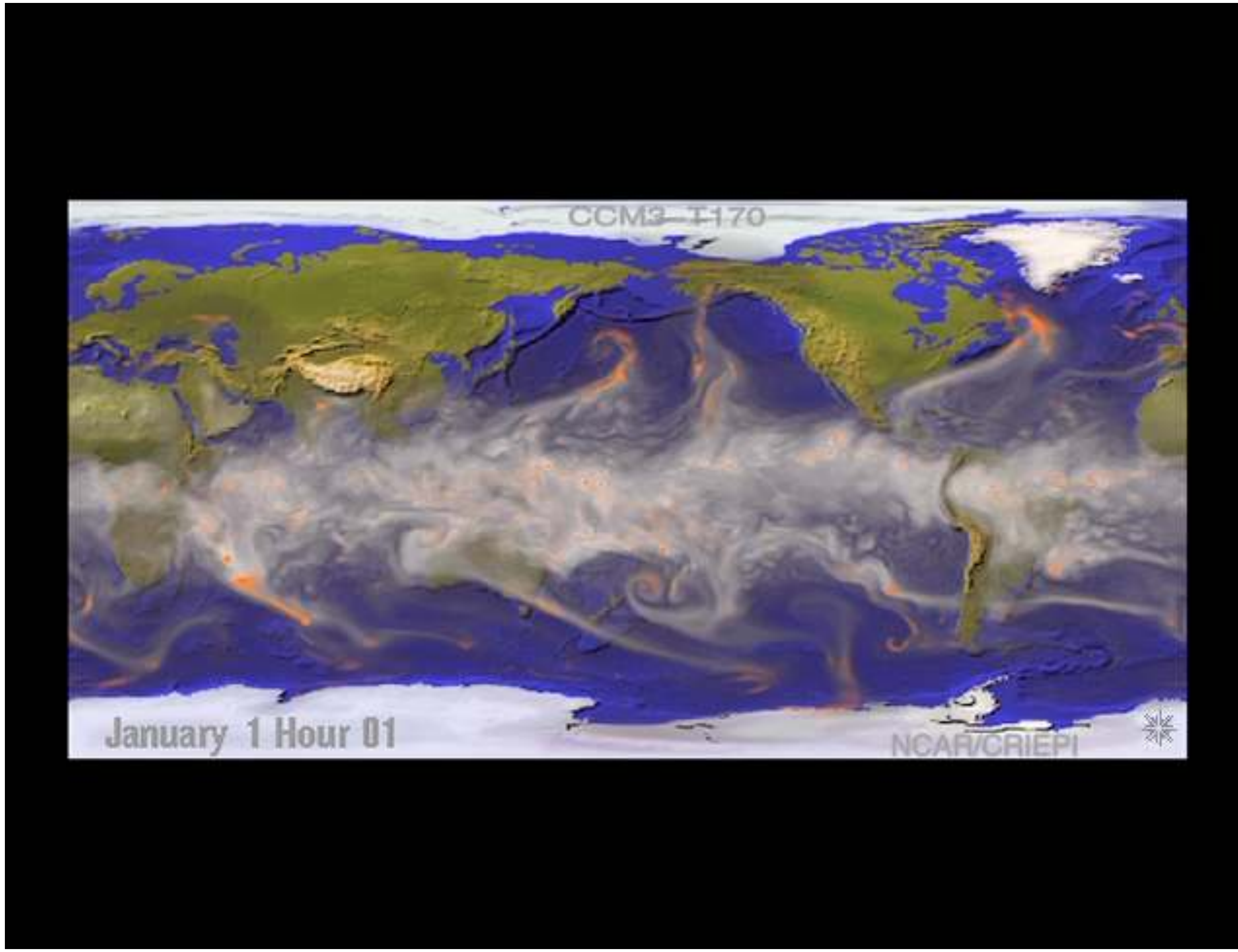
- NB eddies always present…?
  - Baroclinic or barotropic?
- Eddies are mainly barotropic in character for $Ro_T >> 1$ (CK > CE > 0)
- Eddies are mainly baroclinic in character for $Ro_T << 1$ (CK < 0 and |CK| < CE)





Baroclinic instabilities in Earth's atmosphere?

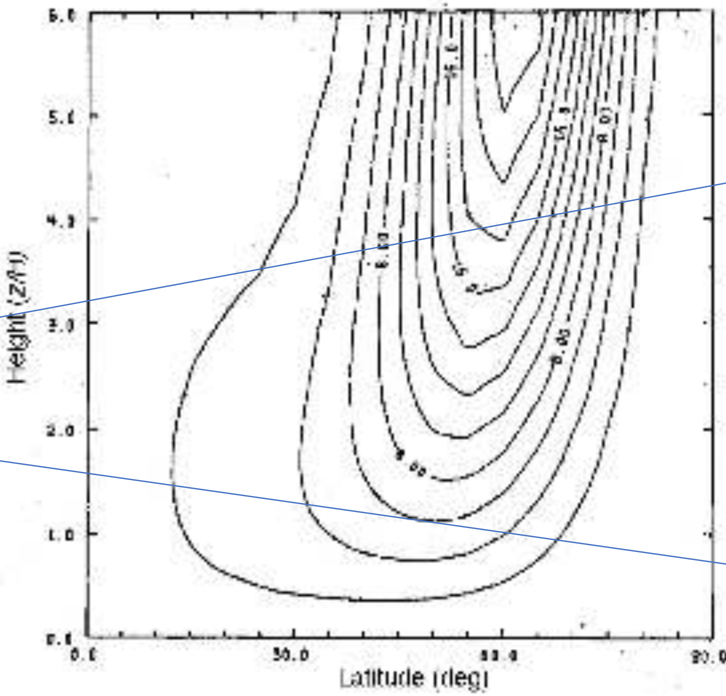




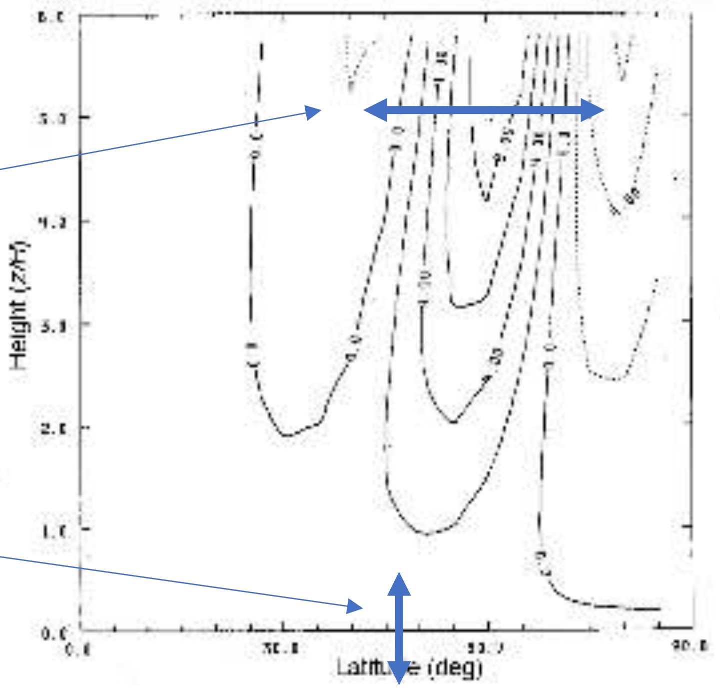
- For Earth's troposphere, U ~ 10 m s^{-1}, H ~ 10 km, N ~ 10^{-2} s^{-1}, f_0 ~ 10^{-4} s^{-1}
- Hence, L_d ~ *1000 km*
- λ_{max} ~ 3.9 L_d ~ 4000 km
- σ_{max} ~ 0.31 $\Lambda f_0/N$ ~ 0.5 day^{-1}

Baroclinic instability in the Martian atmosphere:
Barnes (1984) *J. Atmos. Sci.*, **41**, 1536-1550

- Realistic mid-latitude zonal jet
- Q_y changes sign in latitude, and at the ground
  - Instability criteria i. or iii.
  - Baroclinic and/or barotropic instability?
- For Mars, $N \sim 10^{-2}$ s^{-1}, $H \sim 10$ km, $f_0 \sim 10^{-4}$ s^{-1} and $\Delta U \sim$ 50 m s^{-1}
- -> $\lambda_{max} \approx 3.9\frac{NH}{f_0} \approx$3900 km;
- $\tau_{max} \sim \sigma_{max}^{-1} = 3.2\frac{N}{f_0 \Lambda} \approx$ 17 hours



