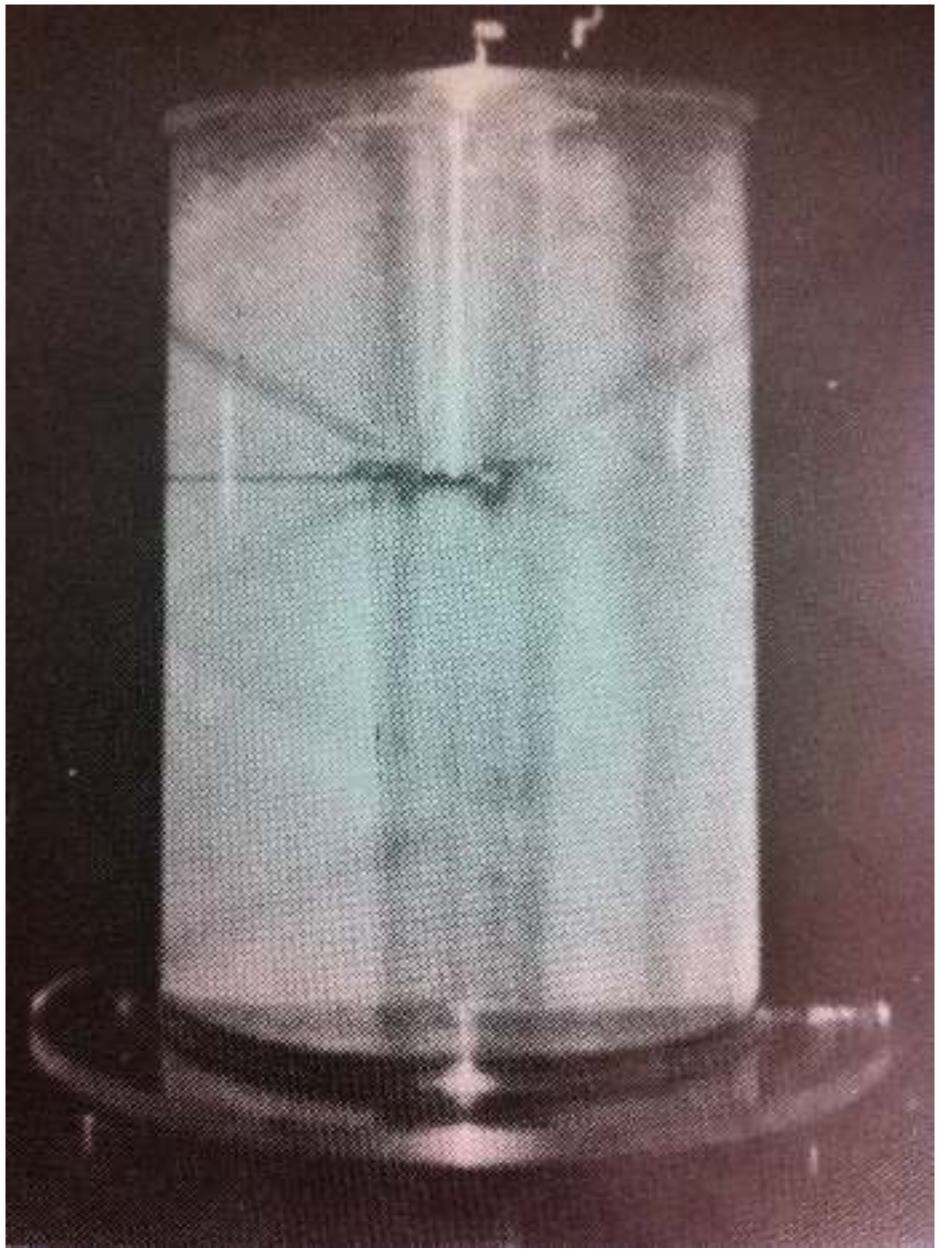
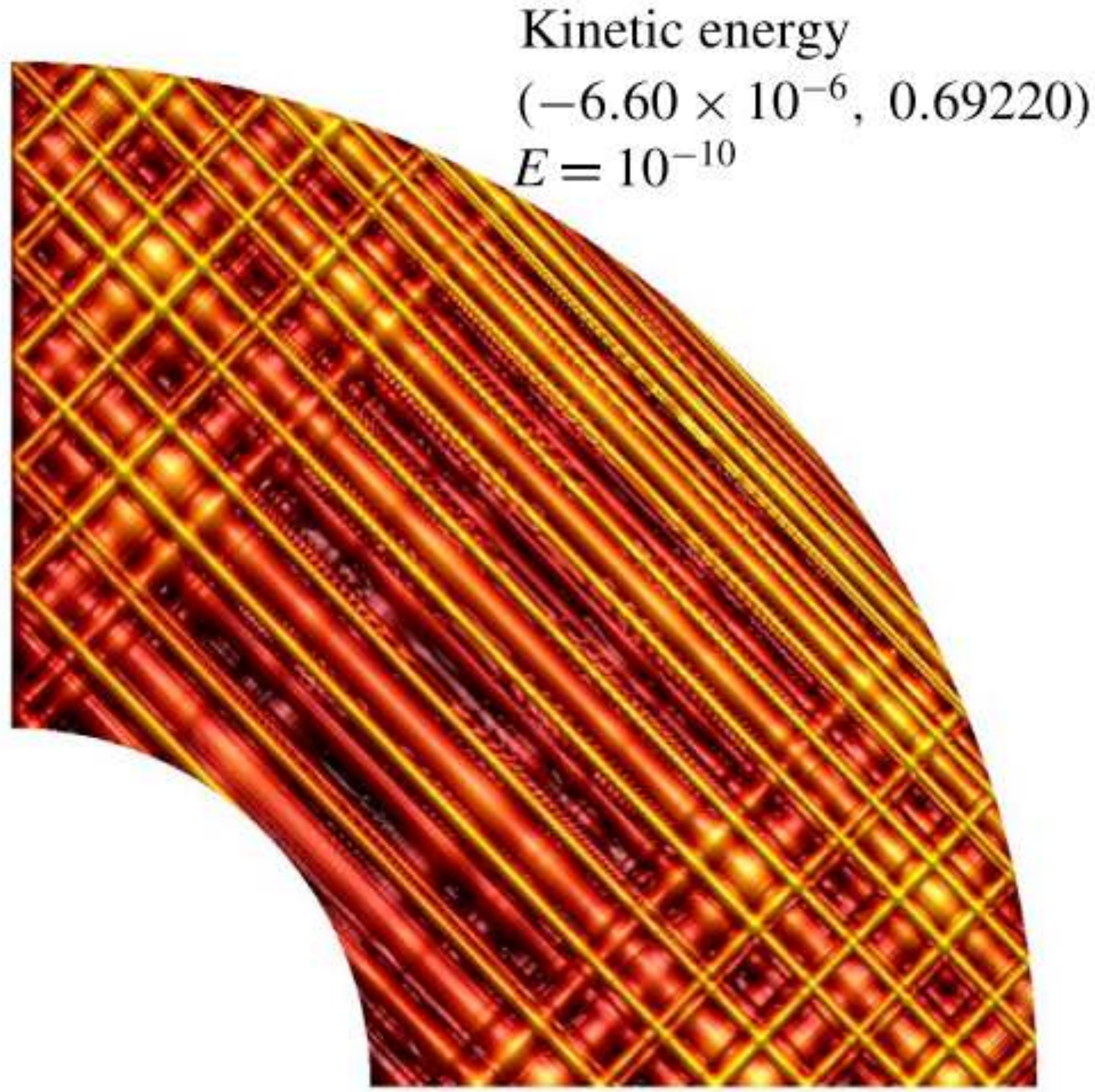
Inertial Waves and Inertial modes in planetary cores and subsurface oceans

Lecture by J. Noir at WITGAF Cargese, Corsica, 2019



Experiment by Goertler 1957, picture from: The theory of rotating fluids by H. P. Greenspan 1968



Axisymmetric inertial modes in a spherical shell at low Ekman numbers: M. Rieutord and L Valdettaro, 2018

Inertial Waves and Inertial modes in planetary cores and subsurface oceans

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- **Introduction to rapidly rotating fluids**
- **Inertial Waves and Inertial Modes**
- **Instabilities**
- **Dissipation in the lunar core**

Inertial Waves and Inertial modes in planetary cores and subsurface oceans

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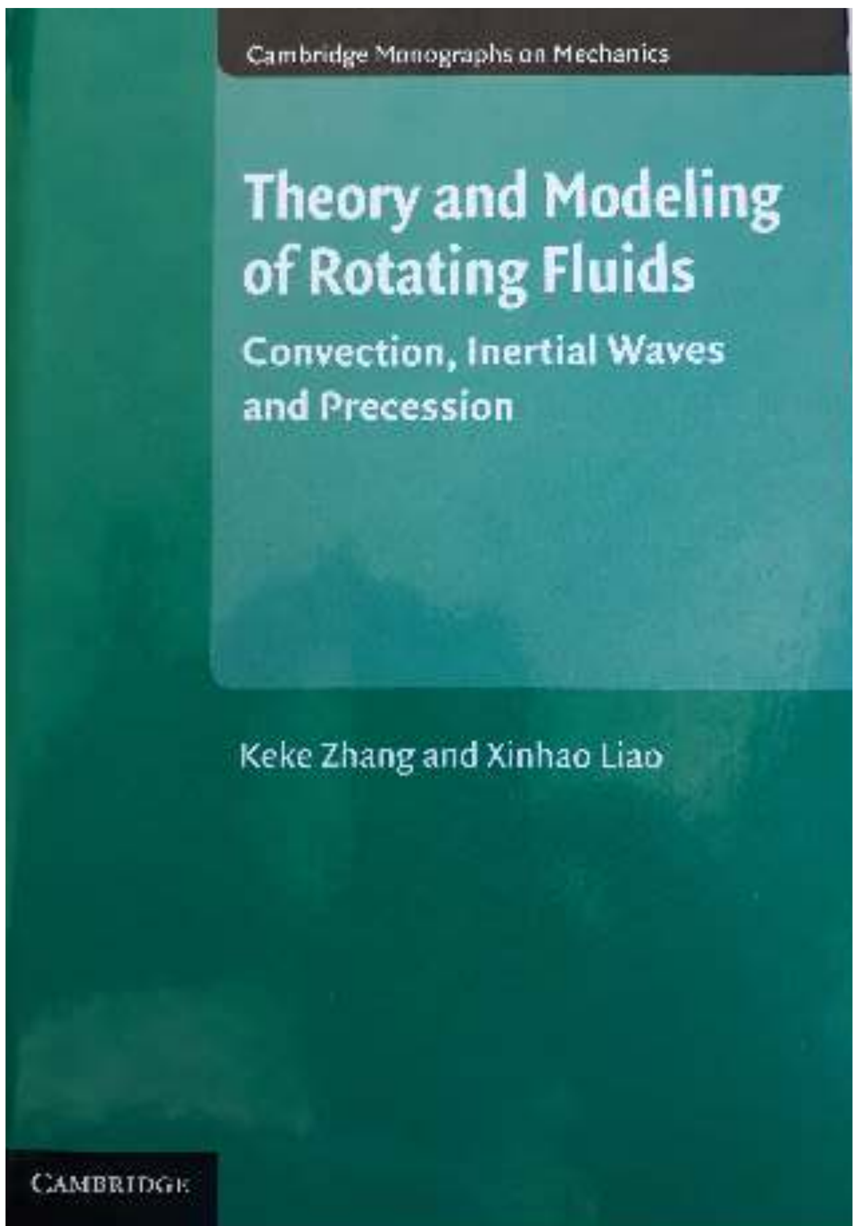
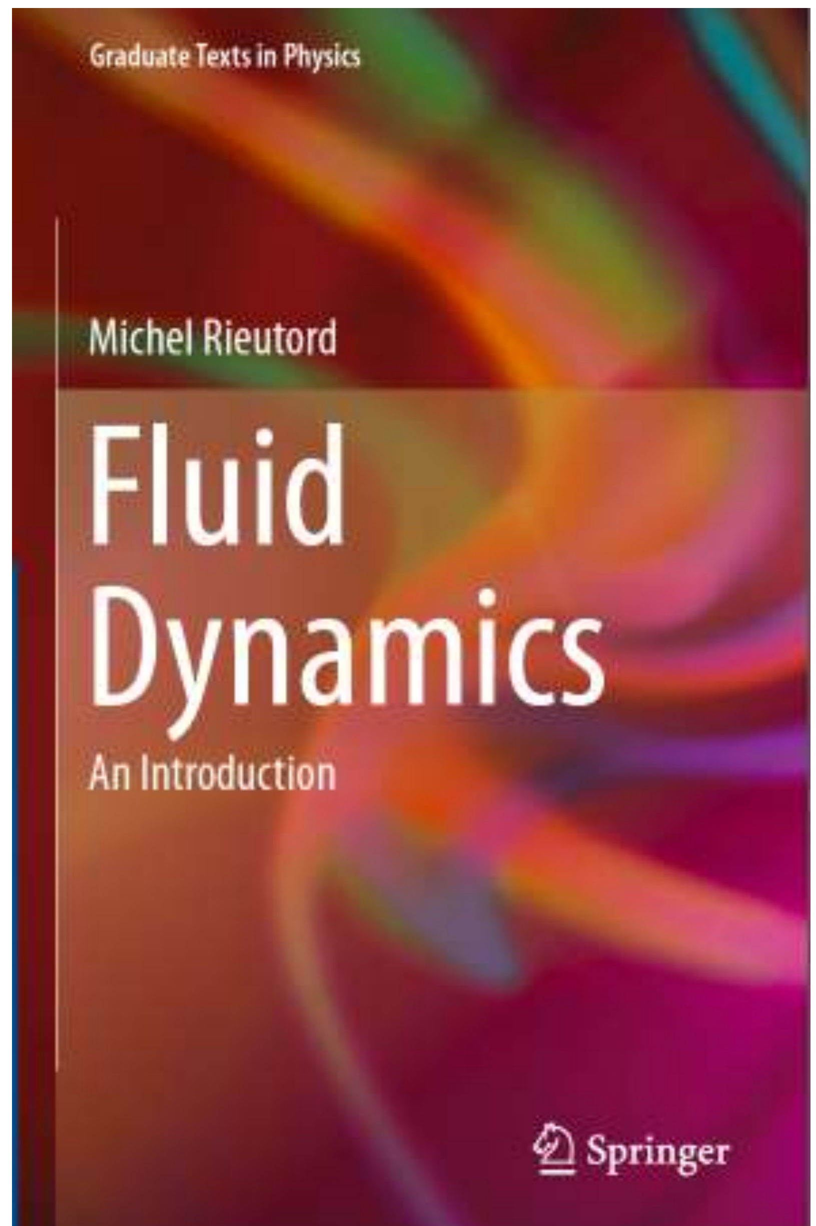




Part I:Introduction to rapidly rotating fluids:

1. Some observations
2. Navier-Stokes in a rotating frame
3. Taylor-Proudman Theorem
4. Earth’s core and mantle.

In many Geophysical and Astrophysical problems, the fluid is primarily rotating. In such situations the fluid dynamics will exhibit some unusual behaviours.

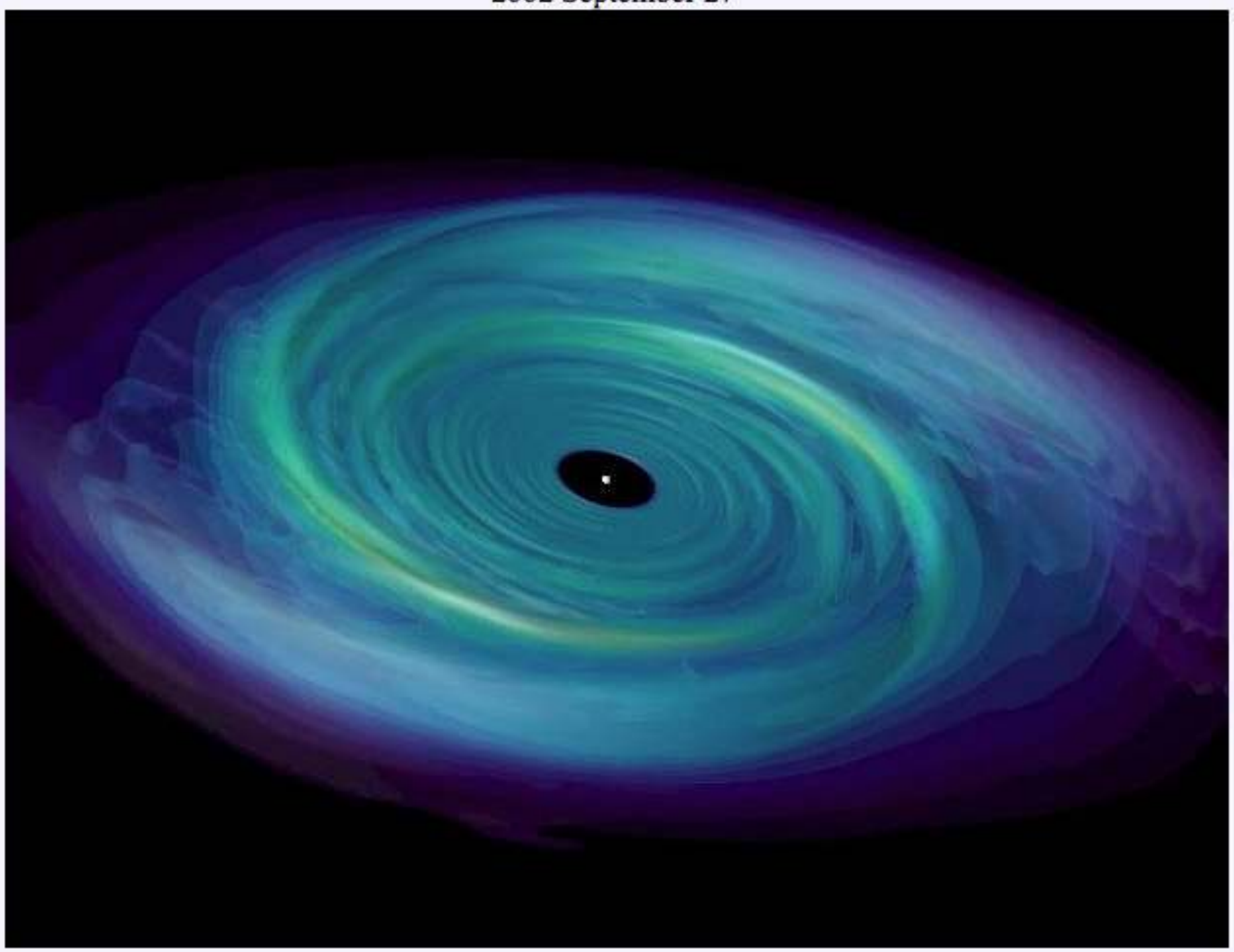
"The SeaWiFS image was provided courtesy of Orbimage, Inc., the SeaWiFS Project (Code 970.2) and the Distributed Active Archive Center (Code 902) at NASA's Goddard Space Flight Center, Greenbelt, MD, 20771."