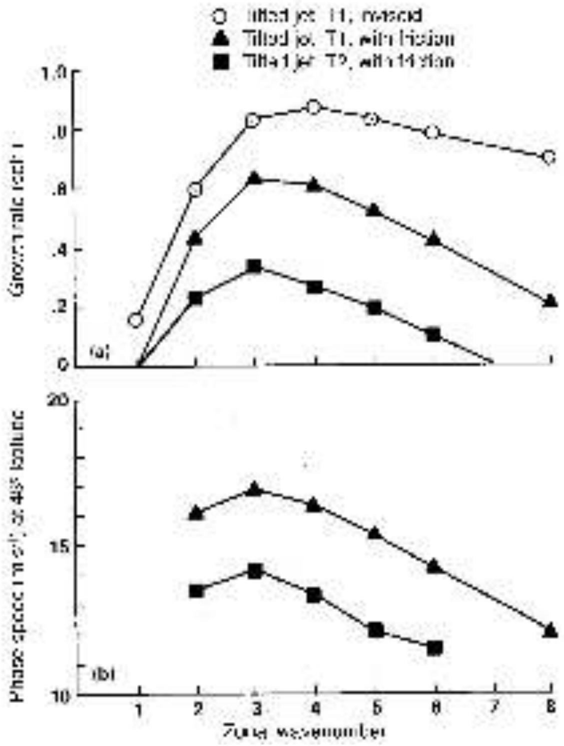
- Zonal wind U

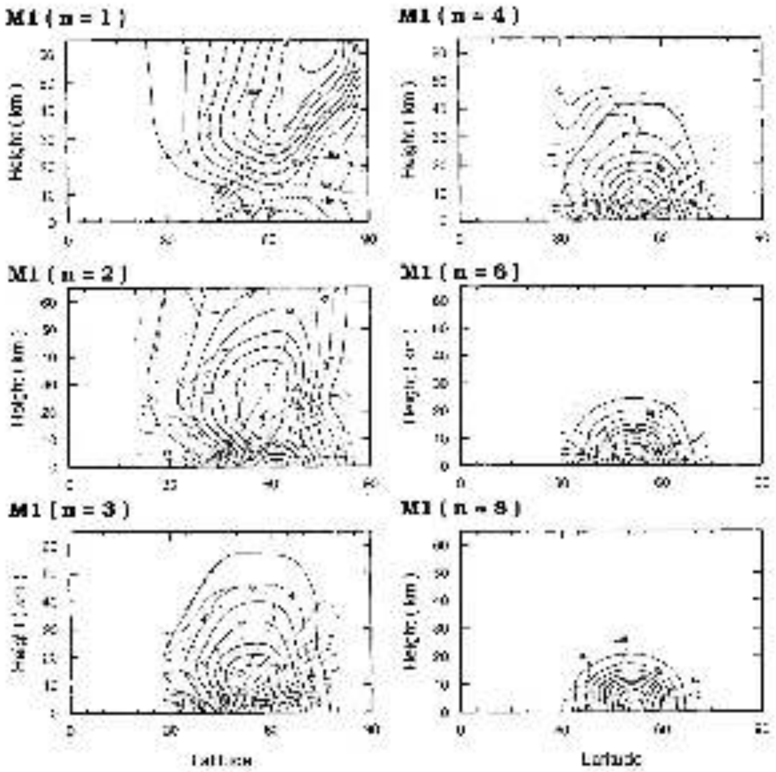


∂Q/∂y

‘Realistic’ baroclinic instability on Mars

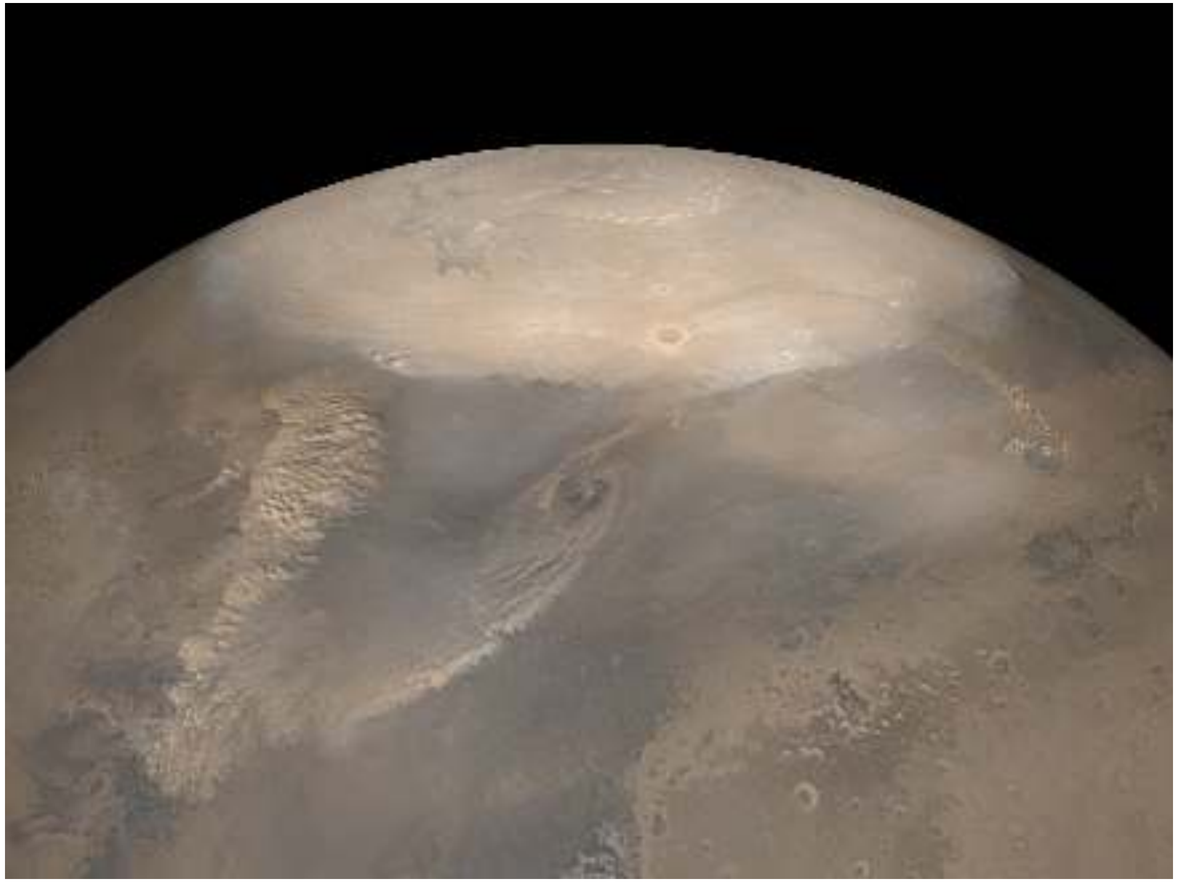
[Tanaka & Arai (1999) *Earth Plan. Space*, **51**, 225-232]

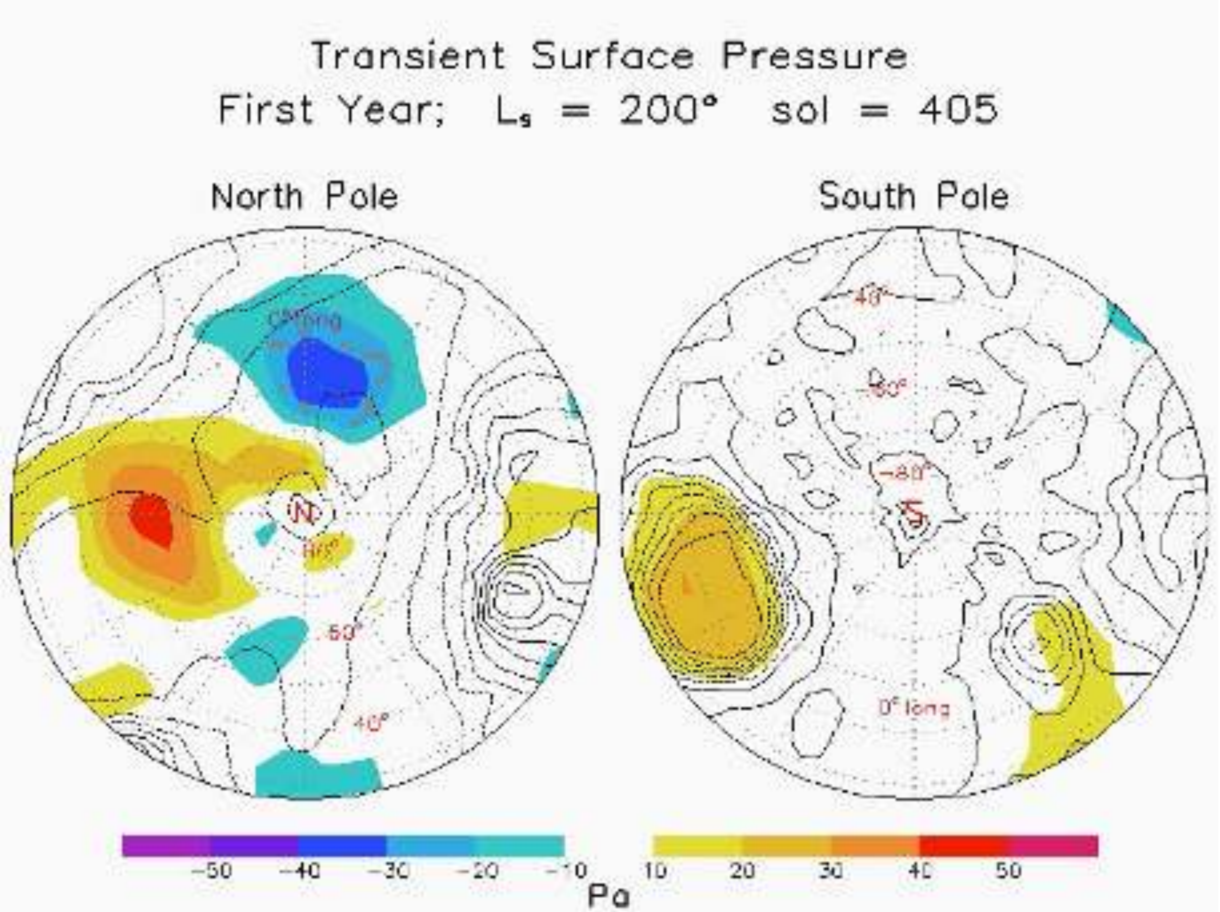




- Maximum growth around *m = 2*-3
- Low *m* growing waves are deep *internal modes*
- Higher *m* growing waves are shallow *external modes*

Baroclinic/barotropic instabilities on Mars

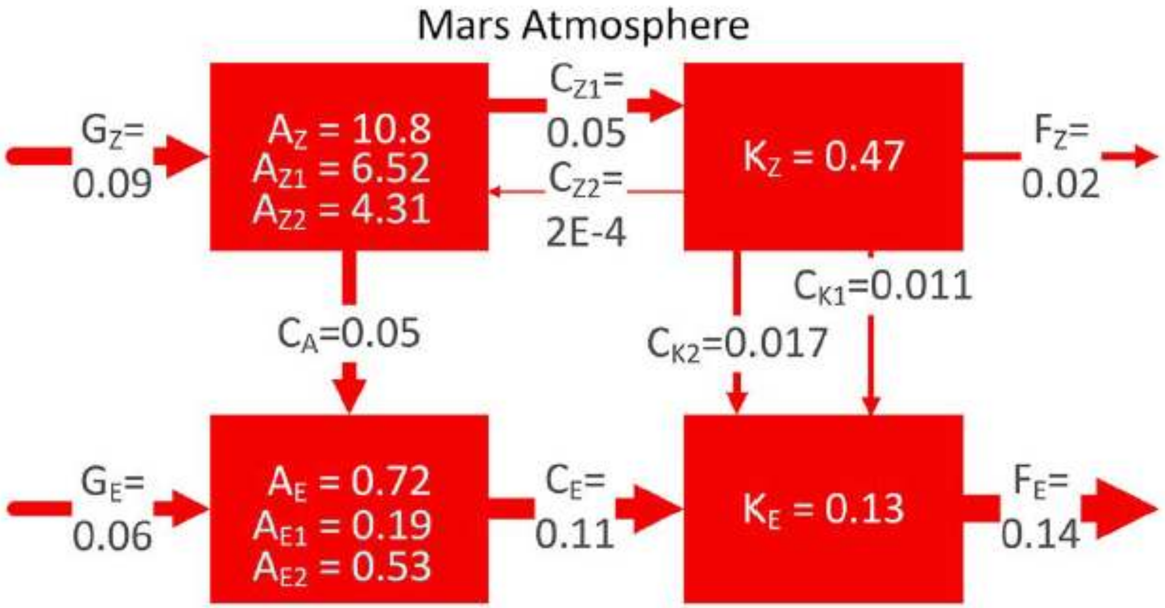


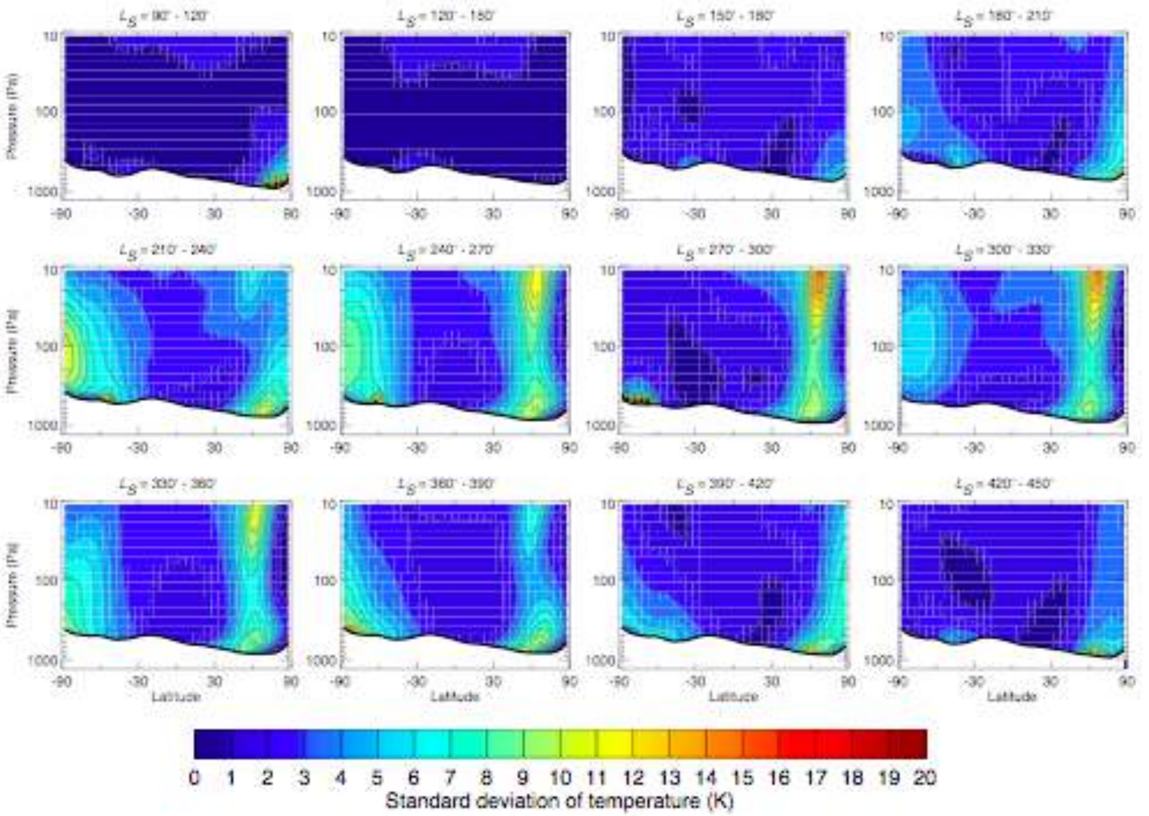


North circumpolar storms (visualized by dust storms!)
[Credit: NASA/JPL]

Travelling cyclonic storms analysed for surface p
[Lewis et al. 2007 *Icarus*]

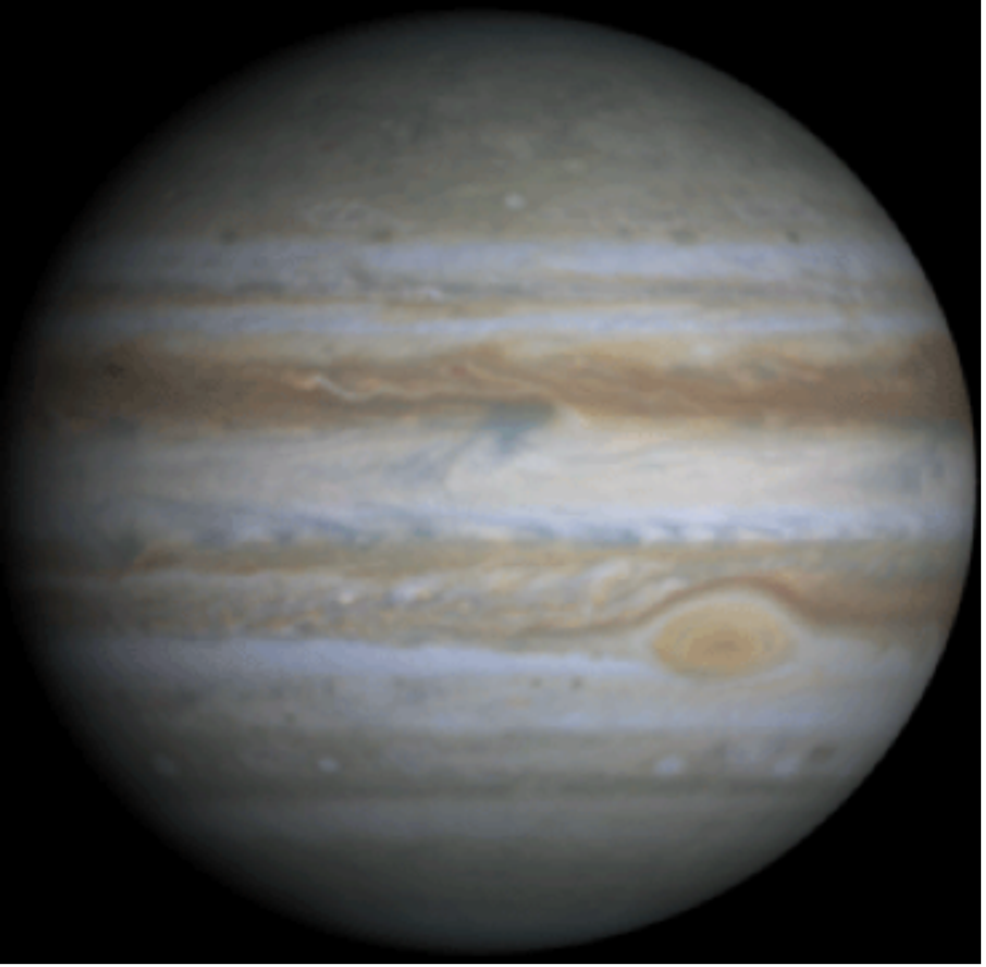
Baroclinic/barotropic instabilities on Mars