Courtesy of NASA

In many Geophysical and Astrophysical problems, the fluid is primarily rotating. In such situations the fluid dynamics will exhibit some unusual behaviours.

Stars, Galaxies, Accretion disks…

2002 September 27



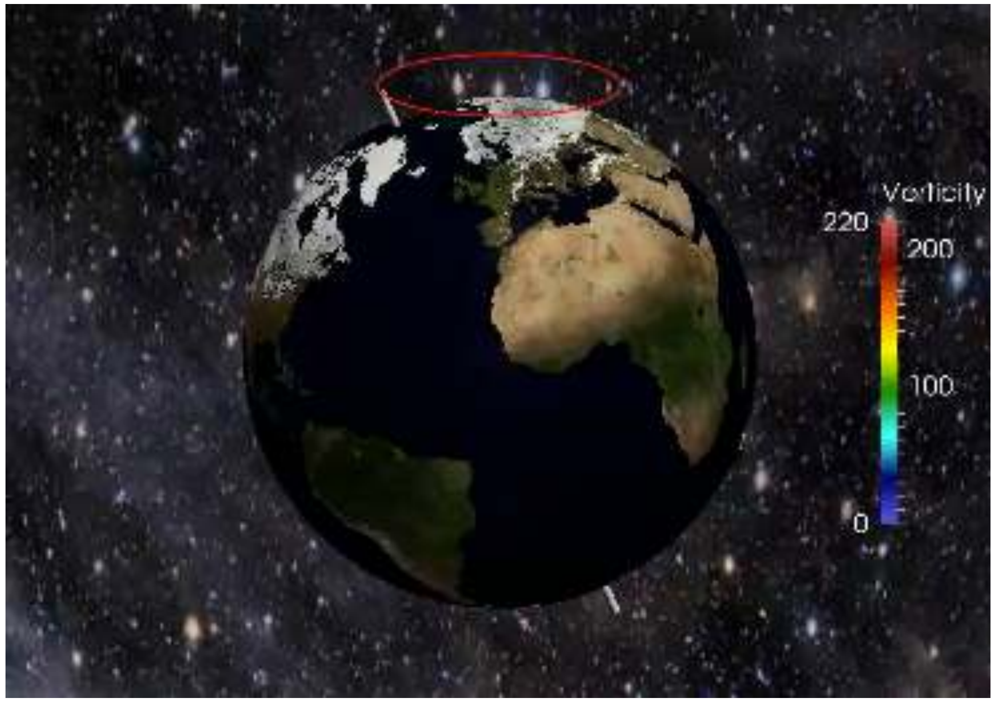
Accretion Disk Simulation
Credit: Michael Owen, John Blondin (North Carolina State Univ.)

Probing planetary interiors through the variation of their rotation.

Librations

nasa.gov

Precession / Nutations



nasa.gov

Tides

nasa.gov

Introduction to rapidly rotating fluids:

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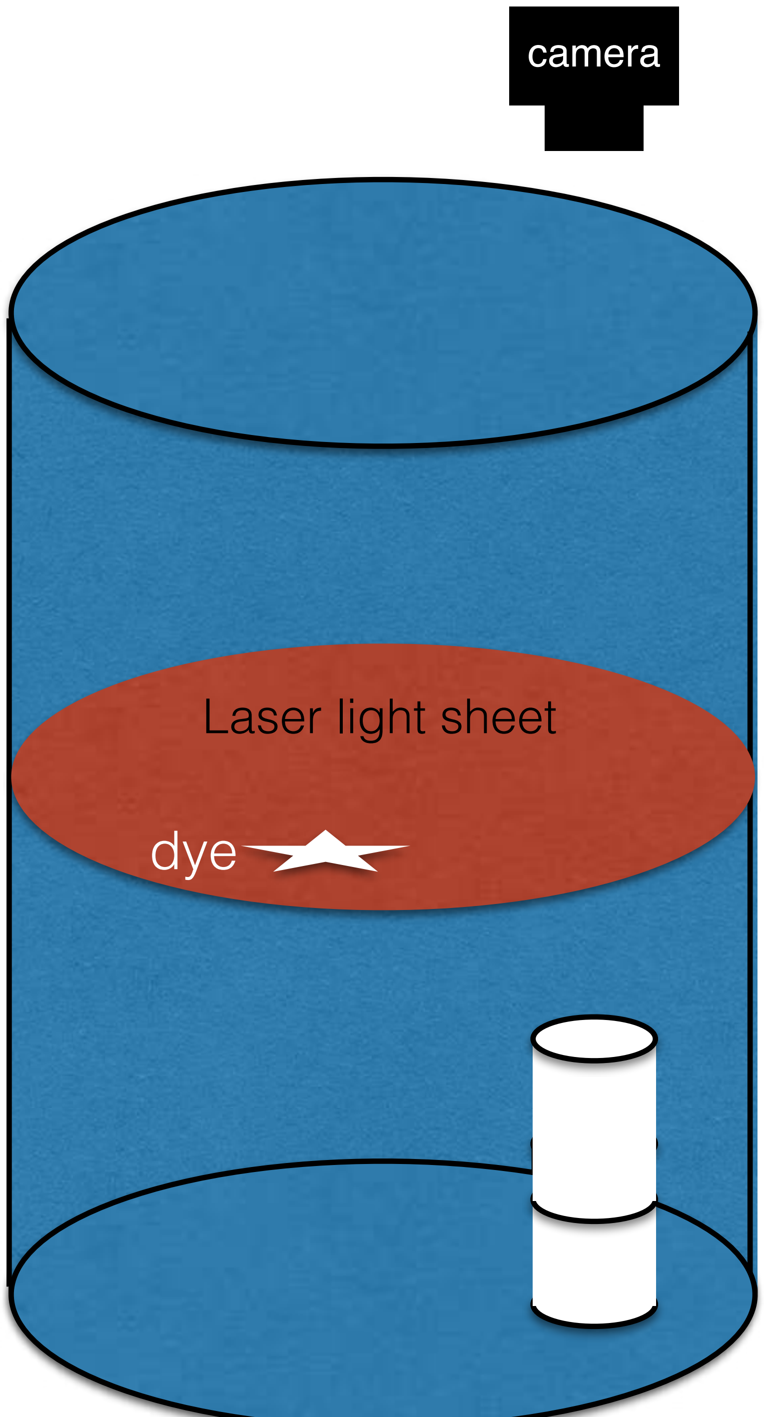
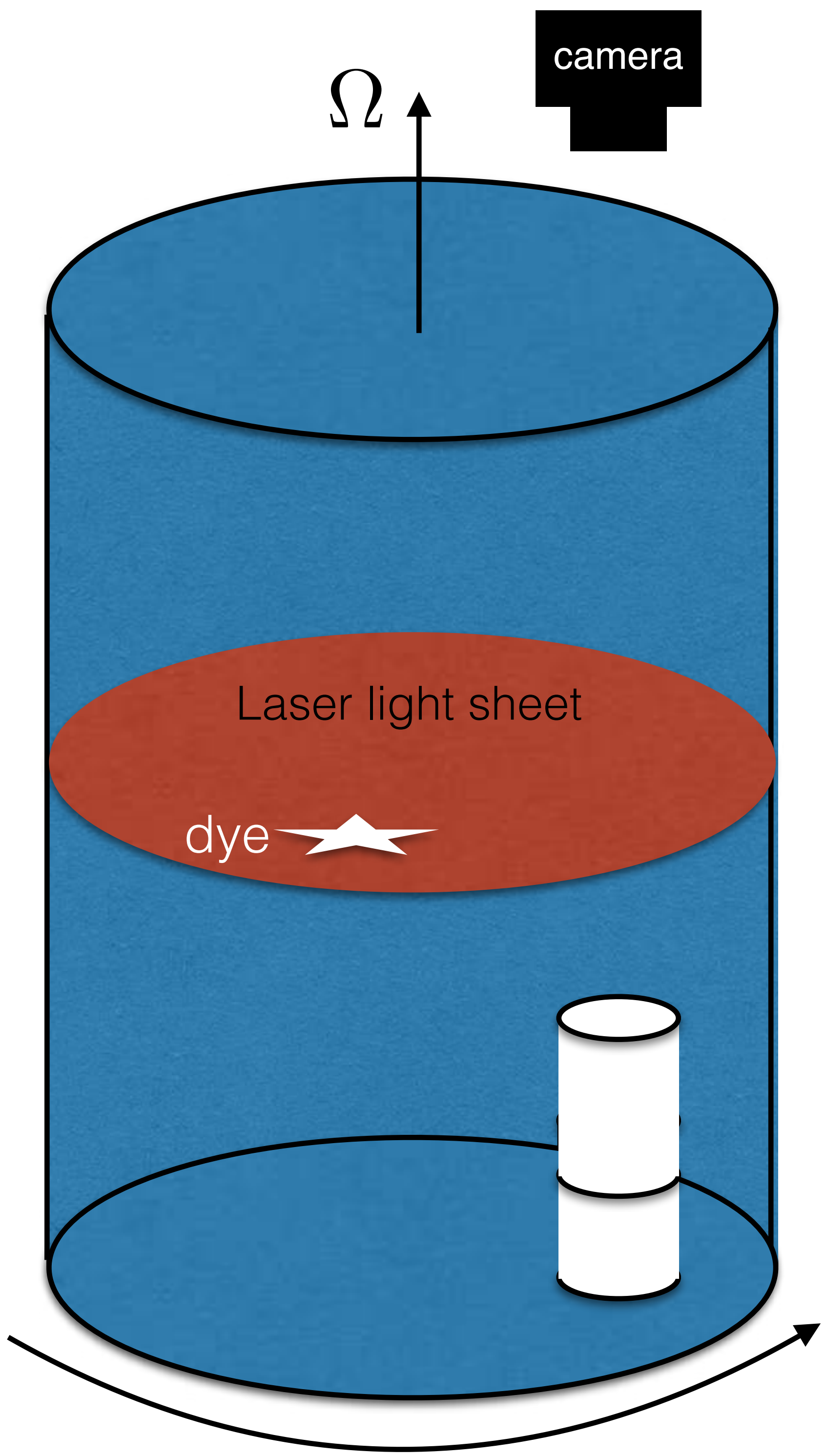
What makes rotating fluids so special…



- We now put a laser light sheet way above the obstacle, a drop of dye.
- **At t<to the fluid is at rest**.
- At t=to, we slowly move the obstacle toward the dye.

What do you expect to see from the camera ?

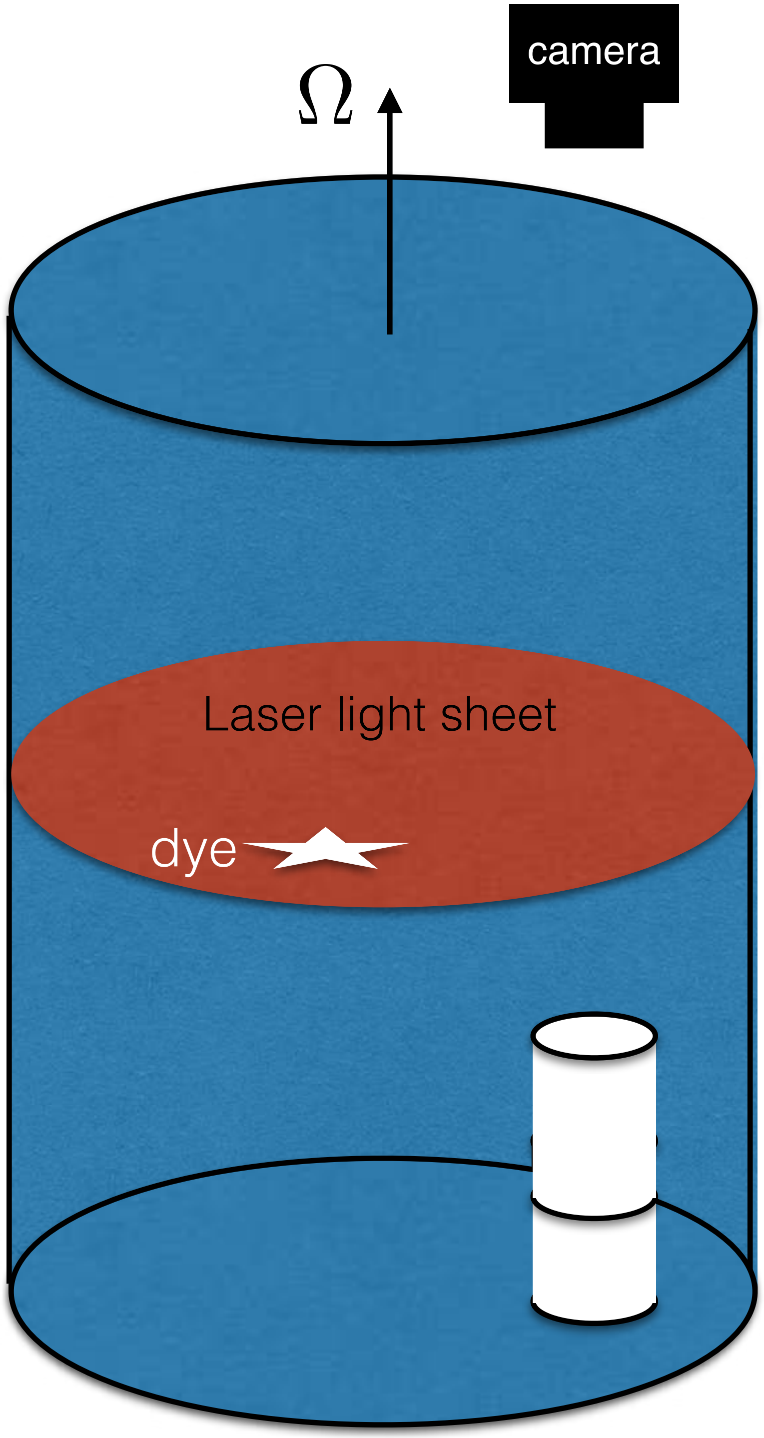
What makes rotating fluids so special…



- We now put a laser light sheet way above the obstacle, a drop of dye.
- **At t<to the fluid rotates**.
- At t=to, we decrease the rotation rate by a small amount such that the obstacle moves toward the dye.

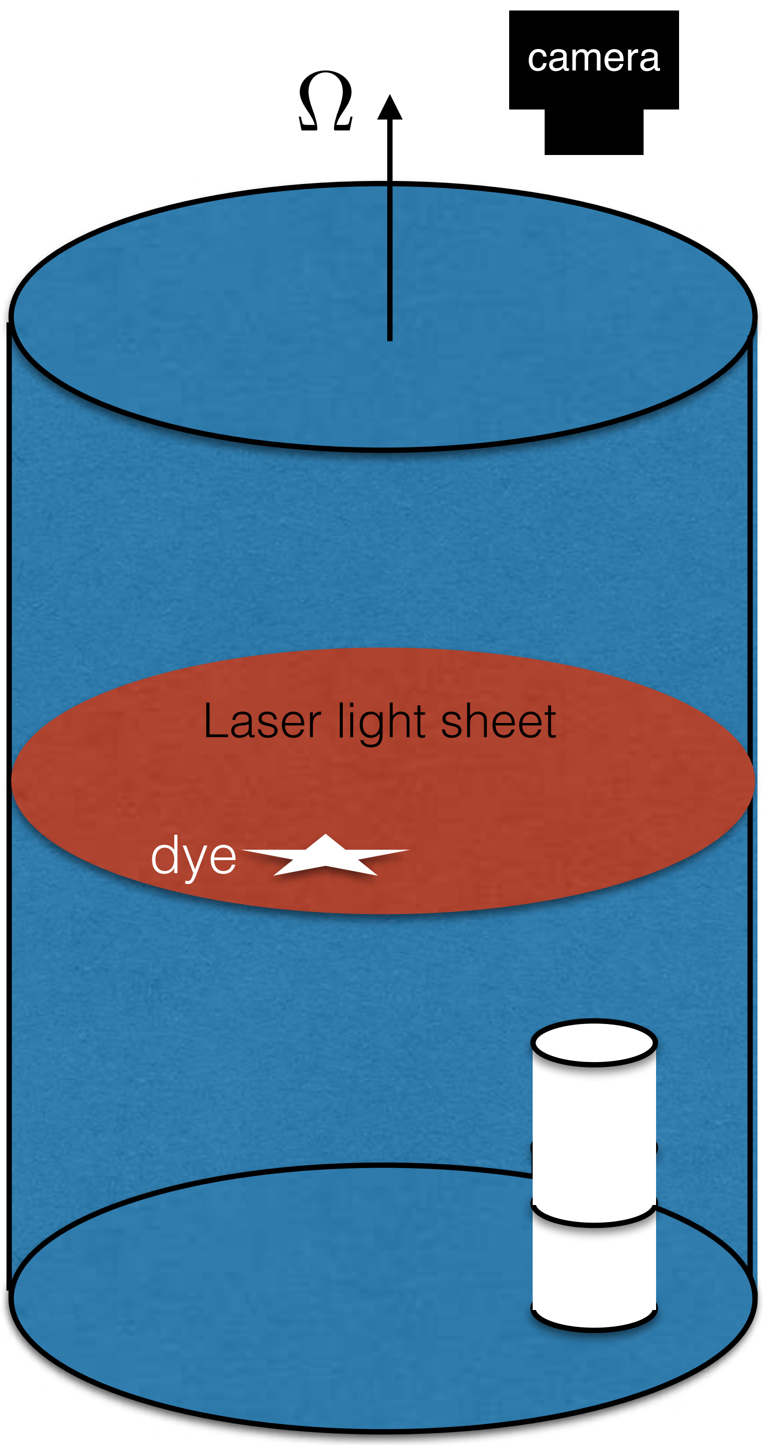
What do you expect to see from the camera ?