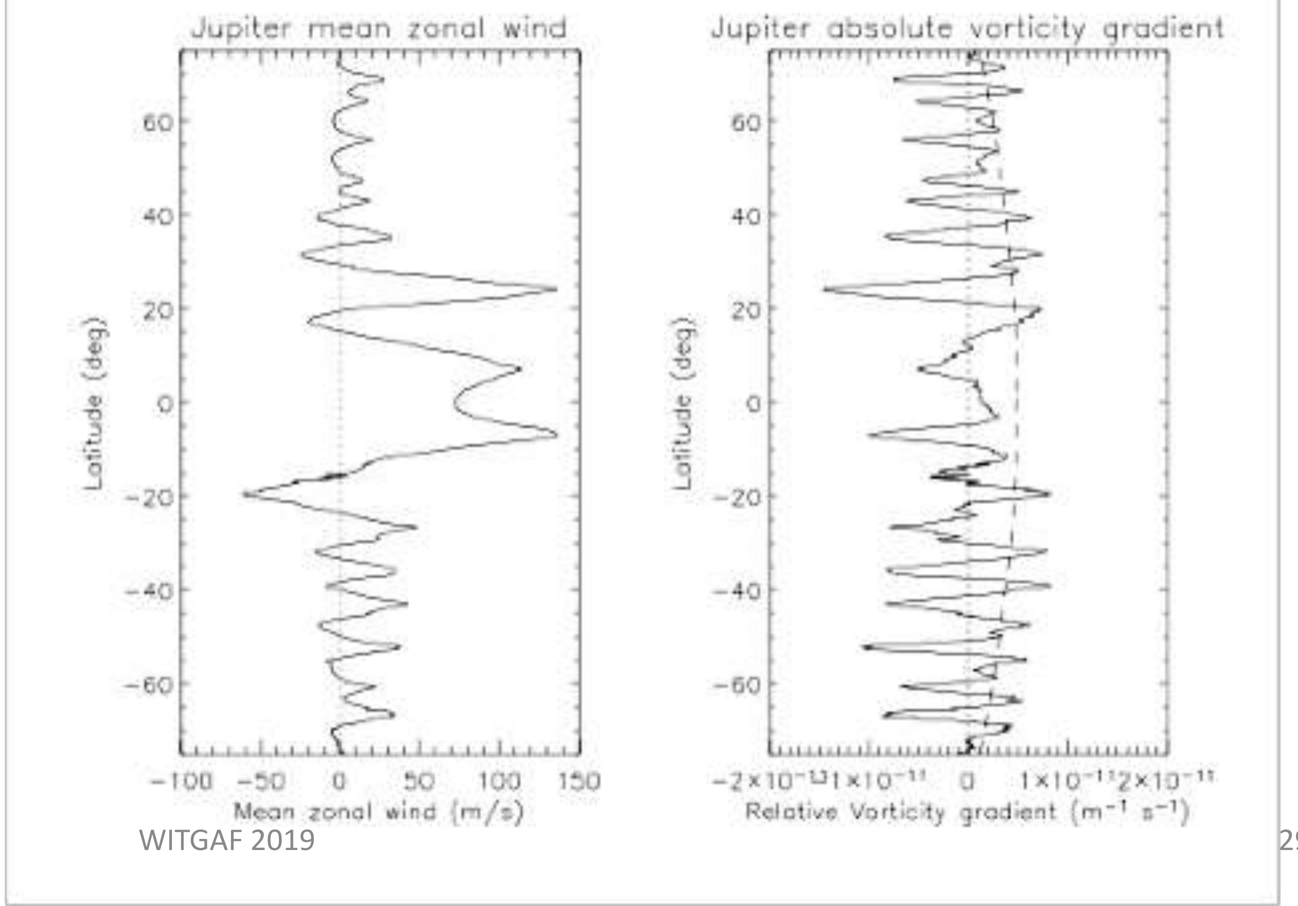
Baroclinic transient $<T'^2>^{1/2}$ vs season

- Lorenz energy budget (from observations – Tabataba-Vakili et al. 2015) in J/W m^{-2}
- Conversions dominated by baroclinic instability terms + barotropic instability –> a mixed case!



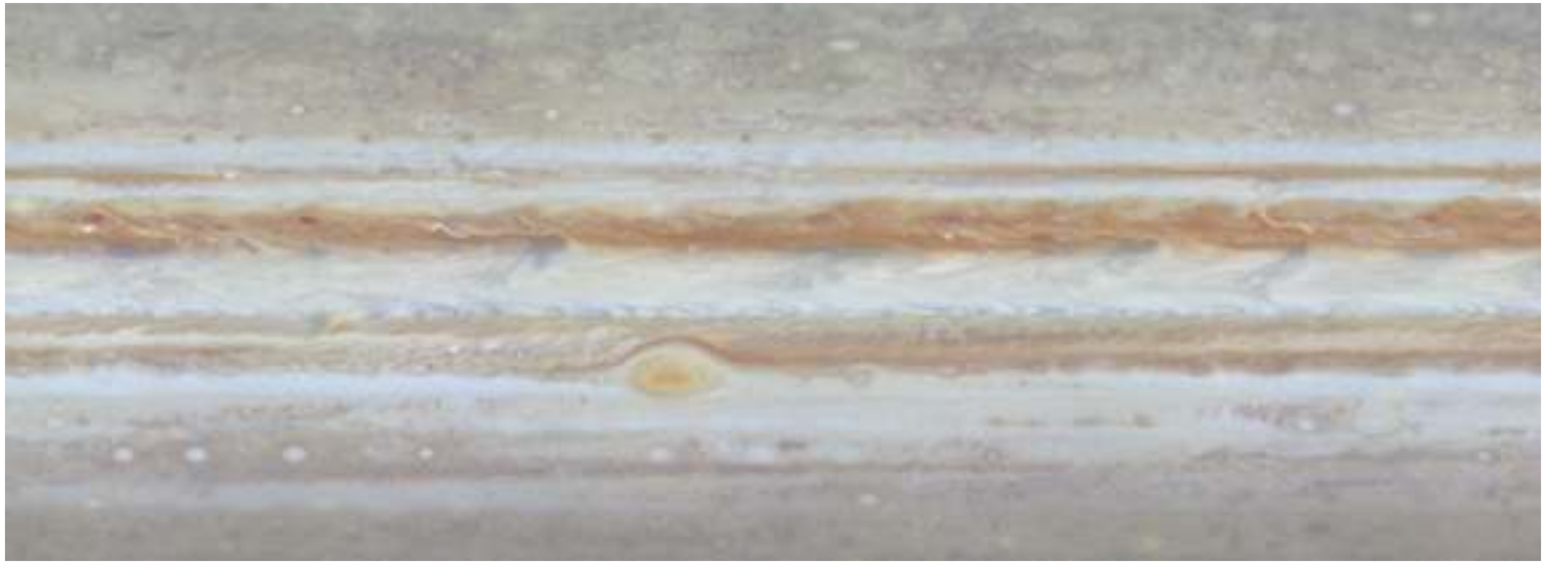
Barotropic and baroclinic instabilities on Jupiter (and Saturn)?

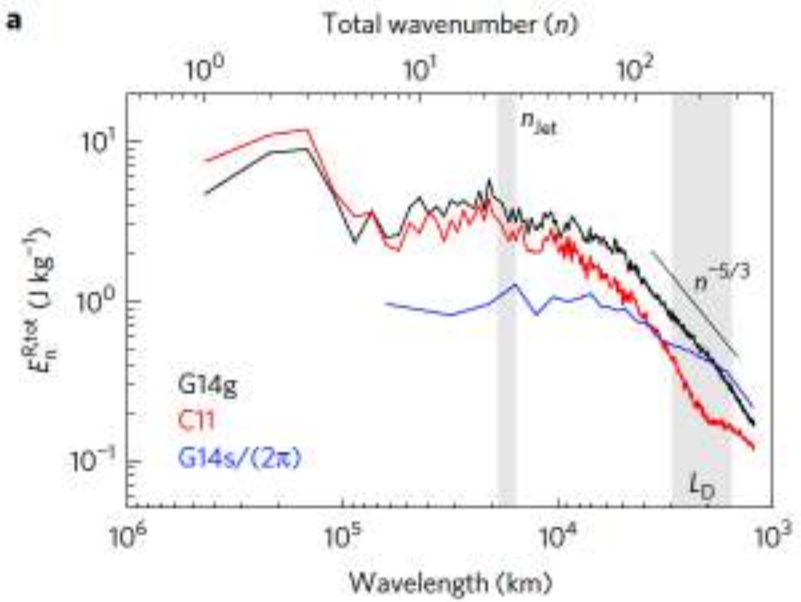
Cassini ISS winds (Porco et al. 2003)

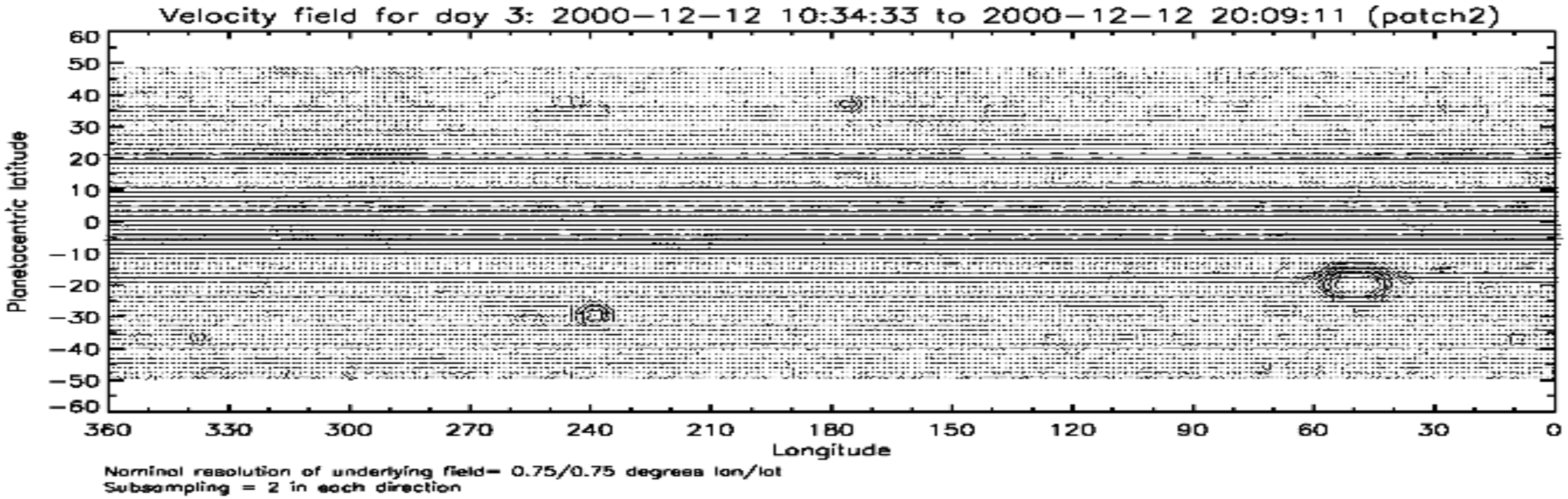


- $\beta - u_{yy} < 0$ in easterly jets
- Barotropically unstable…?

Jupiter cloud motions







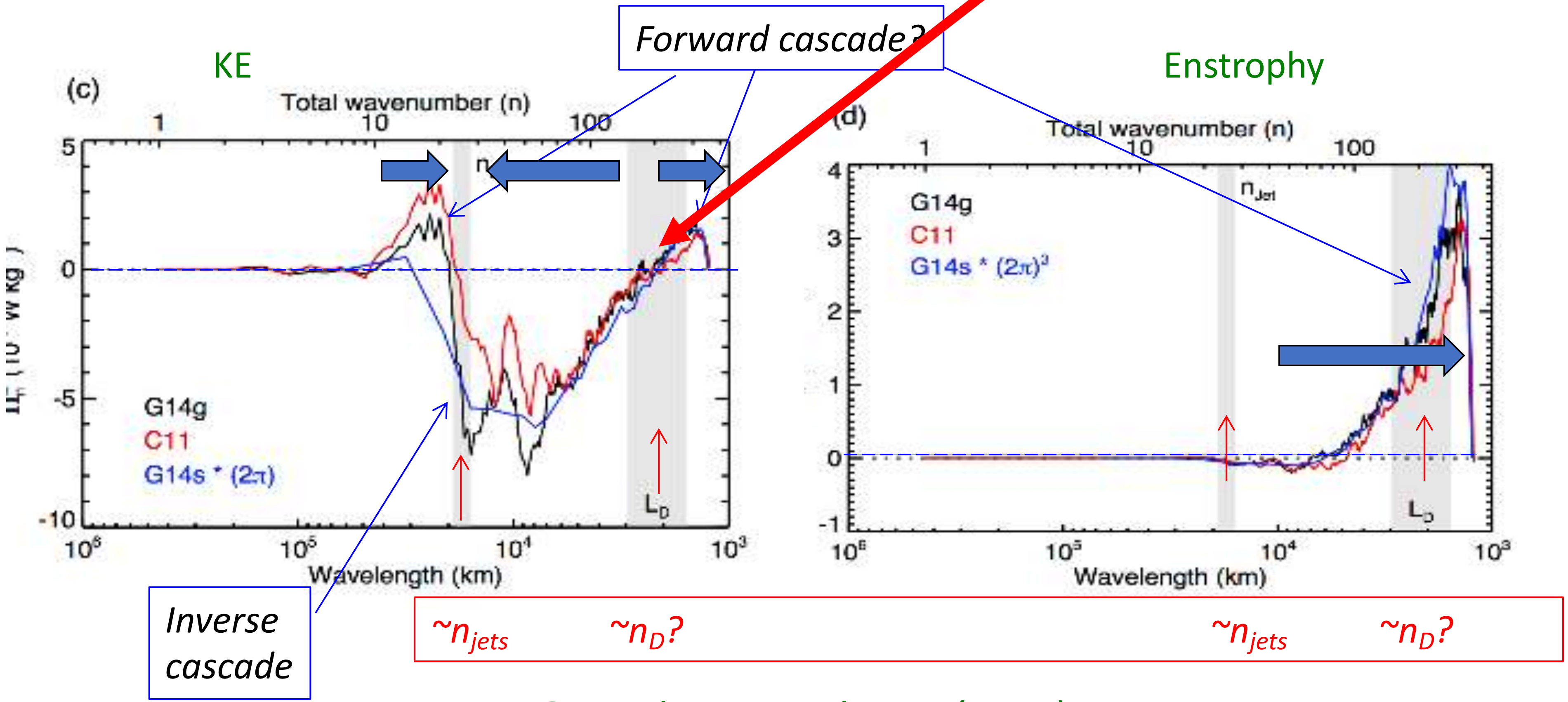
- 2D KE spectrum
- Projected onto spherical harmonics of total order n

Jupiter: KE spectrum & spectral fluxes

NB Source of KE at length scales ~ L_d

- Baroclinic instability?

[Rotational flow: Boer & Shepherd 1987]



Young & Read *Nature Physics (2017)*

Baroclinic instability on Jupiter or Saturn?

- No solid surface, so C-S-P criterion iii. not valid!
- $\partial Q/\partial y \neq 0$ in free atmosphere [Read et al. 2006;2009]
- Strong tropopause (interface between convective troposphere and stable stratosphere around p ~ 0.3 bar)
- $\Longrightarrow$ C-S-P criteria i. or ii. possible….
  - E.g. ii. Satisfied in westward jets at tropopause [Conrath et al. 1991]