What makes rotating fluids so special…

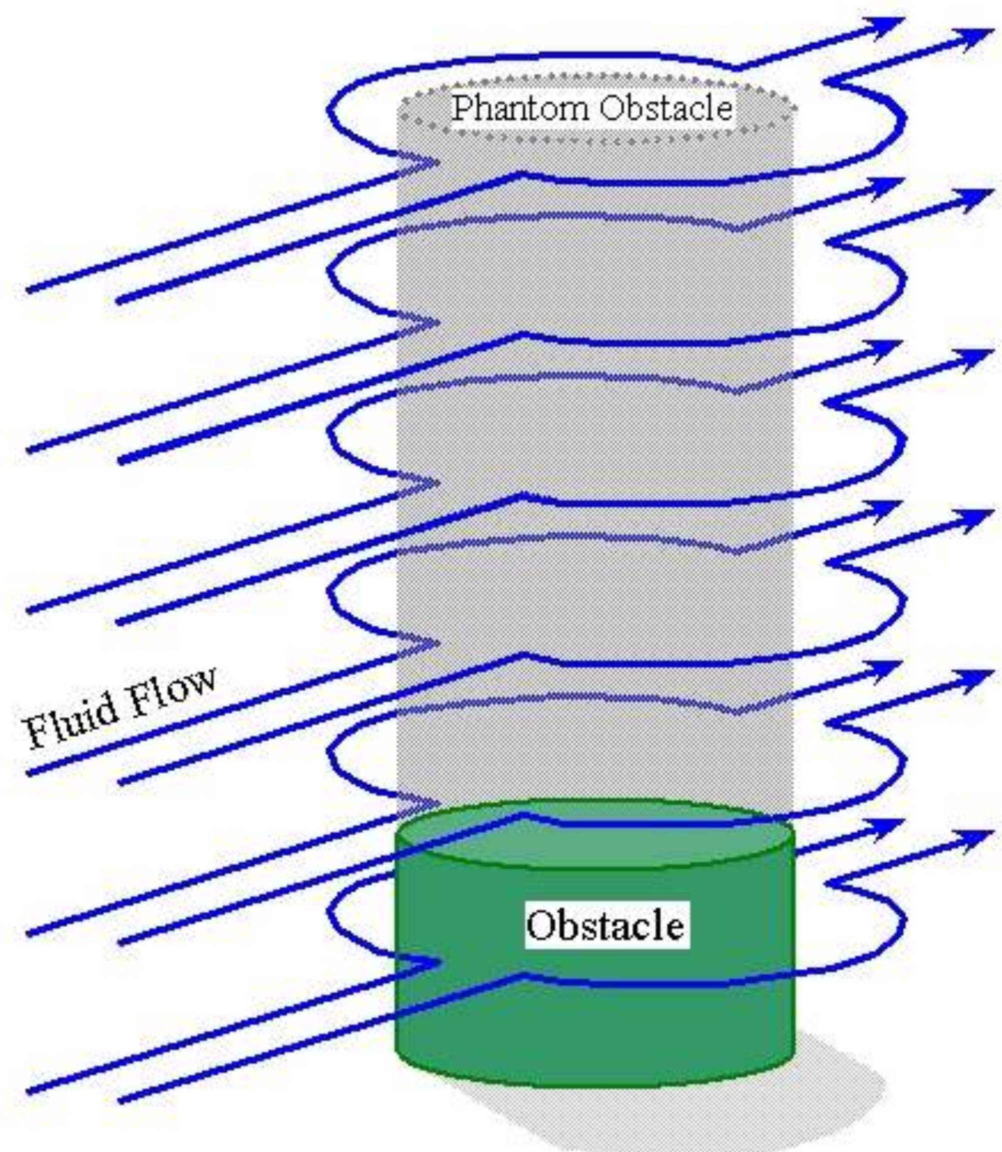


What makes rotating fluids so special…

The fluid, as seen from the rotating frame act as it was rigid in the direction aligned with the axis of rotation when a **small and quasi steady** perturbation is driven.



The phantom obstacle is what we call a Taylor Column



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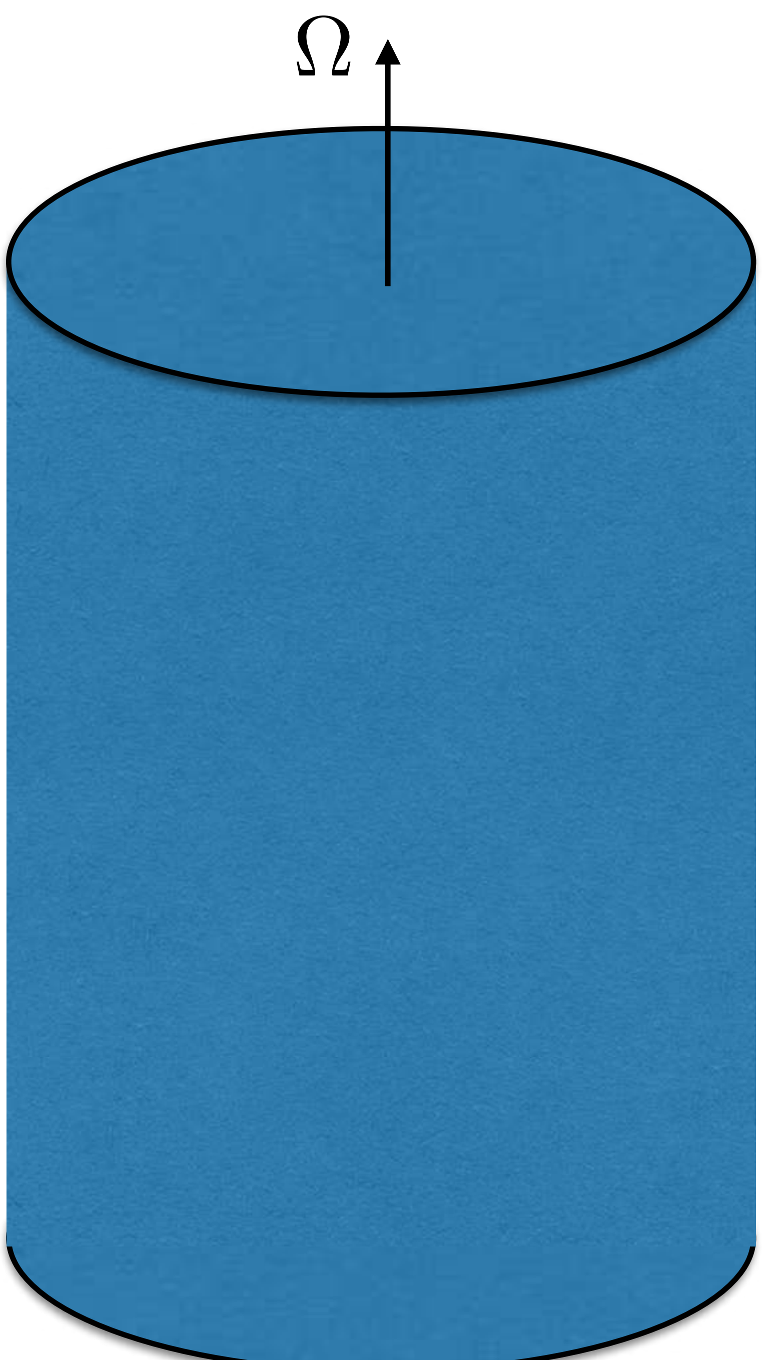
Let's consider a fluid domain bounded or unbounded rotating around an axis Z at a pulsation $\mathbf{\Omega}$.



In this lecture we will consider the fluid to be:

- **Homogeneous**
- **neutrally buoyant**
- **Incompressible**

Viewed from an inertial frame (non rotating) the fluid velocity can be decomposed as:

$$\vec{U}_I = \vec{\Omega} \times \vec{r} + \vec{U}_R$$

$\vec{U}_I$ ↑ In the frame of inertia

$\vec{U}_R$ ↑ In the rotating frame

In the frame of inertia the Navier-Stokes equation and mass conservation equations are written:

$$\rho \frac{d\vec{U}}{dt} = -\vec{\nabla} P + \rho \vec{g} + \mu \vec{\nabla}^2 \vec{U} + \rho \vec{f}$$

$$\vec{\nabla} \cdot \vec{U} = 0$$

Let's denote $\left.\frac{\partial \vec{A}}{\partial t}\right|_I$ the rate of change of a vector viewed by an observer in the frame of inertia and $\left.\frac{\partial \vec{A}}{\partial t}\right|_R$ the rate of change viewed by an observer in the rotating frame.

The two are related by:

$$\left.\frac{\partial \vec{A}}{\partial t}\right|_I = \left.\frac{\partial \vec{A}}{\partial t}\right|_R + \vec{\Omega} \times \vec{A}$$

Applying that to the position vector:

$$\left.\frac{\partial \vec{r}}{\partial t}\right|_I = \left.\frac{\partial \vec{r}}{\partial t}\right|_R + \vec{\Omega} \times \vec{r}$$

$$\vec{U} = \vec{u} + \vec{\Omega} \times \vec{r}$$

Applying the rate of change transformation to the velocity:

$$\left.\frac{d\vec{U}}{dt}\right|_I = \left.\frac{d(\vec{\Omega}\times\vec{r}+\vec{u})}{dt}\right|_R + \vec{\Omega}\times\vec{u}+\vec{\Omega}\times\vec{\Omega}\times\vec{r}$$

$$\left.\frac{d\vec{U}}{dt}\right|_I = \left.\frac{d\vec{\Omega}}{dt}\right|_R \times\vec{r}+\vec{\Omega}\times \left.\frac{dr}{dt}\right|_R + \left.\frac{d\vec{u}}{dt}\right|_R + \vec{\Omega}\times\vec{u}+\vec{\Omega}\times\vec{\Omega}\times\vec{r}$$

$$\left.\frac{d\vec{U}}{dt}\right|_I = \left.\frac{d\vec{u}}{dt}\right|_R + 2\vec{\Omega}\times\vec{u}+\vec{\Omega}\times\vec{\Omega}\times\vec{r}+ \left.\frac{d\vec{\Omega}}{dt}\right|_R \times\vec{r}$$

Noticing that:

$$\left.\frac{d\vec{\Omega}}{dt}\right|_I = \left.\frac{d\vec{\Omega}}{dt}\right|_R + \vec{\Omega}\times\vec{\Omega} = \left.\frac{d\vec{\Omega}}{dt}\right|_R$$

and that:

$$\vec{\nabla}^2\vec{\Omega}\times\vec{r} = 0$$

Dropping the subscript „R” the Navier-Stokes equation for the velocity ***u*** is given by:

$$\rho\left[\frac{d\vec{u}}{dt}+2\vec{\Omega}\times\vec{u}+\vec{\Omega}\times\vec{\Omega}\times\vec{r}+\frac{d\vec{\Omega}}{dt}\times\vec{r}\right]=-\vec{\nabla}p+\rho\vec{g}+\mu\vec{\nabla}^2\vec{u}+\rho\vec{f}$$

Since,

$$\vec{\nabla}\cdot\vec{\Omega}\times\vec{r}=0$$

the mass conservation equation remains simple in the rotating frame for incompressible fluids:

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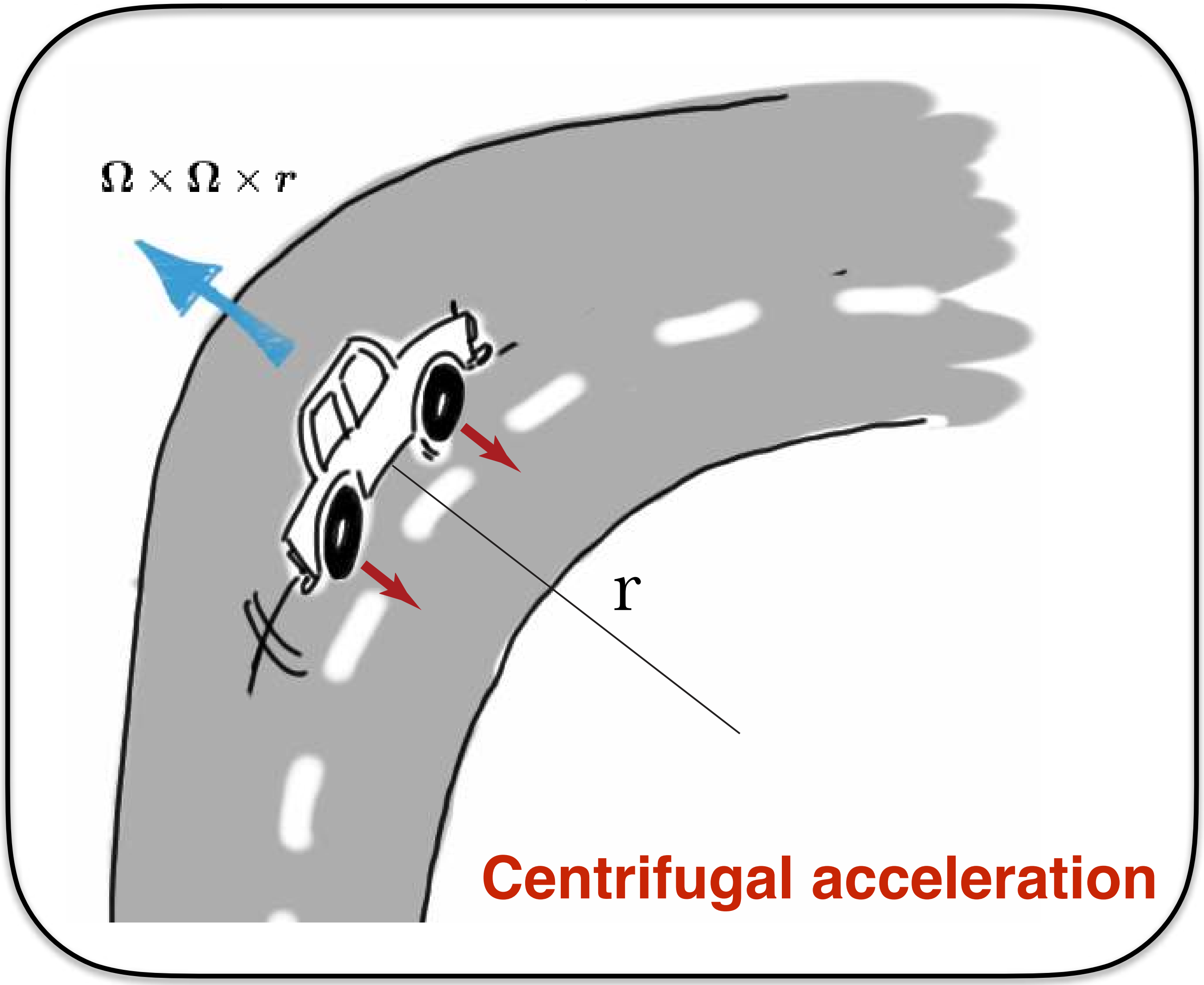