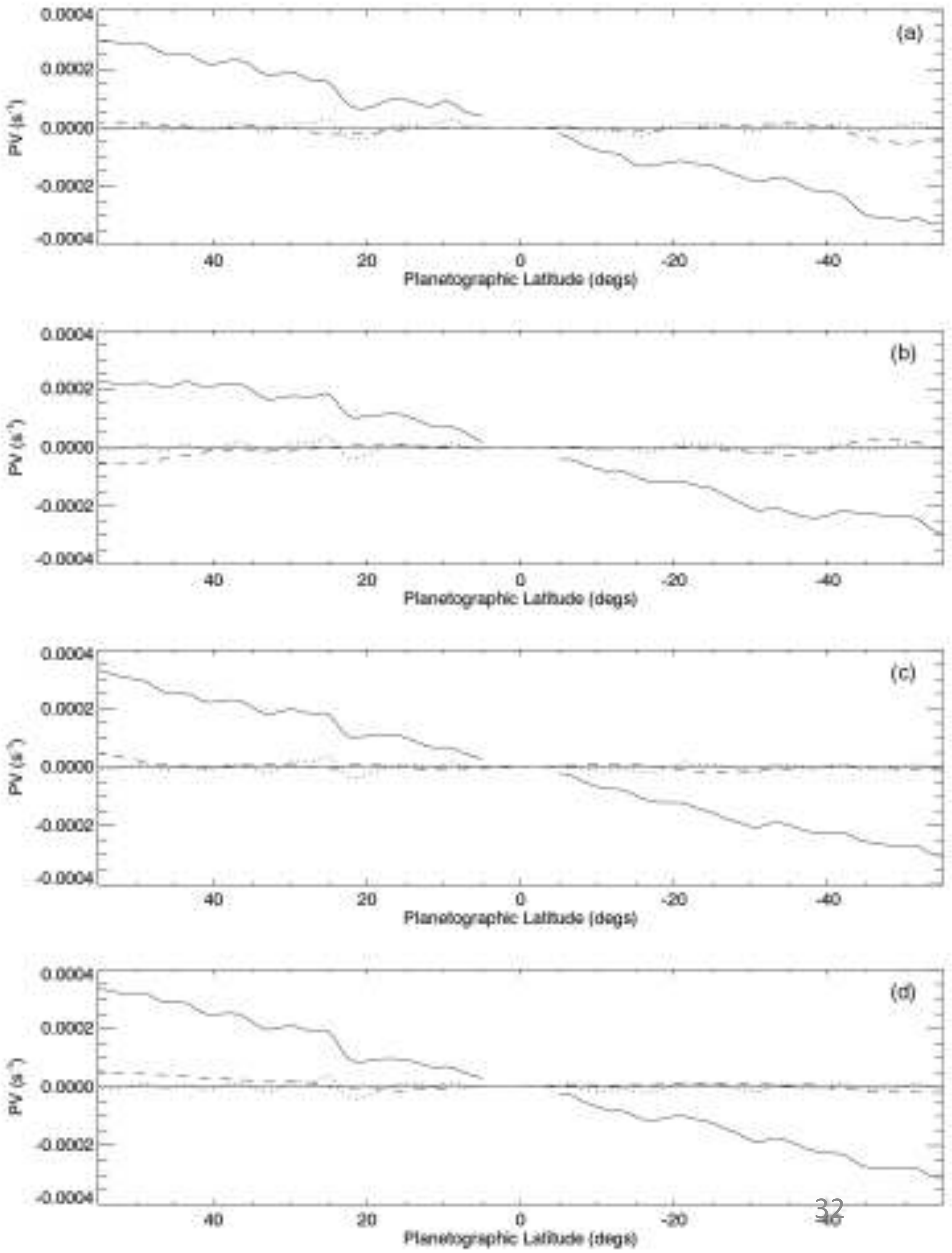
QGPV profiles on pressure levels

2 hPa

10 hPa

118 hPa

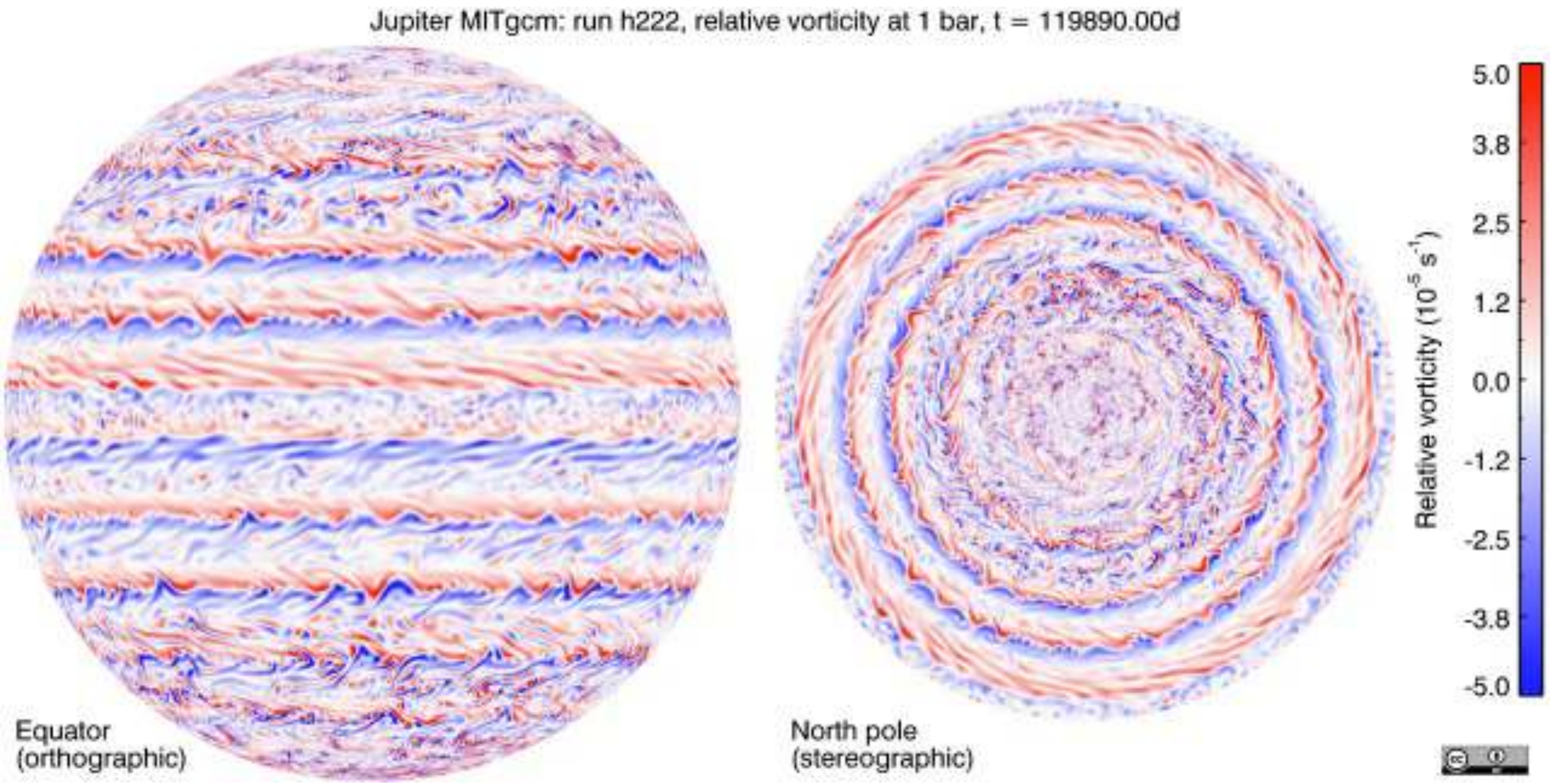
225 hPa



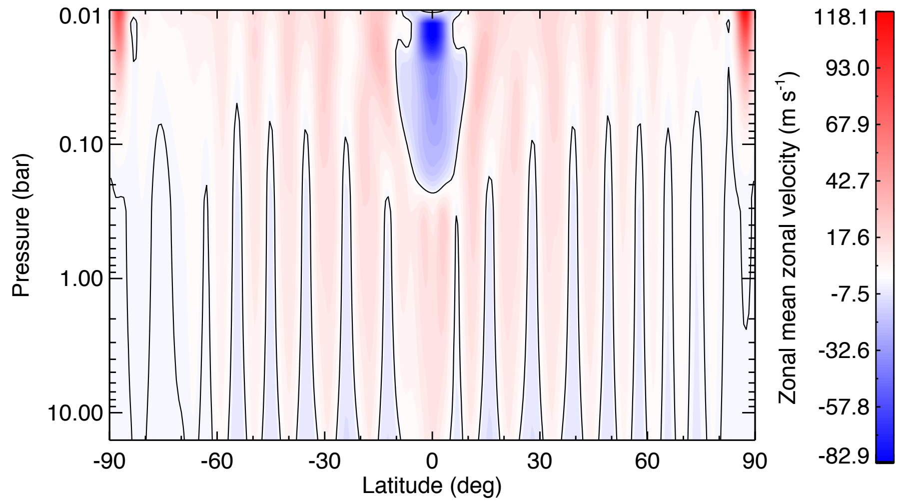
[Run on UK STFC DiRAC supercomputer]

Oxford/MIT-gpm (Young et al. 2019)

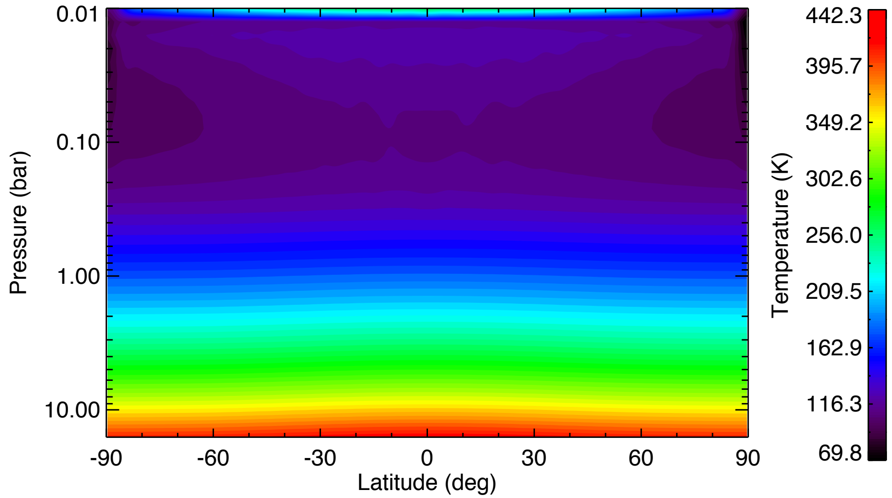
- Global atmospheric circulation model for Jupiter troposphere/stratosphere [~20bar –10mb]
- Based on MITgcm dynamical core
  - 0.7° x 0.7° to 0.3° x 0.3° x 33 vertical levels
  - Weak "MHD" drag at bottom
- 2-band "semi-gray" radiation scheme
- Interior heat flux (uniform w. latitude) = 5.7 W m^{-2}
- Passive condensible clouds
- Moist convection parameterization
  - Zuchowski et al. (2009)



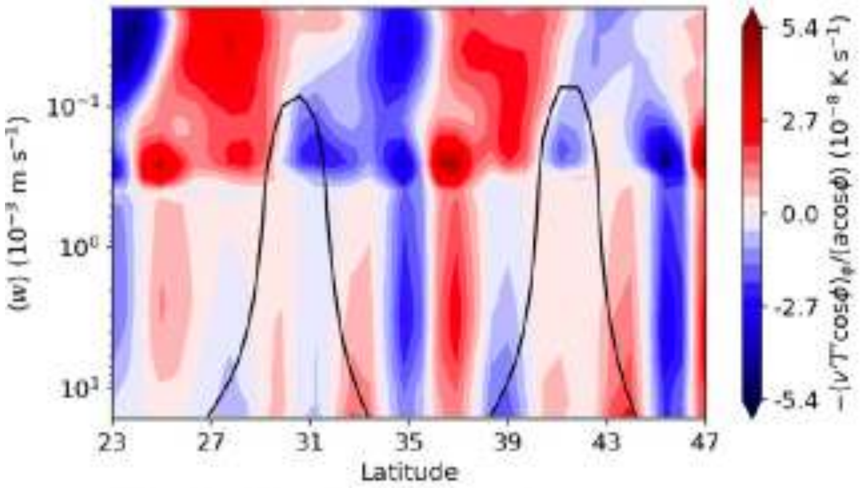
Oxford-gpm: Velocity & temperature



(b) Zonal velocity (B1).



(h) Temperature (B1).



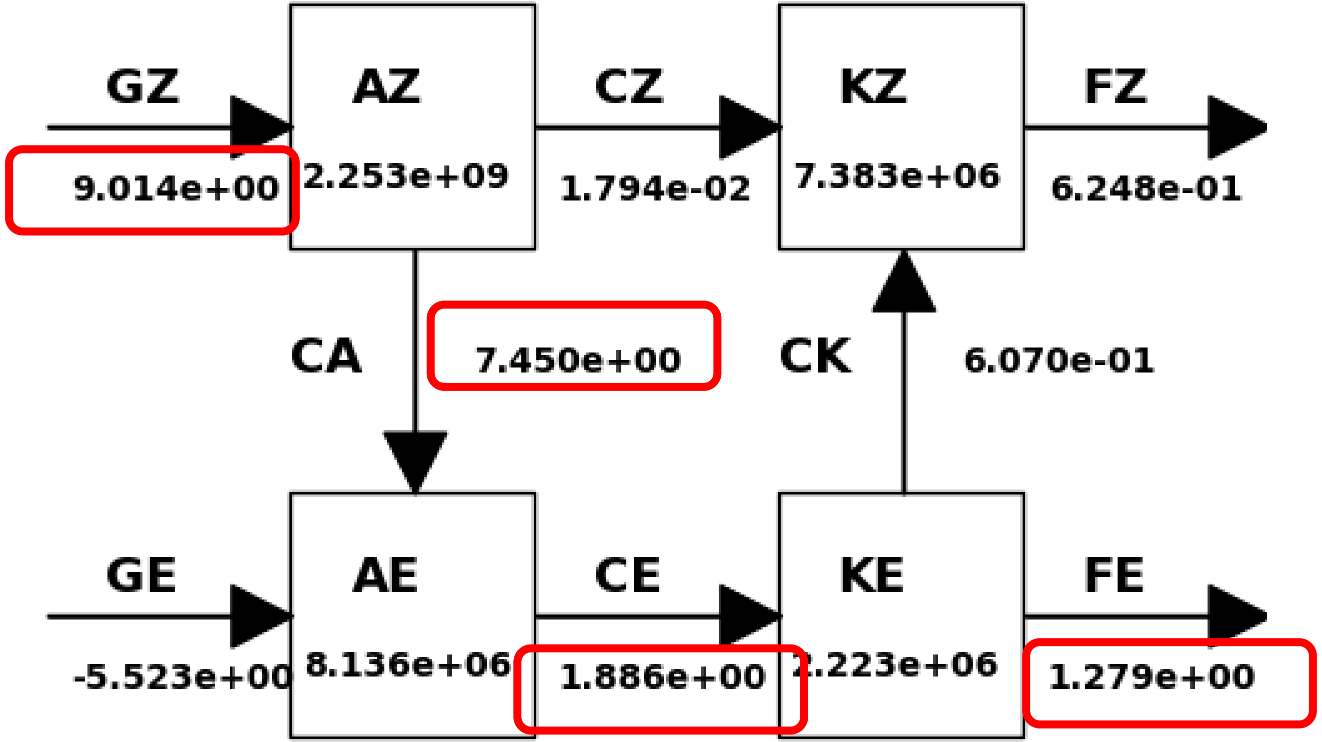
(b) Meridional eddy heat flux convergence.