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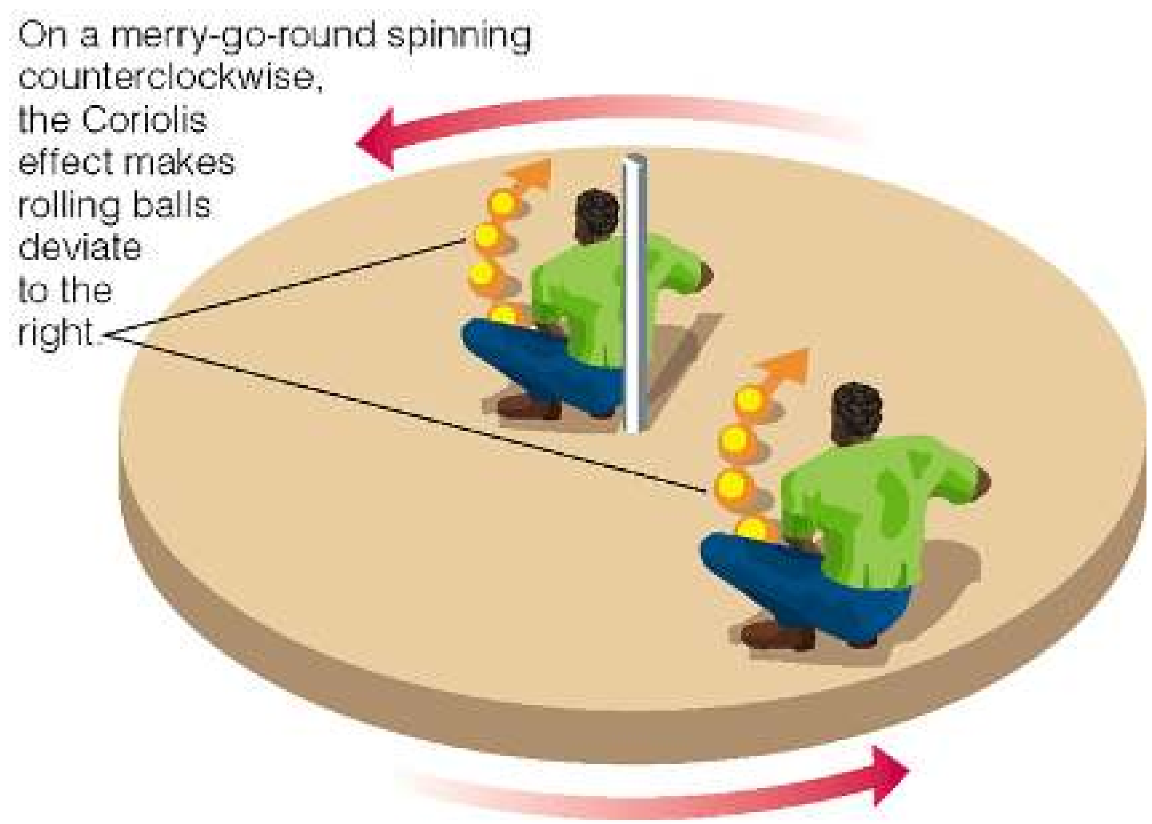

Coriolis acceleration

Dropping the subscript „R” the Navier-Stokes equation for the velocity ***u*** is given by:

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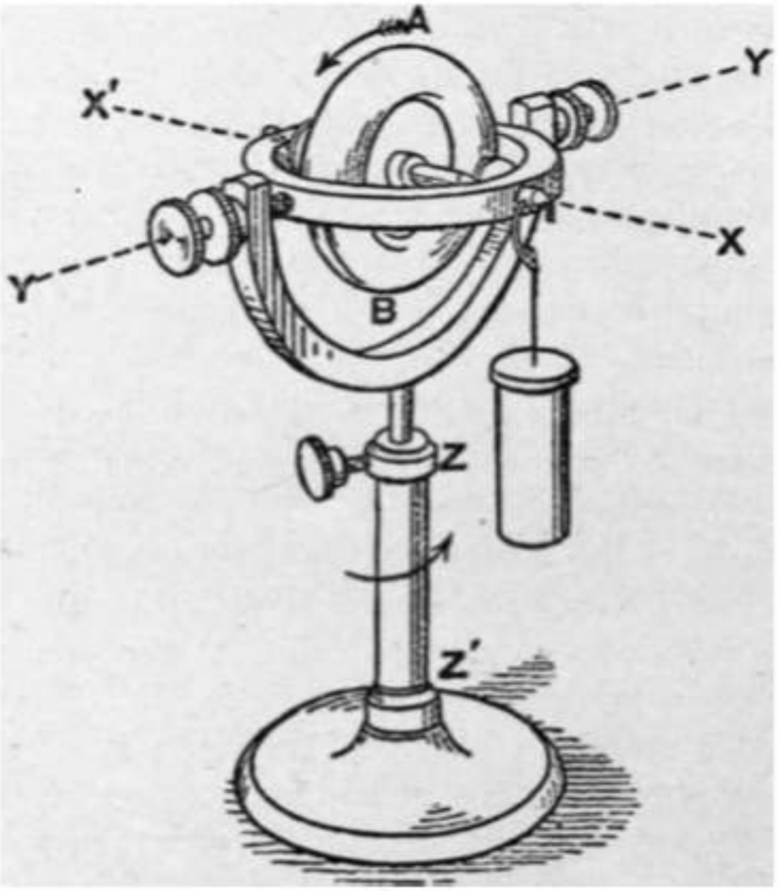

Poincaré’s acceleration

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$$\vec{\Omega}\times\vec{\Omega}\times\vec{r} \rightarrow \vec{\nabla}(\vec{\Omega}\times\vec{r})^2$$

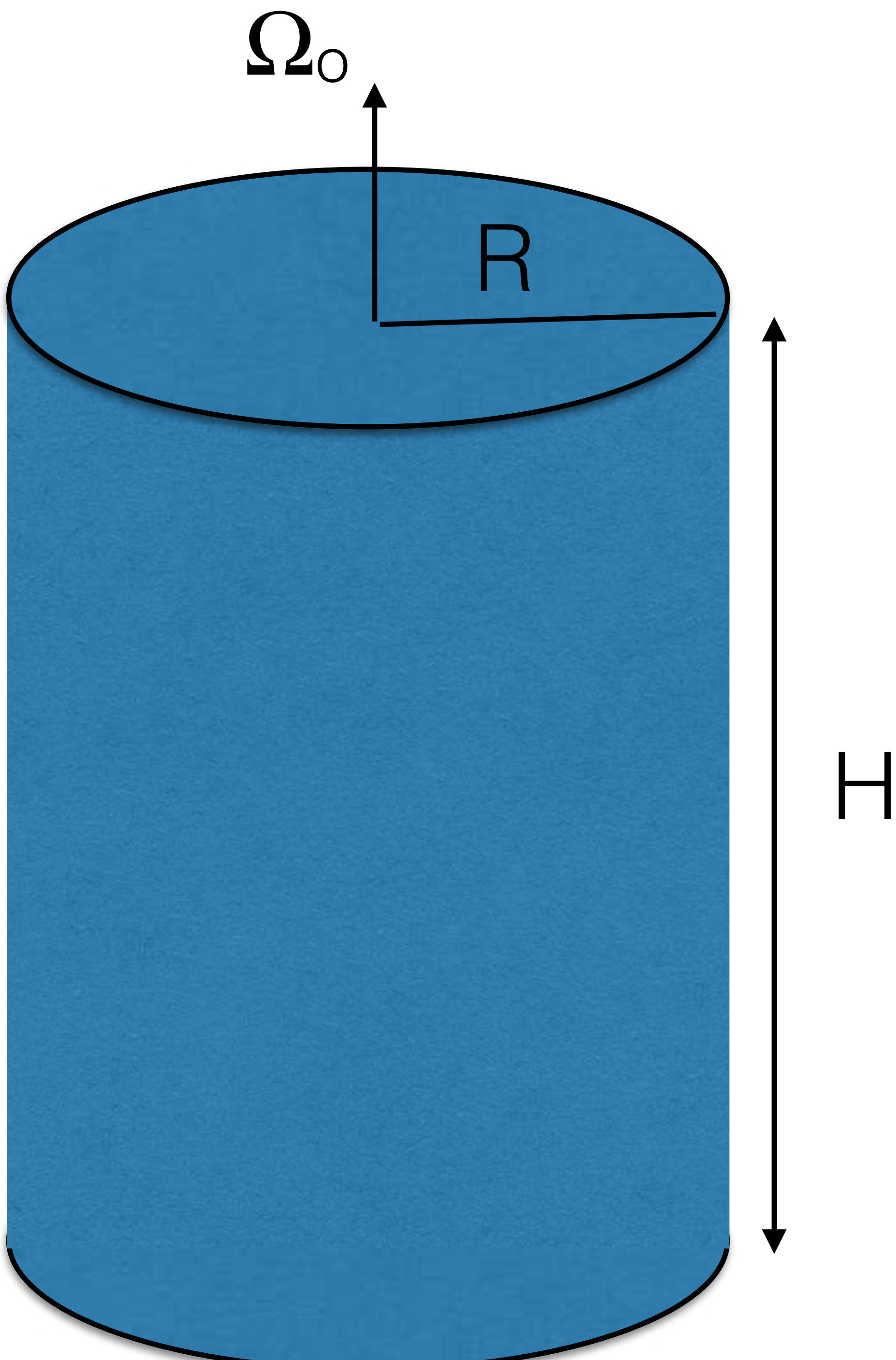
$$\rho\vec{g} \rightarrow \vec{\nabla}(\rho\Phi)$$

Dividing by the density that is uniform under our assumptions and adopting a Eulerian formulation we finally get:

$$\frac{\partial\vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u} + 2\vec{\Omega}\times\vec{u} = -\vec{\nabla}\Pi + \nu\vec{\nabla}^2\vec{u} + \vec{r}\times\frac{d\vec{\Omega}}{dt} + \vec{f}$$

$$\Pi = \frac{p}{\rho} + \left(\vec{\Omega}\times\vec{r}\right)^2 + \Phi$$

Non-dimensional form of the equations:



rescaling of the variables:

$$t = t'[\Omega_o^{-1}]$$

$$l = l'[R]$$

$$\nabla = \nabla'[R^{-1}]$$

$$\vec{\Omega} = \vec{\Omega}'[\Omega_o]$$

$$\vec{u} = \vec{u}'[\Omega_o R]$$

$$\vec{f} = \vec{f}'[\Omega_o^2 R]$$

Non-dimensional form of the equations:

$$\frac{\partial \vec{u}}{\partial t} + 2\vec{\Omega} \times \vec{u} + \vec{u} \cdot \vec{\nabla}\vec{u} = -\vec{\nabla}\Pi + E\vec{\nabla}^2\vec{u} + \vec{r} \times \frac{d\vec{\Omega}}{dt} + \vec{f}$$

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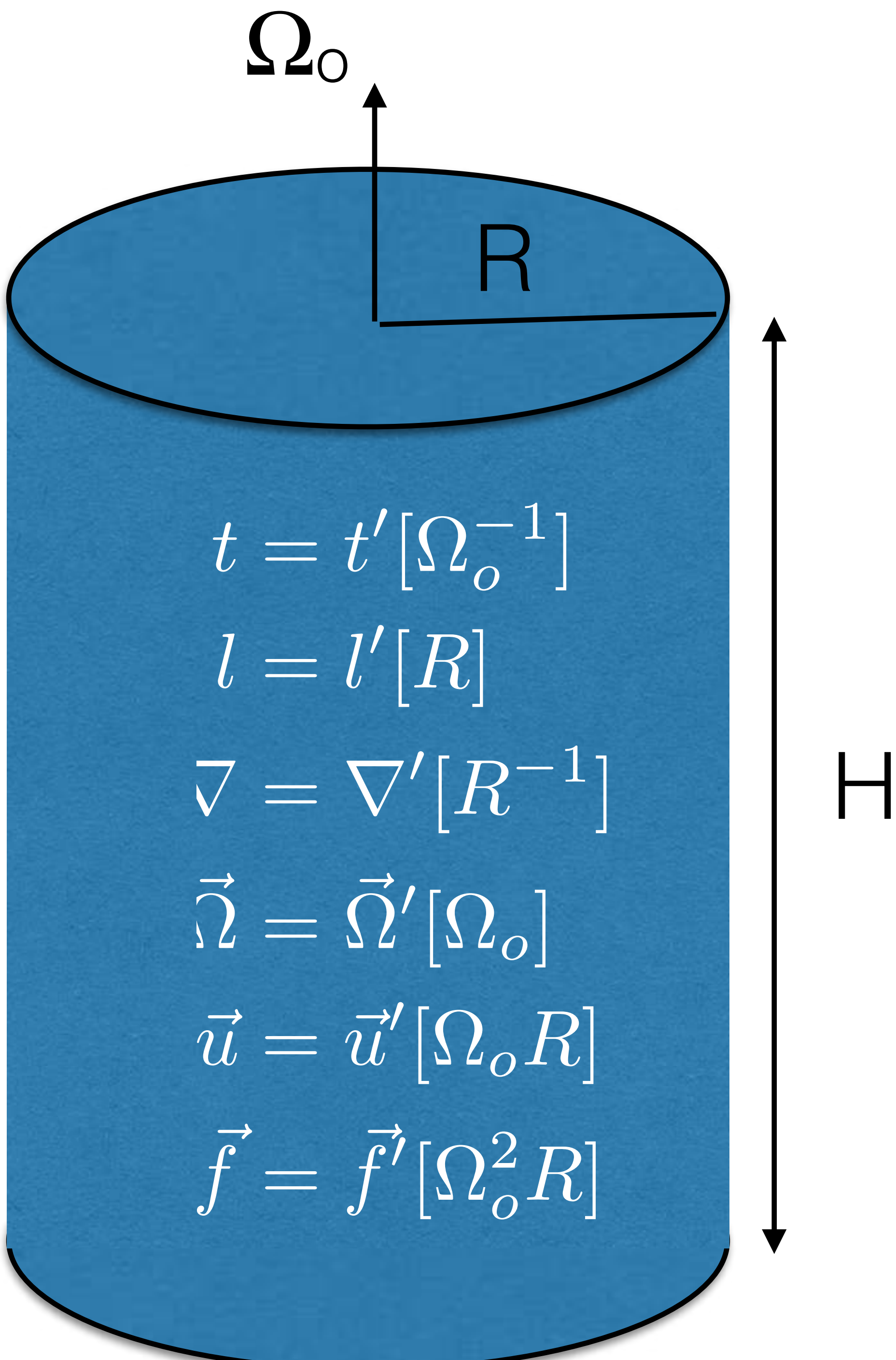

Ekman number:

$$E = \frac{\nu}{\Omega_0 R^2} = \frac{Viscosity}{Coriolis}$$

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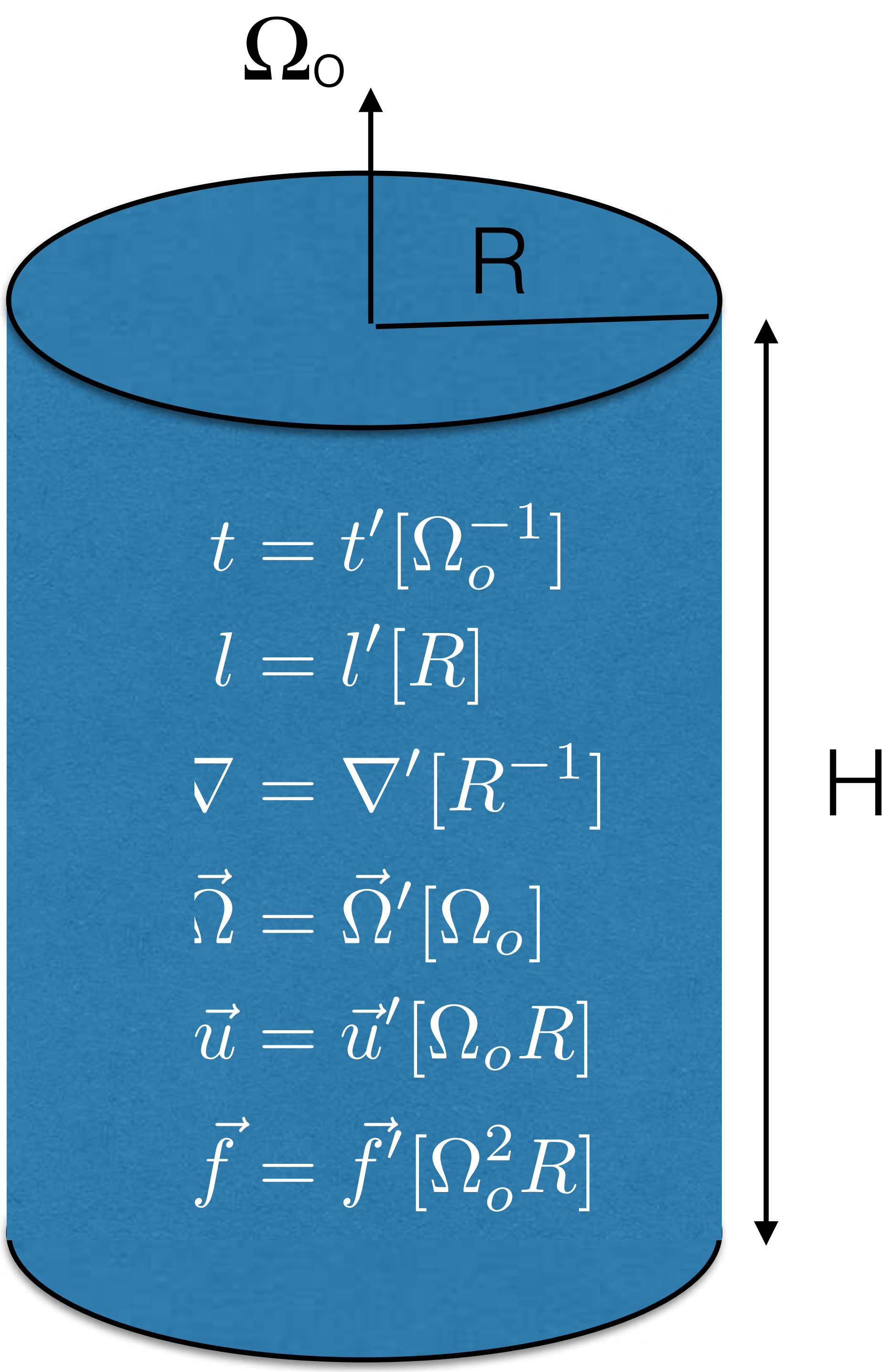

Ekman number:

$$E = \frac{\nu}{\Omega_0 R^2} = \frac{Viscosity}{Coriolis}$$

Rossby number:

$$Ro = \frac{|\vec{u} \cdot \nabla \vec{u}|}{|\vec{\Omega} \times \vec{u}|} = \frac{u}{R\Omega} = \frac{NL}{Coriolis}$$

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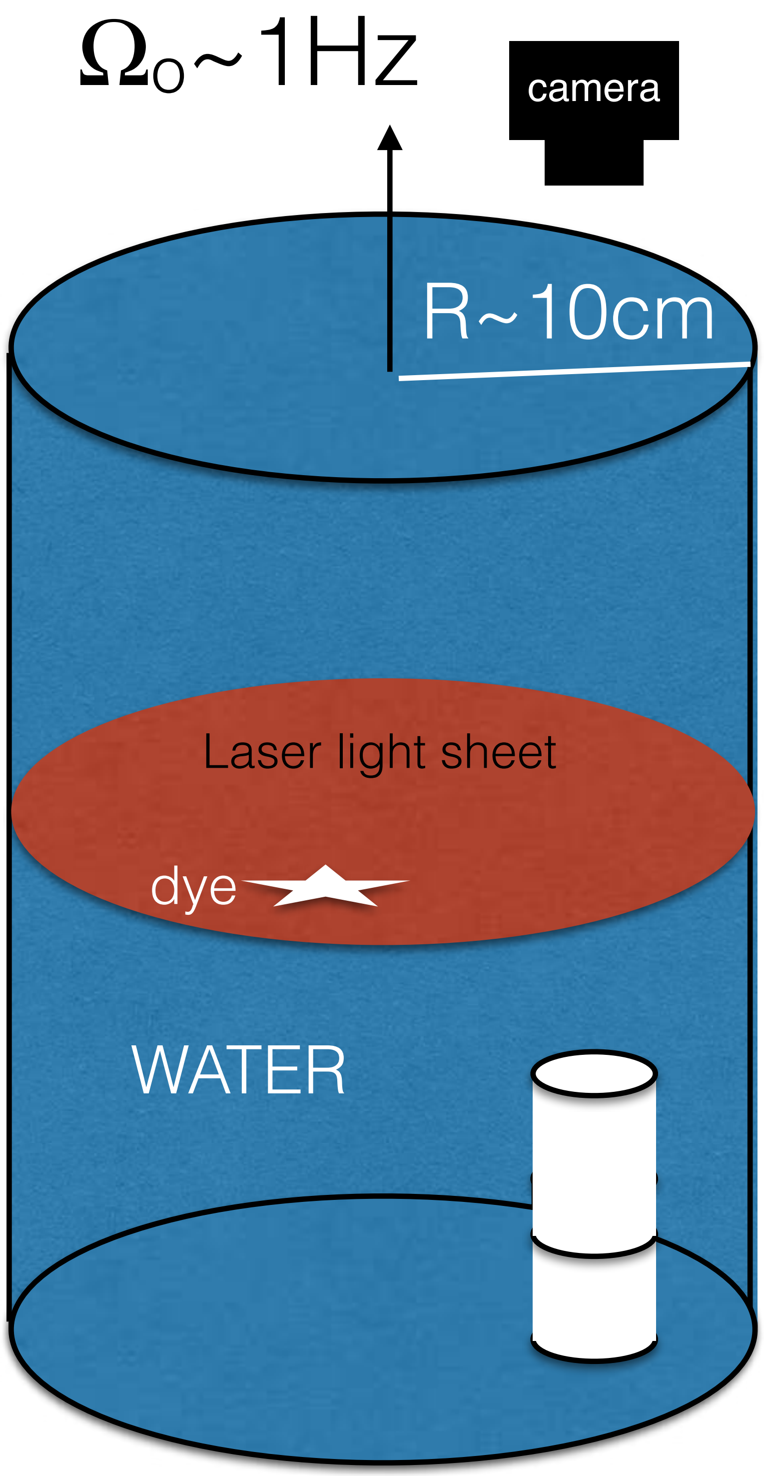
Let's come back to our simple problem of an obstacle moving relative to the fluid, or the opposite…



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No external body forces

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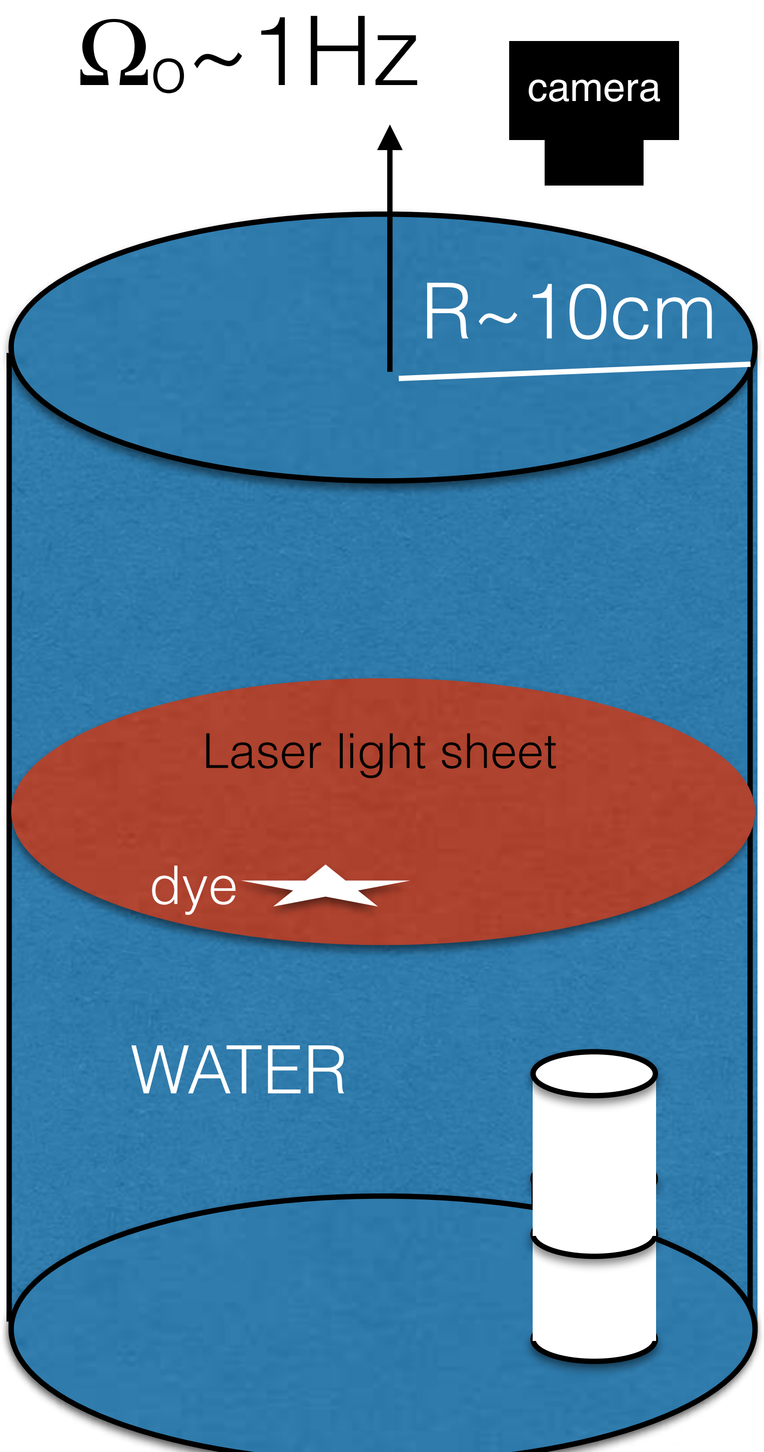

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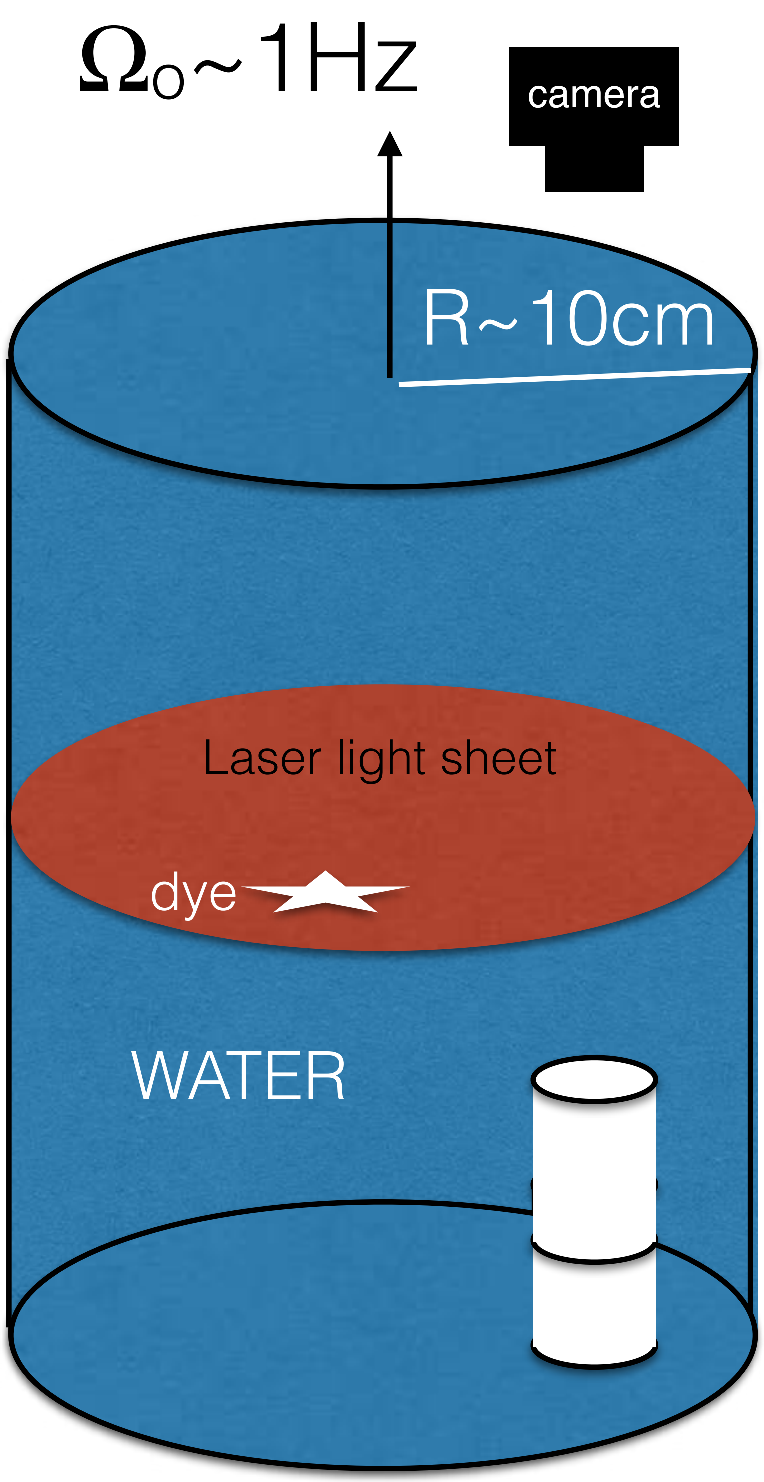

$$\frac{\partial \vec{u}}{\partial t} + 2\vec{\Omega} \times \vec{u} + \vec{u} \cdot \vec{\nabla} \vec{u} = -\vec{\nabla} \Pi + E \vec{\nabla}^2 \vec{u} + \vec{r} \times \frac{d\vec{\Omega}}{dt}$$

$$E = \frac{10^{-6}[m^2/s]}{2\pi 10^{-2}[m^2/s]} \sim 1.6 \times 10^{-5}$$

Viscosity forces << Coriolis acceleration

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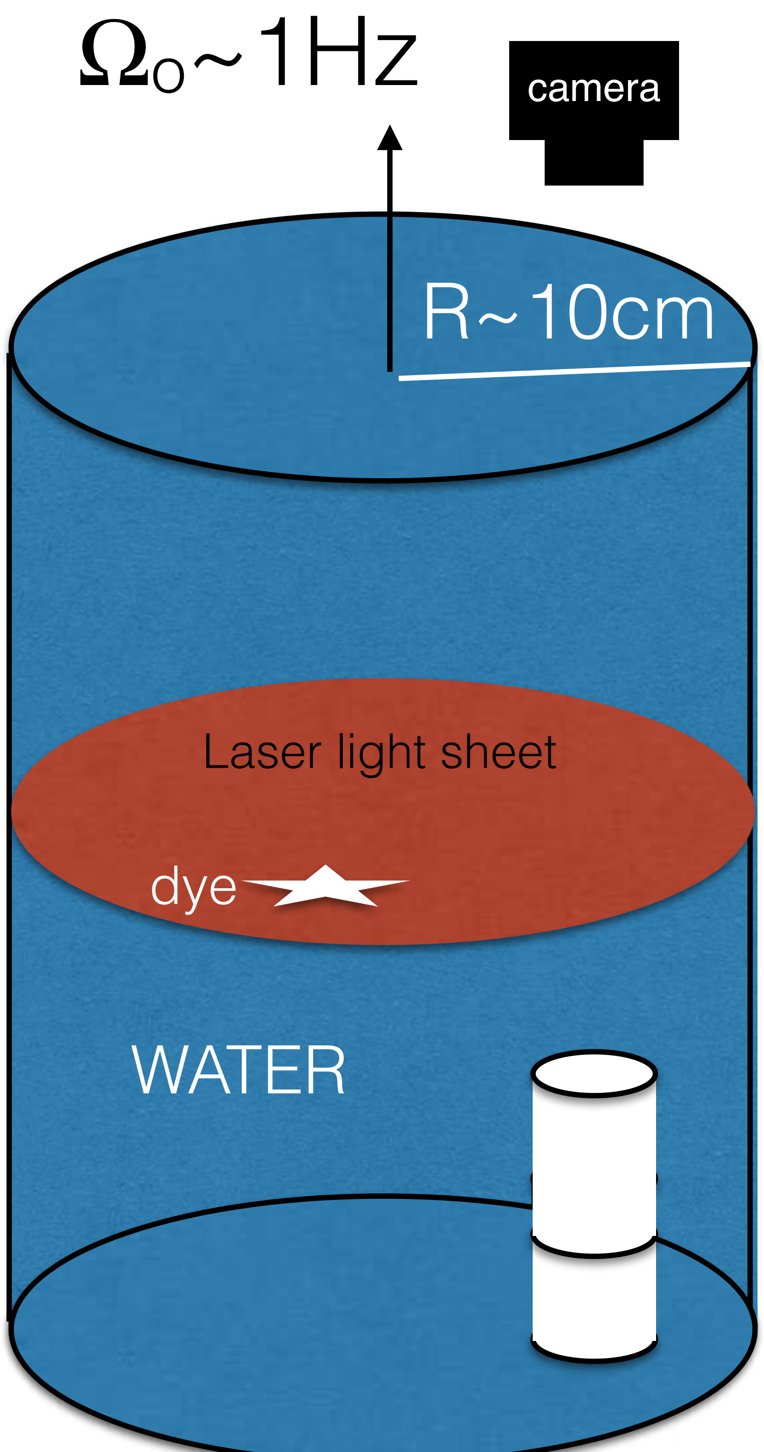

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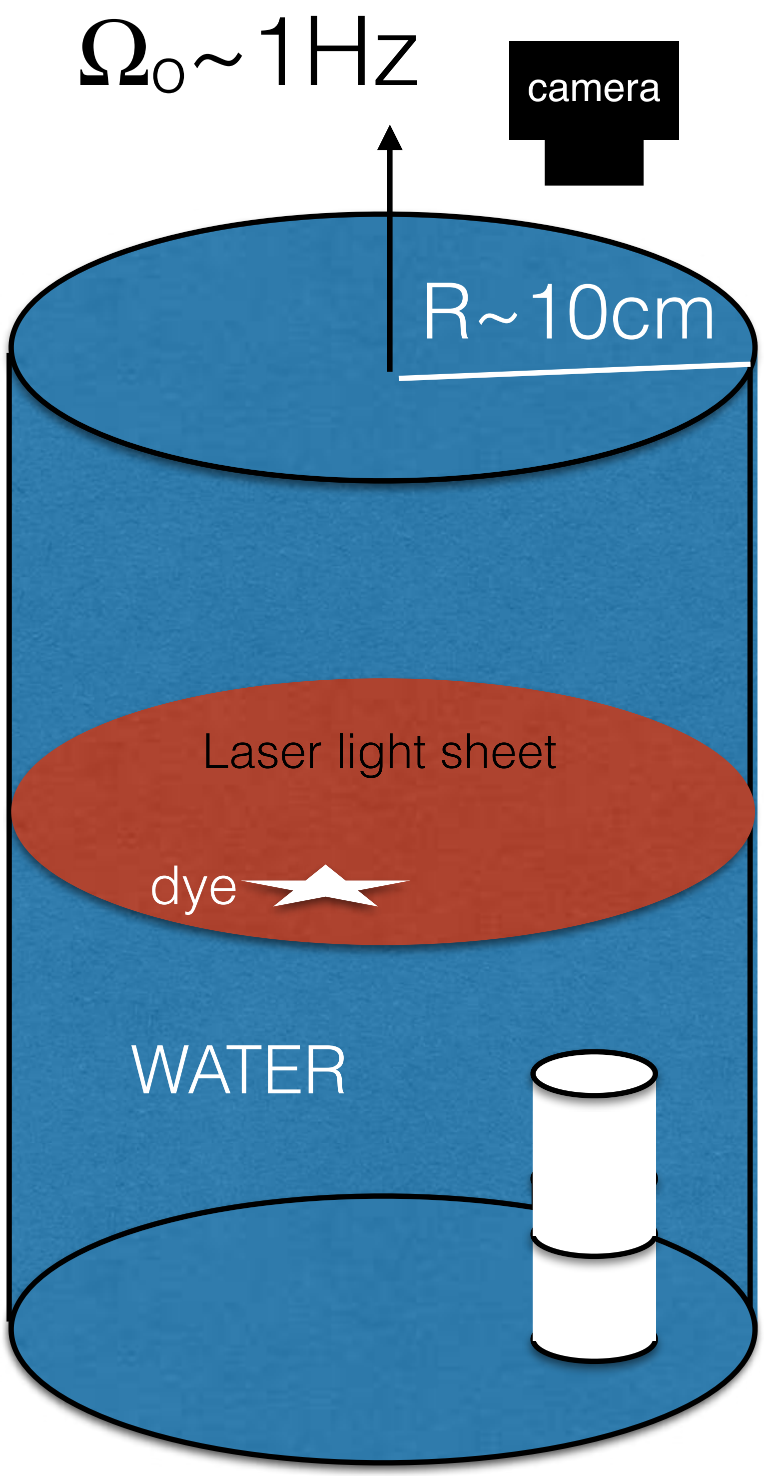