- Meridional eddy heat flux convergence strongest near tropopause (~300-500 hPa)
- Internal/interfacial baroclinic instability?

Oxford-gpm (Jupiter): Energetics

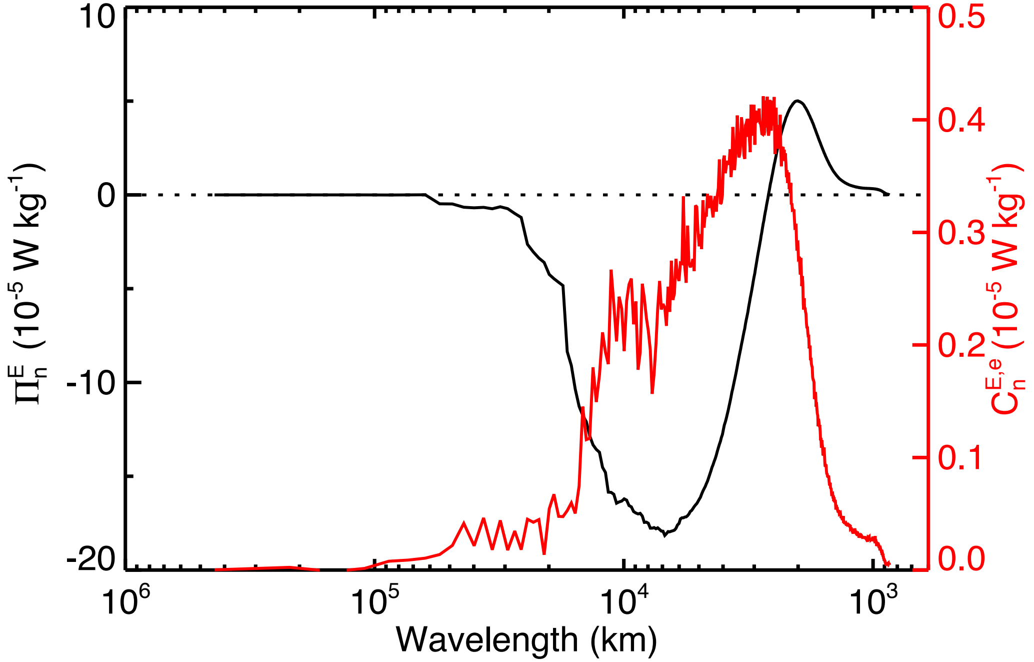
Lorenz energy budget

- Energies in J m^{-2}
- Conversions in W m^{-2}



Kinetic energy spectral flux

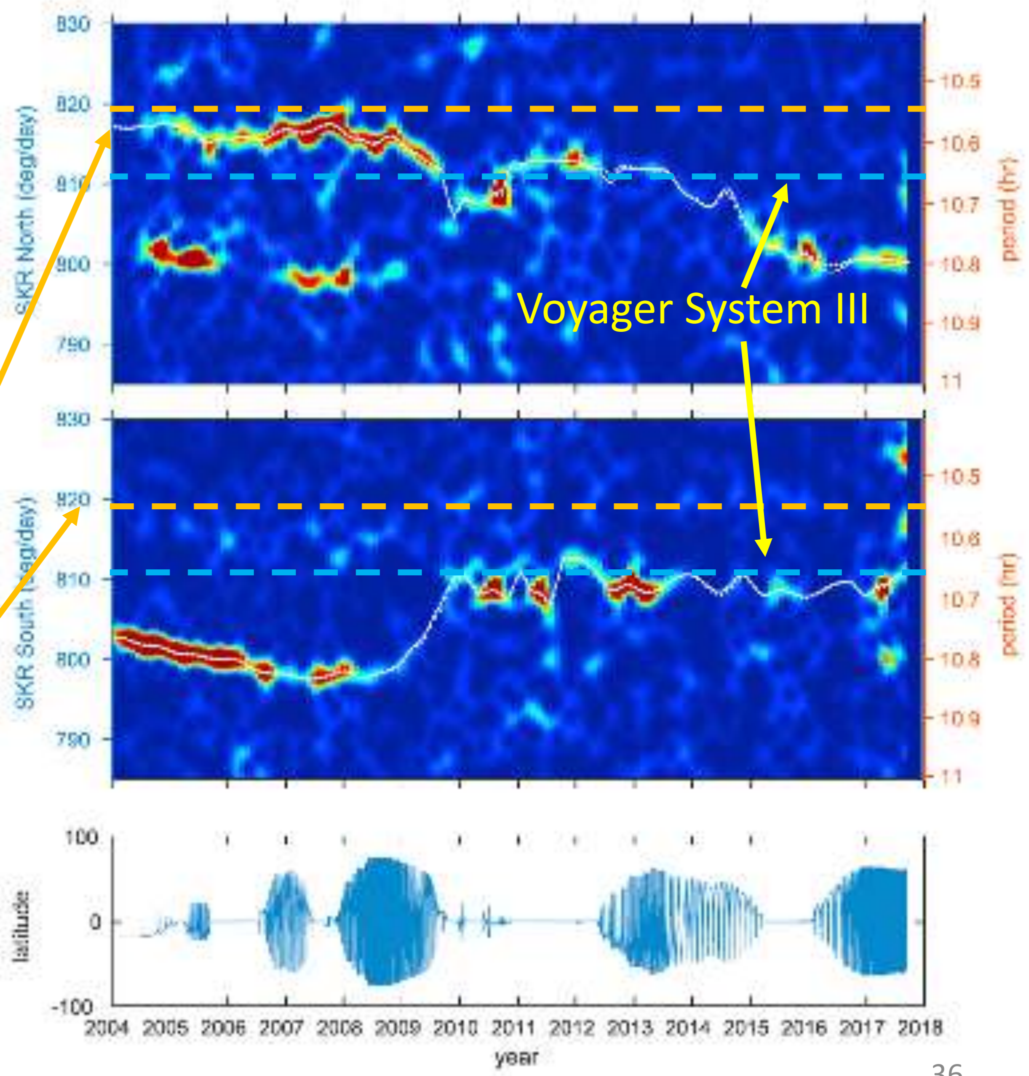
Potential→kinetic energy conversion



- Upscale turbulent cascades at large scales
  - Spectral flux < 0
- DOWNscale at scales < L_D...
  - Spectral flux > 0
- Energised by PE->KE (baroclinic) conversion
  - **Baroclinic instability**

Saturn's interior rotation rate - a mystery?

- Jupiter's interior rotation rate determined to high precision by precession rate of magnetic field
- Saturn's magnetic field dominated by a dipole aligned with its rotation axis (±0.06°)!
  - Periodicity only in very low radio frequency emissions – locked to the interior…..?
  - First measured by Voyager fly-by in 1982
  - Monitored by Cassini orbiter from 2004-2017 and found to vary in time!!
  - Cf rotation period estimated from gravity field and oblateness (Anderson & Schubert 2007)?



From Ye et al. (2018) GRL

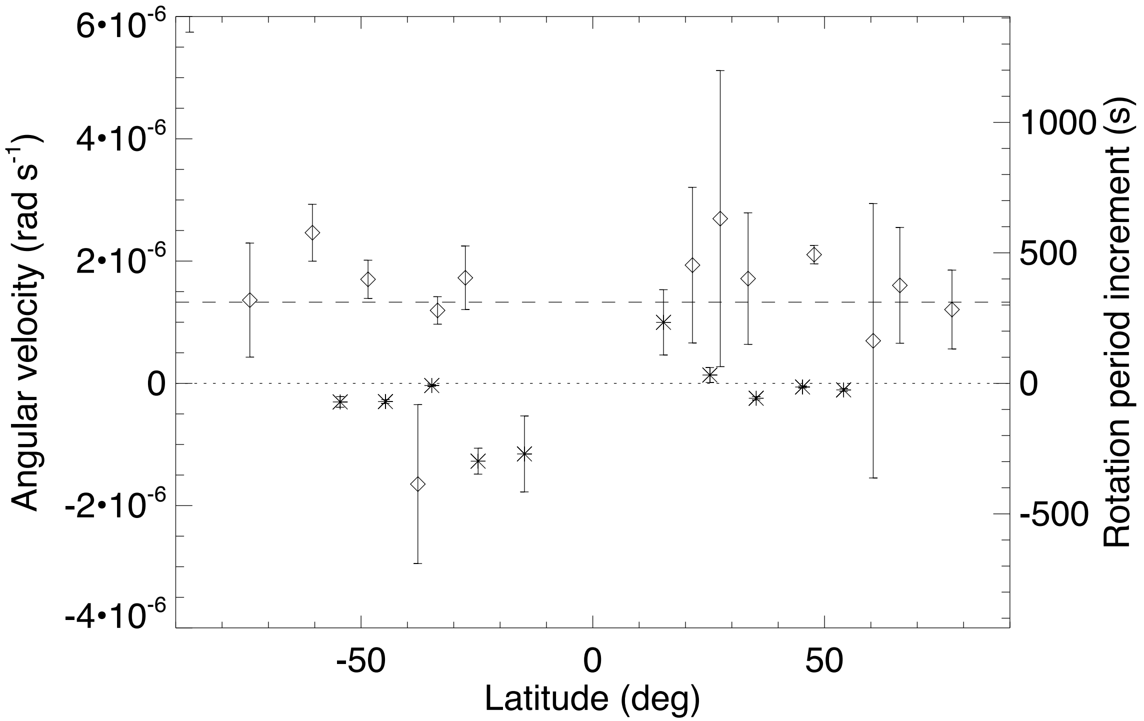
Coda: measuring Saturn's interior rotation using hydrodynamic stability!

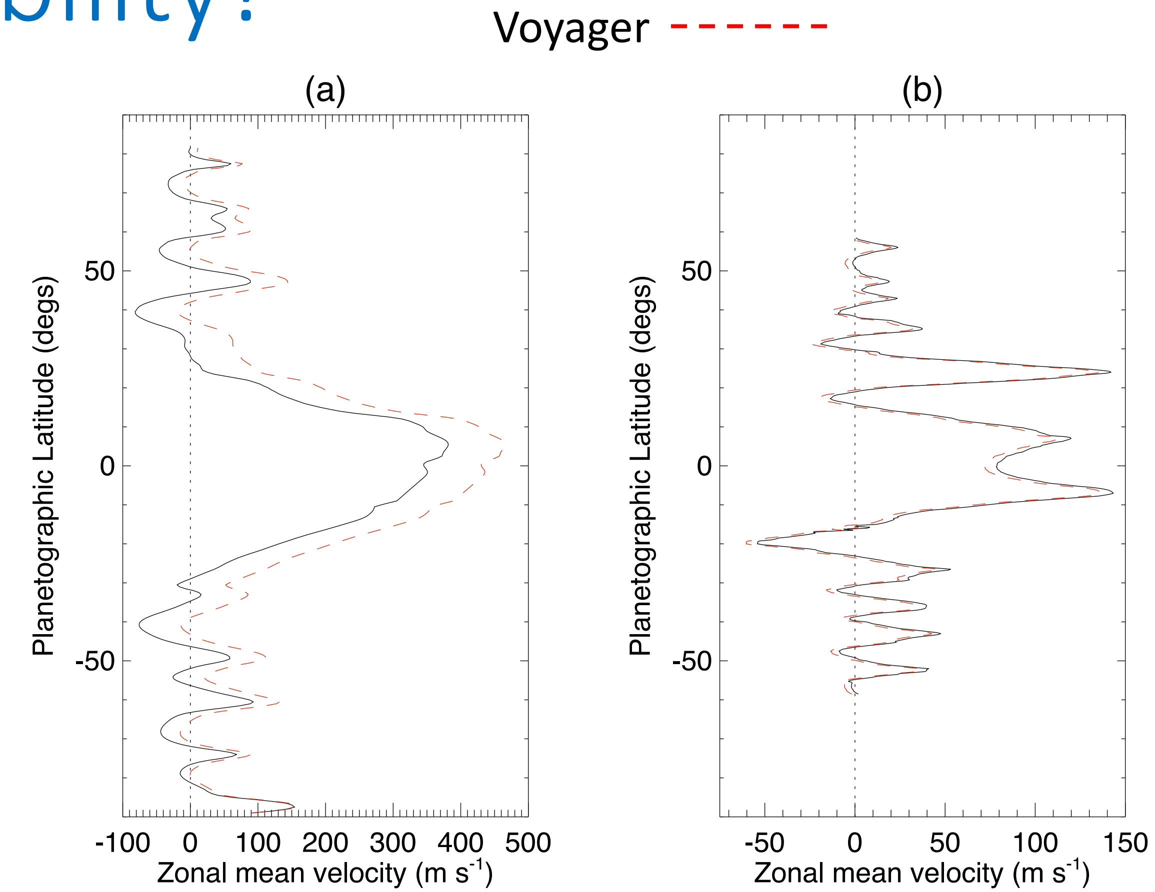
- Stability argument based on pseudo-energy $\mathcal{H}$: stability implied if $\mathcal{H}$ is negative-definite.
  - Leads to sufficient condition for stability (Arnol'd 1966 – known as "Arnol'd II")

$$-\frac{d\Psi}{dQ} = -\frac{d\Psi/dy}{dQ/dy} = \frac{U-\alpha}{\frac{dQ}{dy}} \geq L_d^2$$

  - where α is a constant
- At marginal stability, $\geq \rightarrow =$ and α defines unique reference frame where the gravest edge waves (largest L_d) can just phase-lock....
- Does barotropic adjustment apply to Jupiter and Saturn....?
  - [Dowling 1993 *J. Atmos. Sci.*]

Coda: measuring Saturn's interior rotation using hydrodynamic stability!





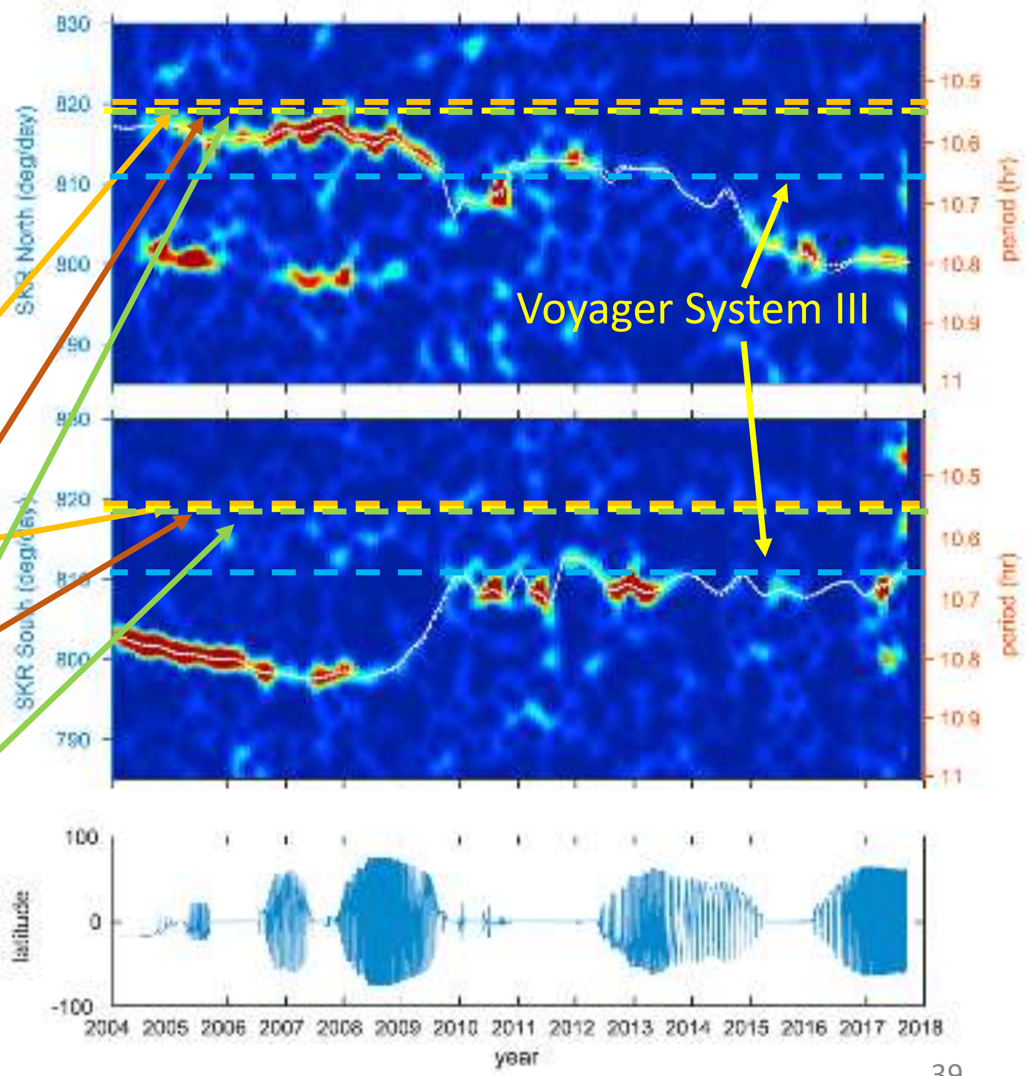
Saturn

Jupiter

- Correlate Q vs Ψ and L_d in latitude bands to determine $\alpha(\phi)$ and corresponding $\Omega(\phi)$
- **Result: a unique Ω for each planet (to within statistical errors)!** [Read et al. 2009 Nature]

Saturn’s interior rotation rate - a mystery?

- Saturn’s magnetic field dominated by a dipole aligned with its rotation axis (±0.06°)!
  - Periodicity only in very low radio frequency emissions – locked to the interior…..?
  - First measured by Voyager fly-by in 1982
  - Monitored by Cassini orbiter from 2004-2017 and found to vary in time!!
  - Cf rotation period estimated from gravity field and oblateness (Anderson & Schubert 2007)?
- Hydrodynamic marginal stability value (Read et al. 2009)
  - agrees with Anderson & Schubert (2007)
- Recent confirmation from Cassini “ring seismology” (Mankovitch et al. 2019)



From Ye et al. (2018) GRL

Conclusions

- Common stability criteria (necessary but not sufficient) for baroclinic and barotropic instabilities
- Rapid rotator planets dominated by waves energized by baroclinic instabilities on scales ~ L_d
- Slow rotator planets dominated by waves energized by barotropic instabilities
- Baroclinic/barotropic adjustment may act as self-organized criticality in some circumstances [not too super-critical]
  - Regulates total equator-pole heat transport [eddy/mean-flow compensation]
  - Controls structure and strength of eddy-driven zonal jets?
  - Earth is close to margins of applicability…..?
- Roles of instabilities in deep gas giant atmospheres….?