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differential rotation<< rotation

Ro~0.05

$$\vec{u} \cdot \vec{\nabla} \vec{u} << 2\vec{\Omega} \times \vec{u}$$

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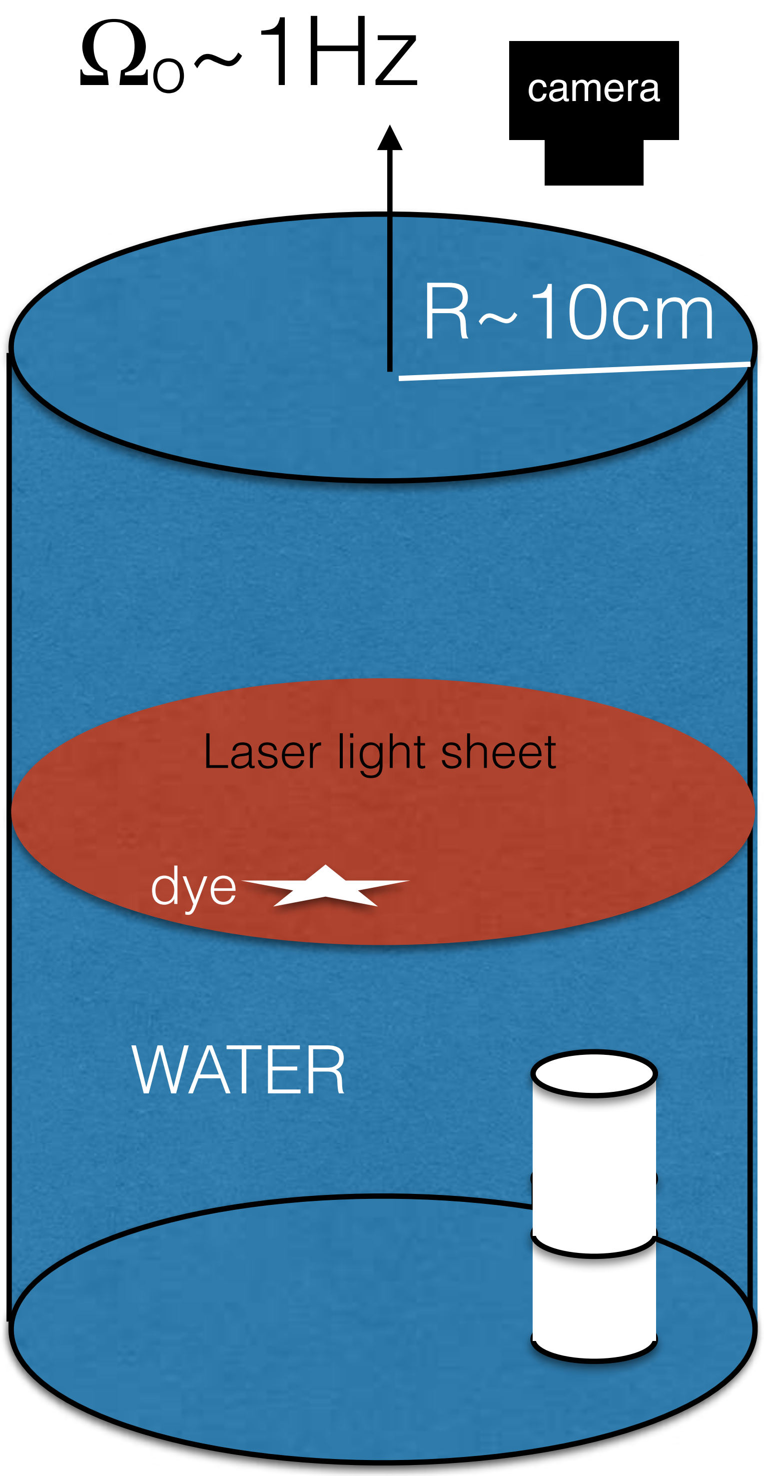

$$\frac{\partial \vec{u}}{\partial t} + 2\vec{\Omega} \times \vec{u} = -\vec{\nabla}\Pi + \vec{r} \times \frac{d\vec{\Omega}}{dt}$$

Very slow changes of the rotation rate

$$\frac{\partial .}{\partial t} << 1$$

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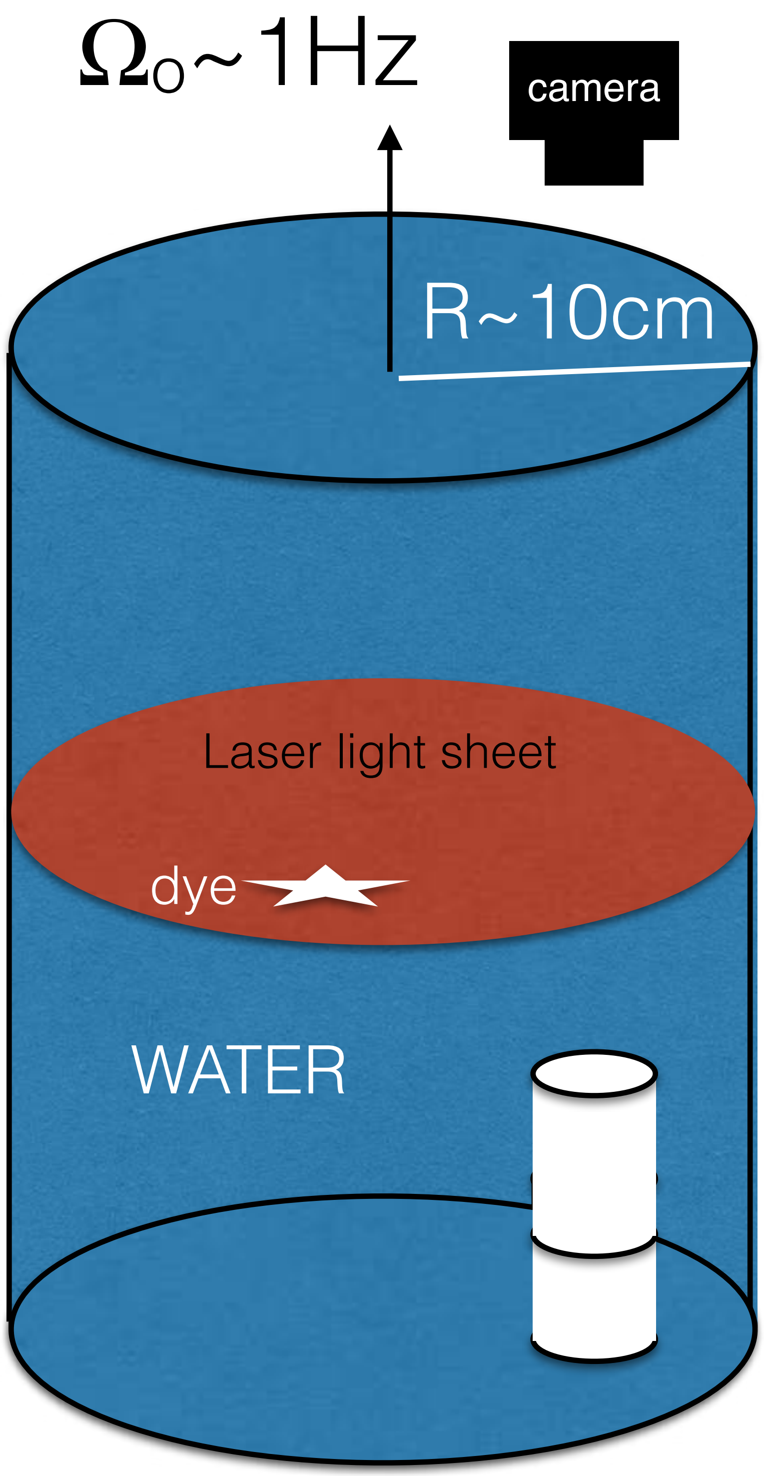

$$2\vec{\Omega} \times \vec{u} \qquad = -\vec{\nabla}\Pi$$

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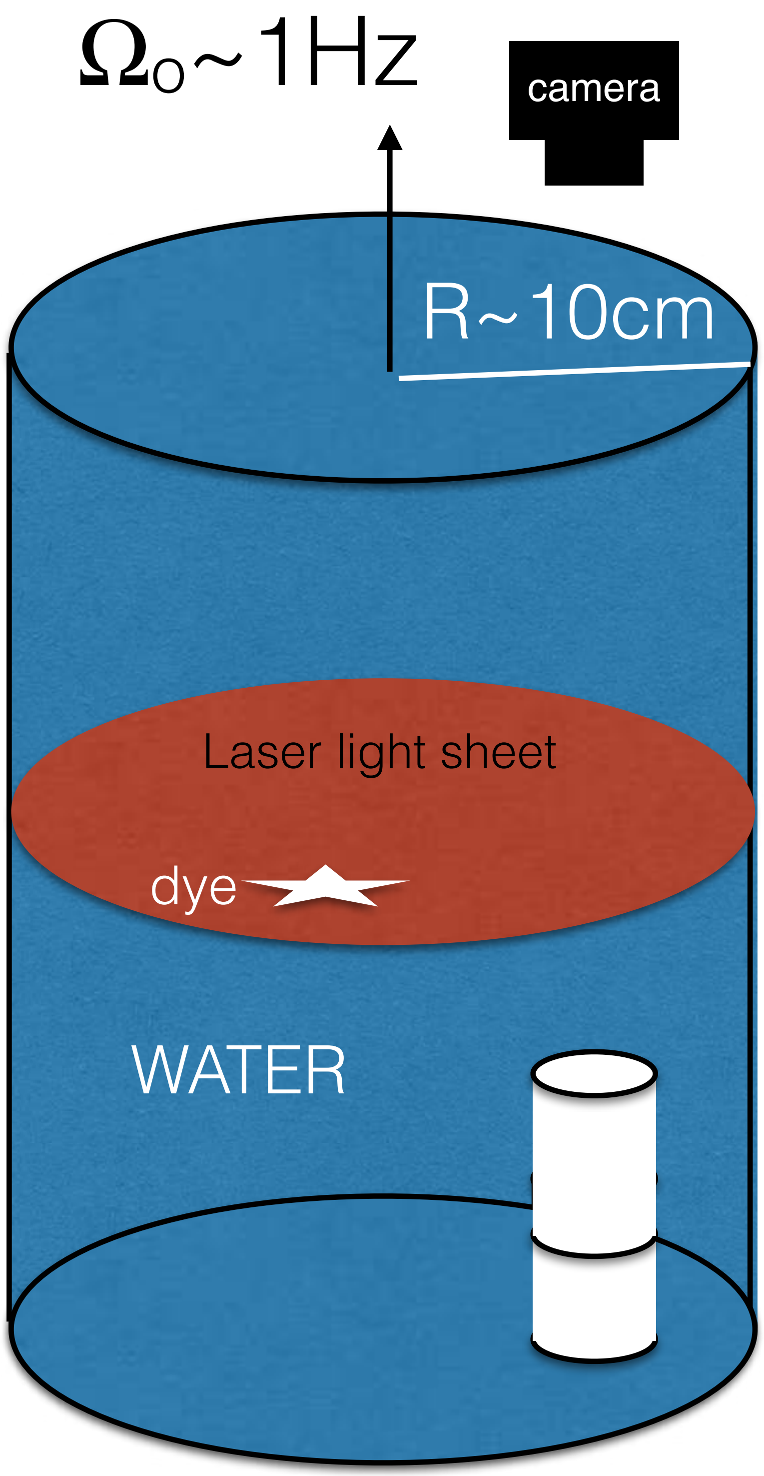

$$2\vec{\Omega} \times \vec{u} \qquad = -\vec{\nabla}\Pi$$

The reduced force balance between Coriolis and pressure is called the geostrophic balance, it is fundamental in planetary core dynamics.

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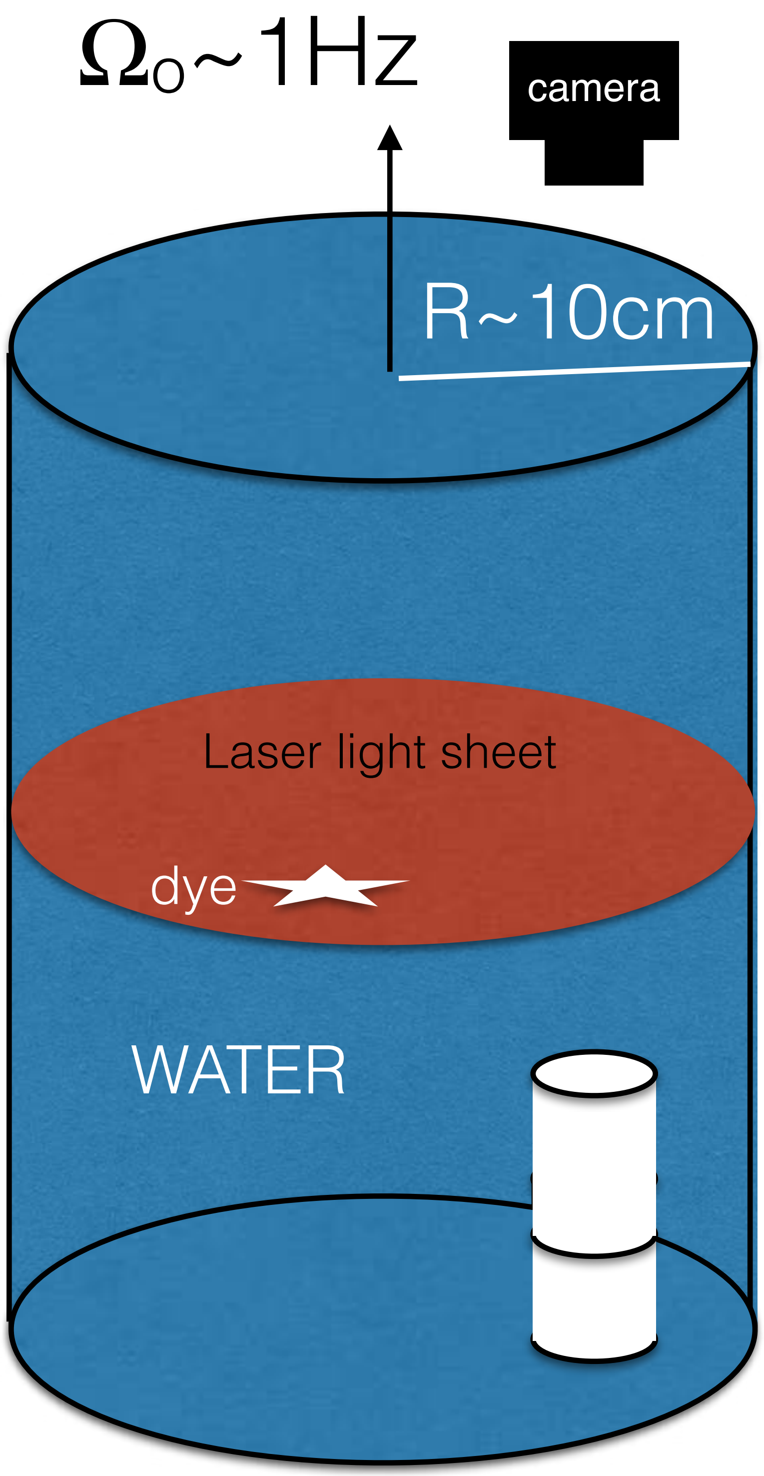

$$\vec{\nabla} \times \left[2\vec{\Omega} \times \vec{u} = \vec{\nabla}\Pi\right]$$

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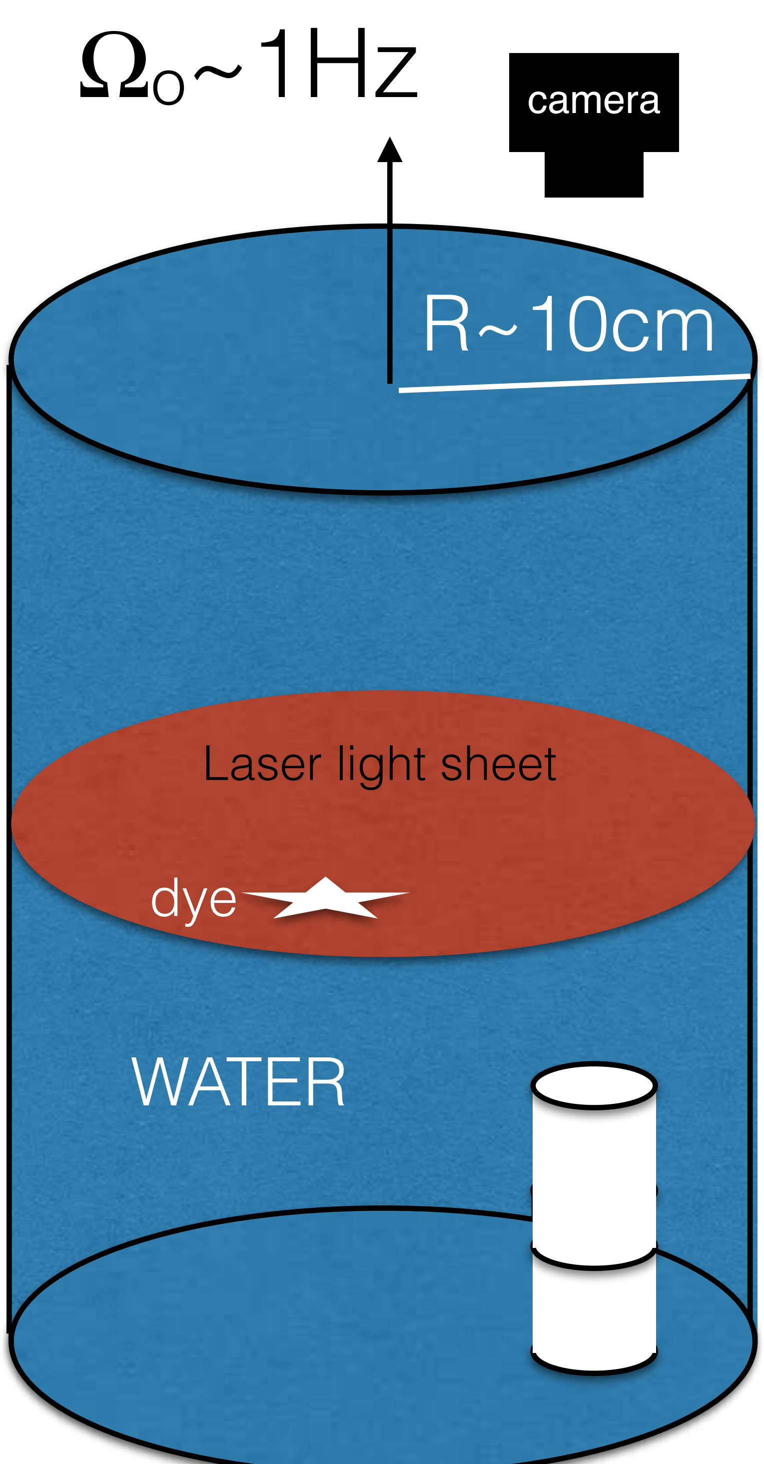

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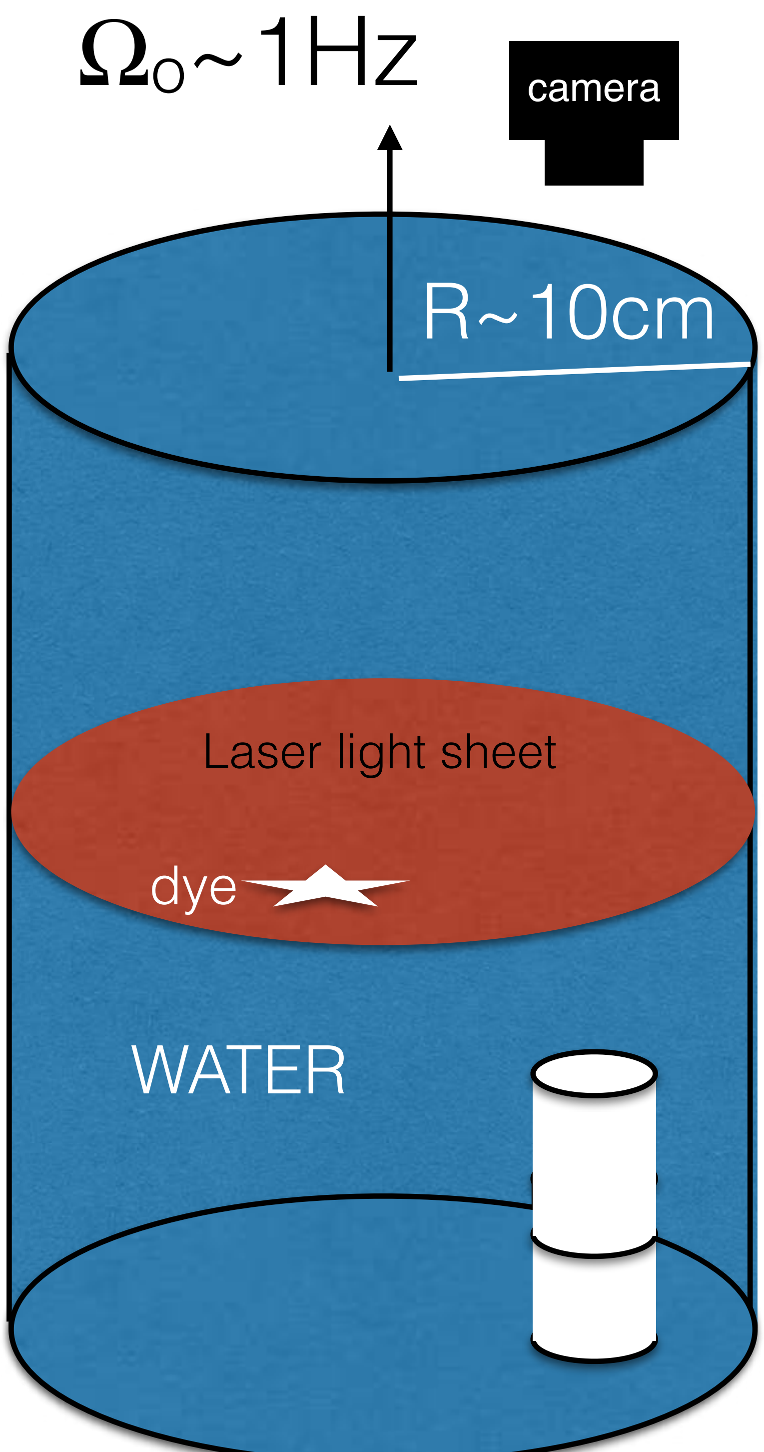

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Taylor-Proudman theorem:

Inviscid steady flows subject to rapid rotation are invariant in the direction aligned with the fluid rotation axis.

Let's come back to our simple problem of an obstacle moving relative to the fluid, or the opposite…

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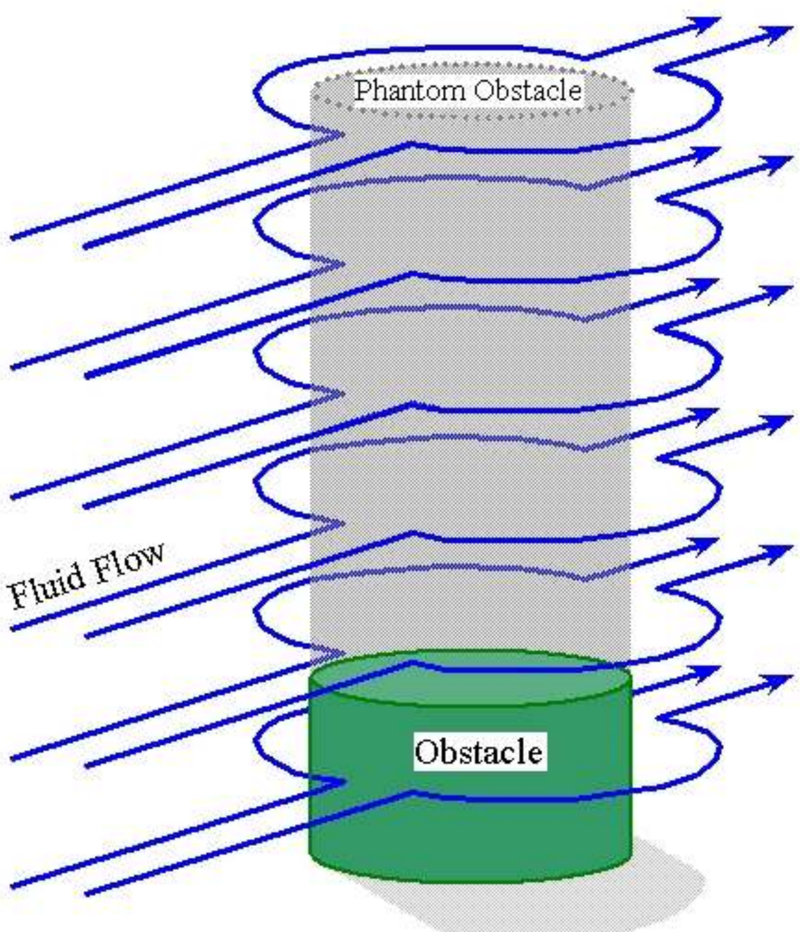

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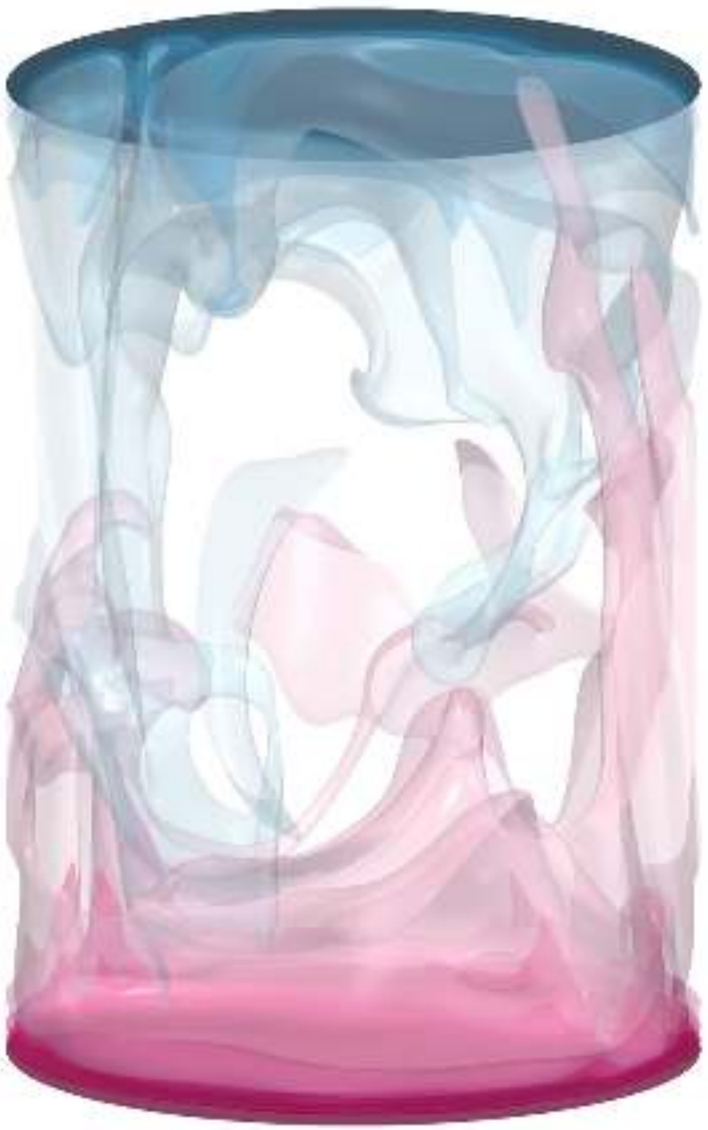
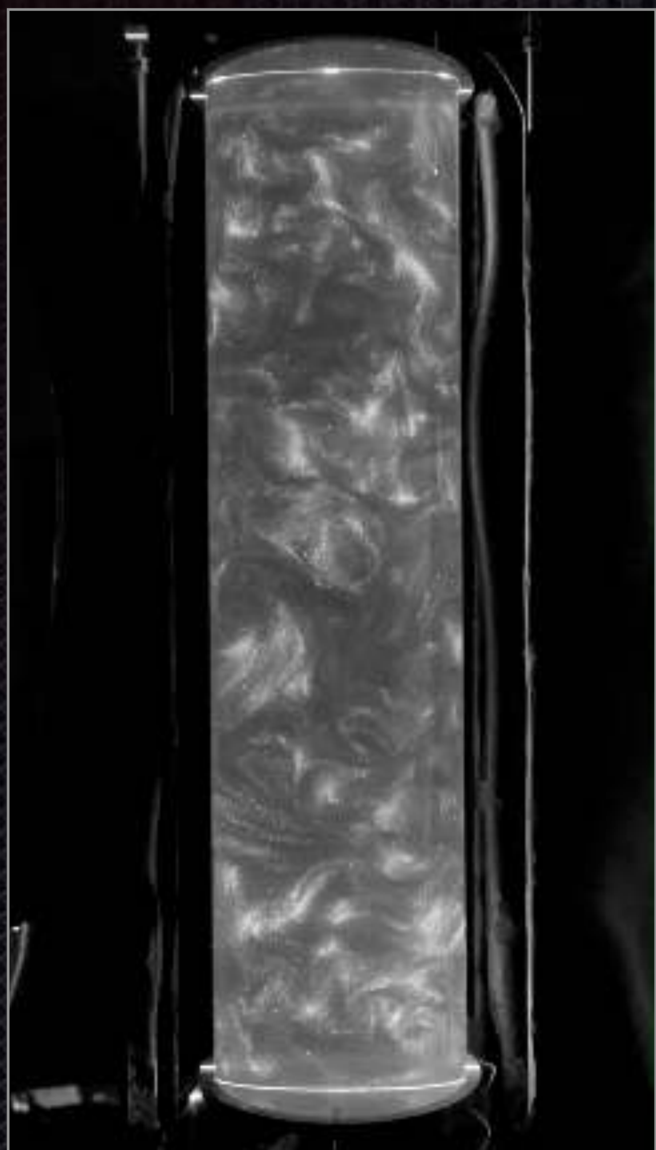
No Rotation



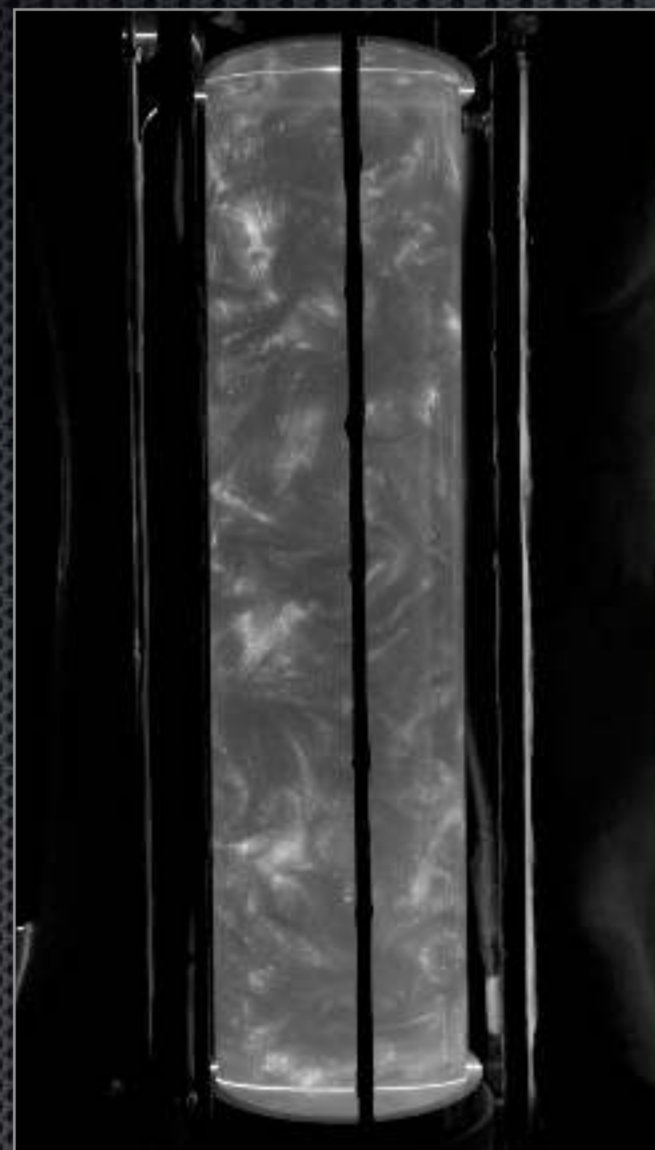
Rotation

SpinLab UCLA:Susanne Horn

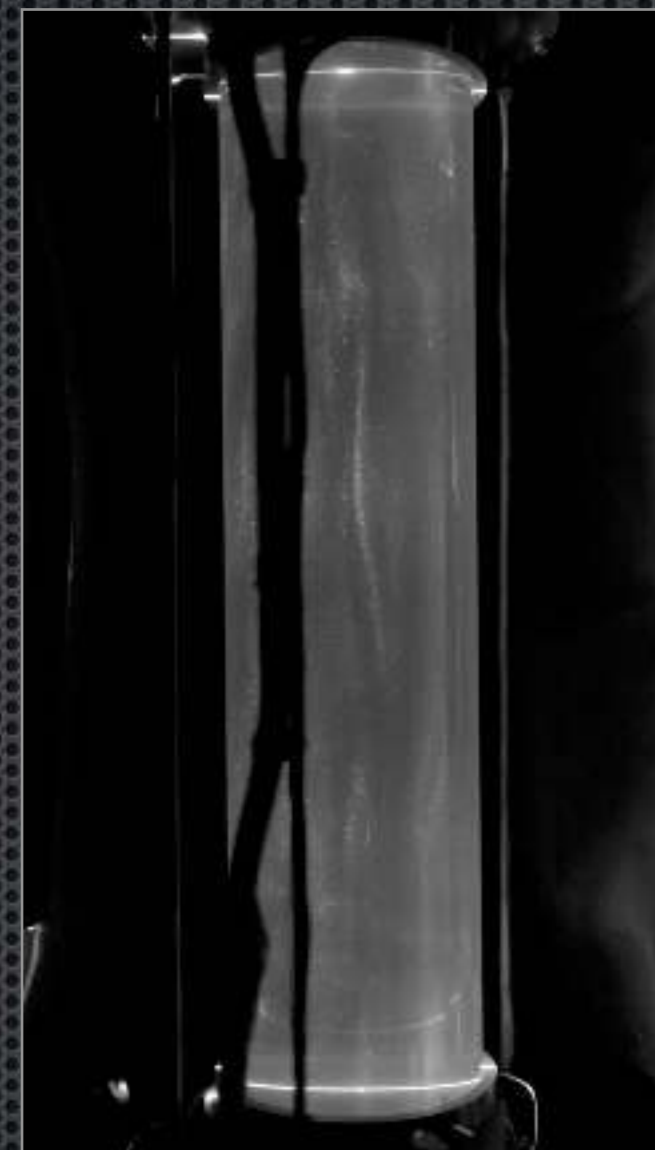
RoMag: 80 cm tanks



E = inf.
$Ra \sim$ 2.7e10
Ro_C = inf.



E = 1.5e-5
$Ra \sim$ 2.9e10
$Ro_C \sim$ 0.97

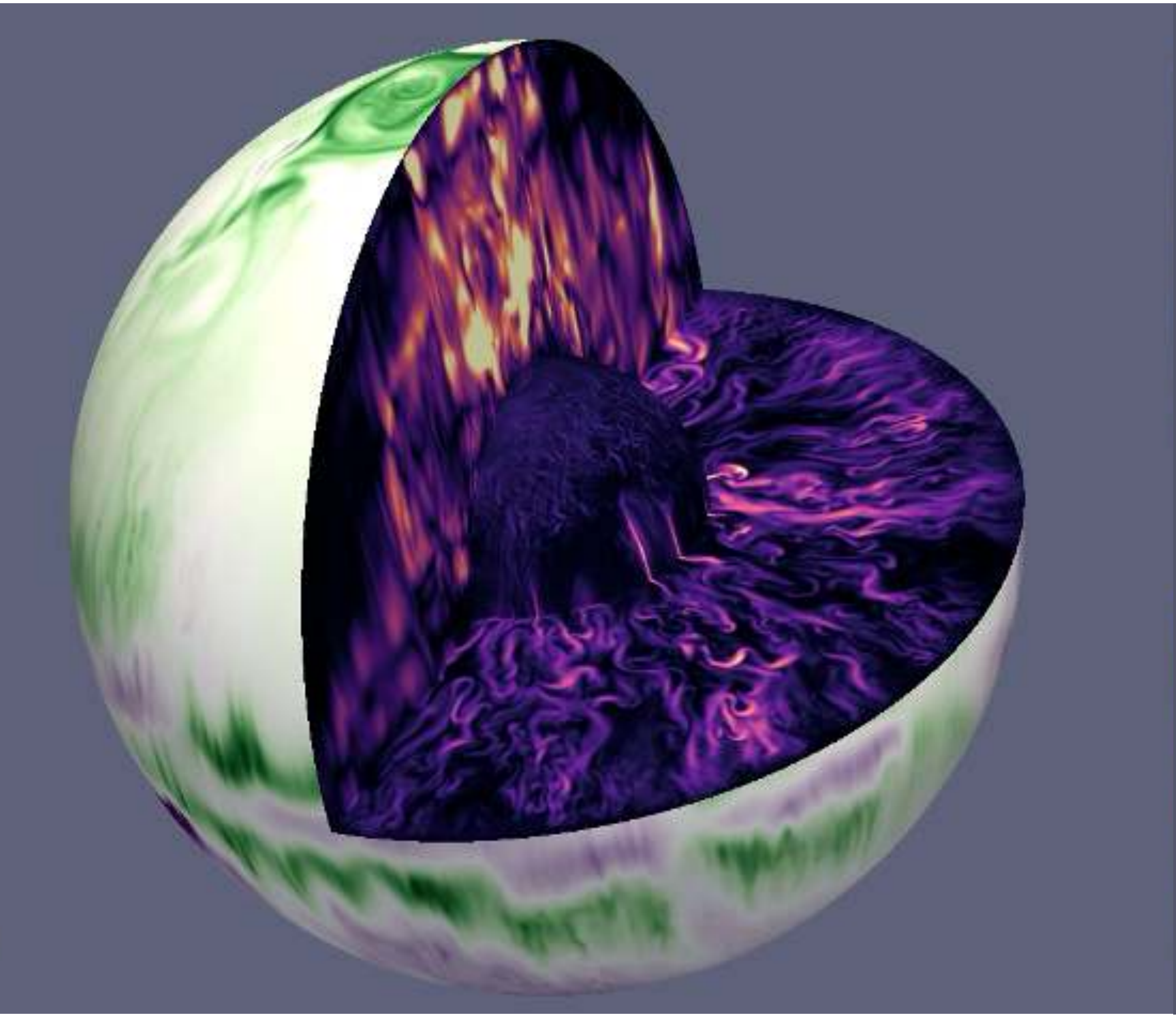


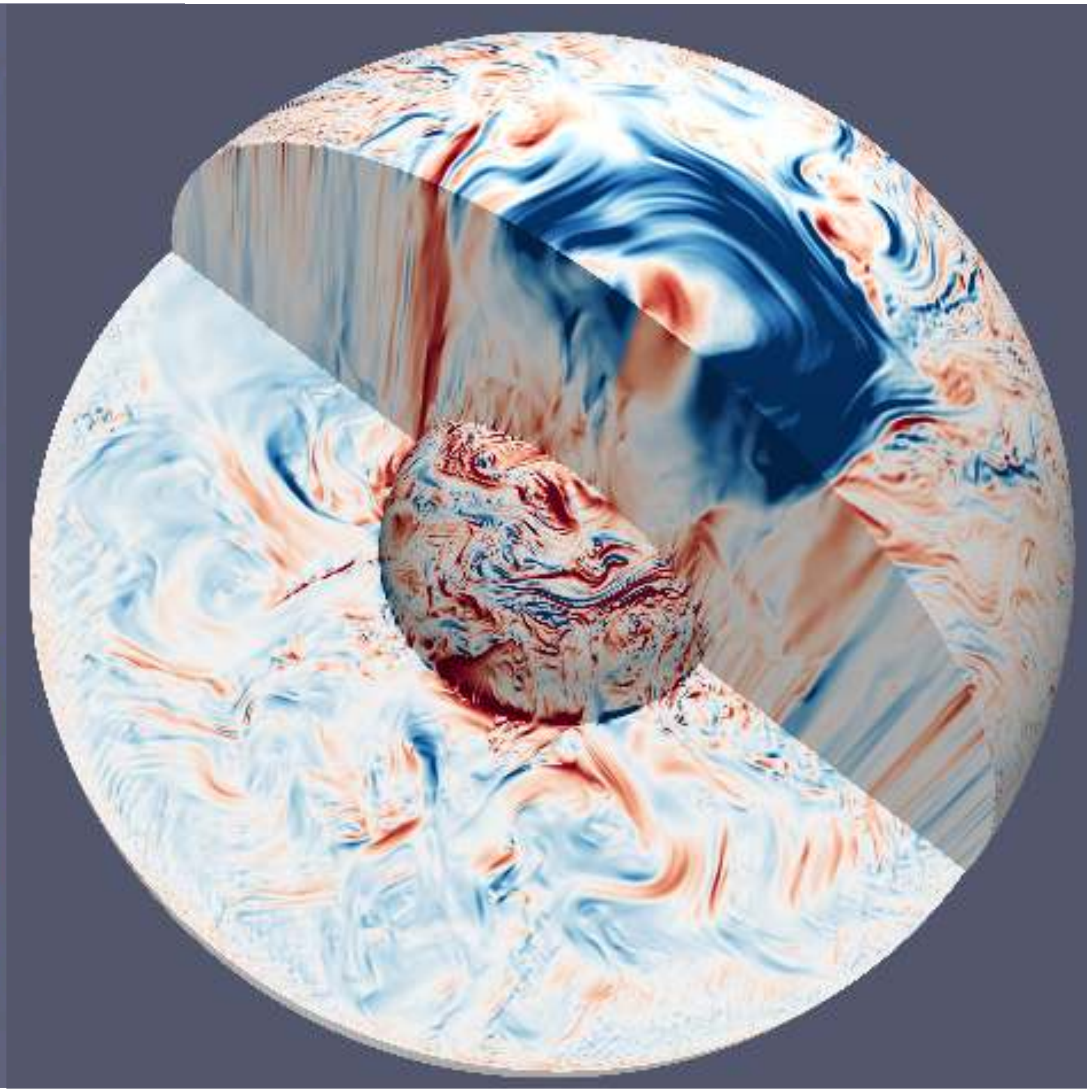
E = 6.0e-7
$Ra \sim$ 3.0e10
$Ro_C \sim$ 0.039

E = 1.0e-7
$Ra \sim$ 5.3e10
$Ro_C \sim$ 0.0087

Cheng et al. *GJI* 2015

Rapidly rotating convection





courtesy of Nathanael Schaeffer, rotating convection simulations performed with XSHELL

Flows predominantly governed by the geostrophic balance will be quasi 2D over a certain extend in the direction parallel to this axis of rotation. As shown by Keith Julien, it does not necessarily means that fully developed Taylor columns form in the volume.

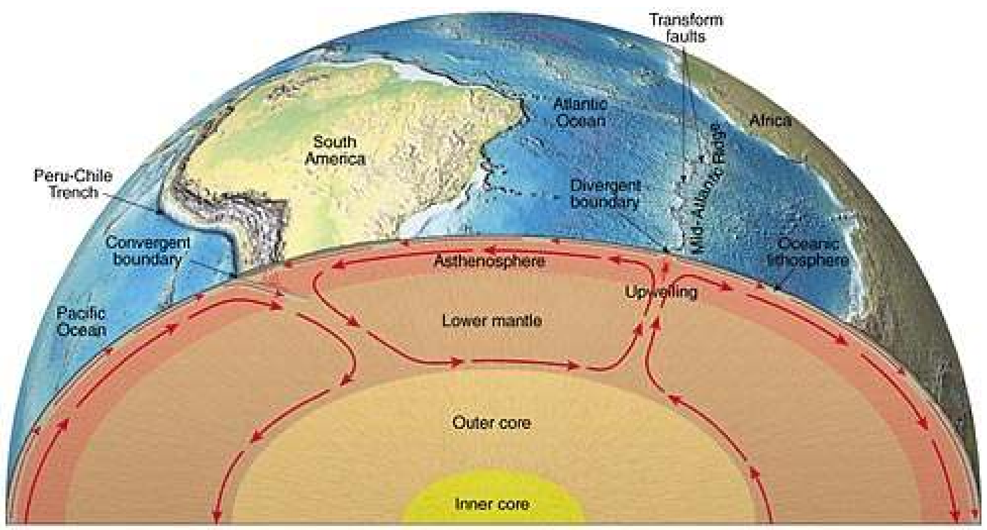
Numerical and theoretical efforts are devoted to the development of very efficient quasi 2D models to go extend the accessible range of parameters of the 3D-DNS, getting now close to planetary conditions. (See Keith Julien lectures)

Introduction to rapidly rotating fluids:

1. Some observations
2. Navier-Stokes in a rotating frame
3. Taylor-Proudman Theorem
4. Earth’s core and mantle.

Is the Earth's mantle a rotating fluid ?

The Earth mantle behaves as a fluid on geological time scales (1-10Myrs). Due to heat flux released at the CMB by the Core and radioactive sources in the bulk, Thermo-chemical convection cells are driven in the mantle which couple to the plate tectonics via viscous shear in the Asthenosphere.



When investigating Earth's mantle convection from the point of view of an observer rotating at the surface of the Earth, shall we take into account the effect associated to the rotation, i.e. Coriolis, centrifuge...?

Flow velocity in the mantle.

At first order we can consider the rotation of the Earth to be constant, such that the Poincaré acceleration can be neglected. Using the values proposed by Y. Ricard in the Treatise of geophysics section 7.02

- a typical velocity scale given by the plate tectonics,

$$U \sim 10cm/yr\,(3 \times 10^{-10} m/s), \qquad (81)$$

- a typical length scale based on the depth of the mantle,

$$L \sim 3 \times 10^{6} m, \qquad (82)$$

- a typical viscosity in the mantle,

$$\nu \sim 10^{17}. \qquad (83)$$

In the mantle:

- the inertia is negligible compare to the Coriolis acceleration,

$$\boxed{Ro \sim \frac{1}{2\pi} 10^{-11},} \qquad (84)$$

- the Coriolis acceleration is negligible compare to the viscous force,

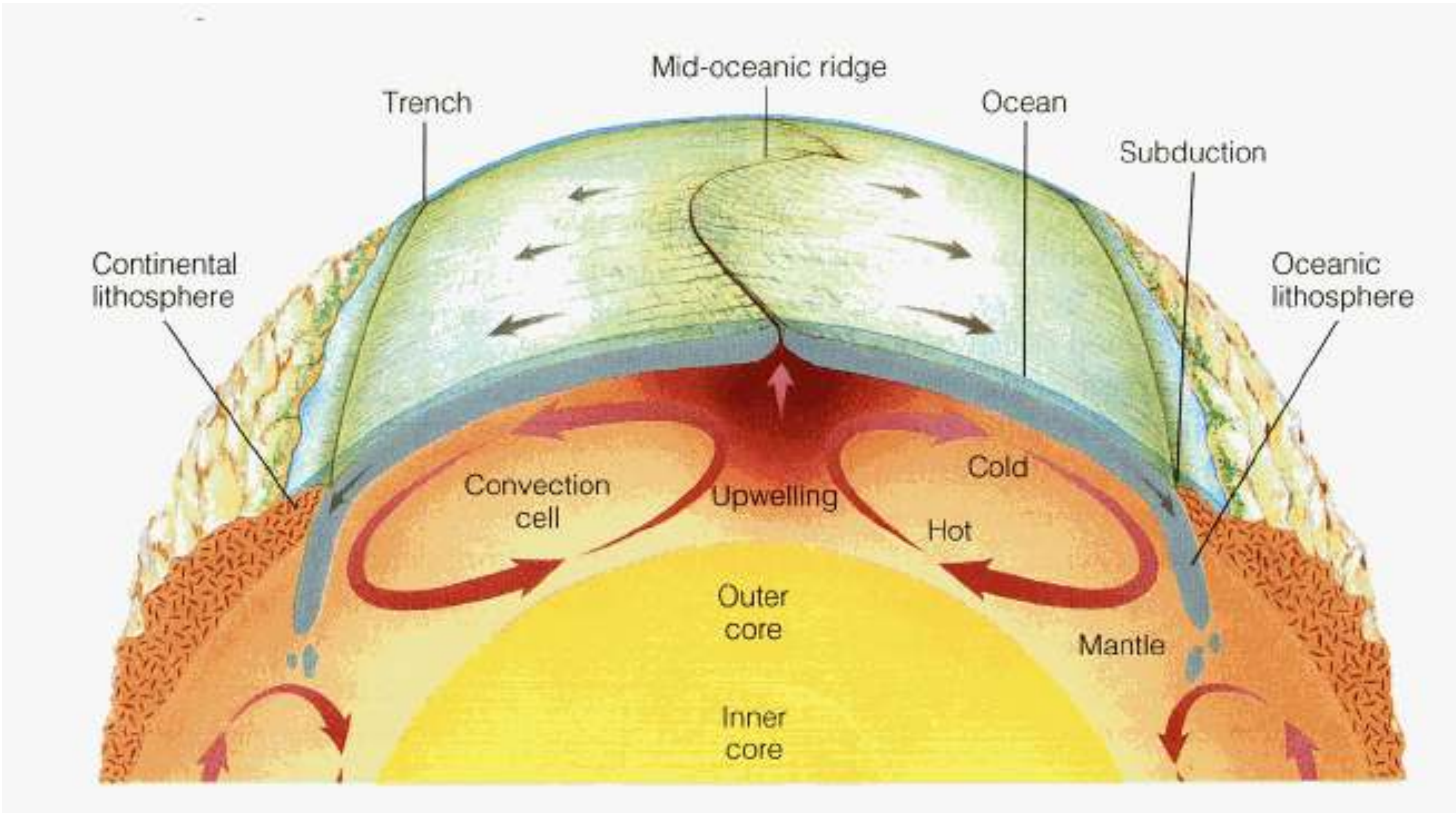
$$\boxed{E \sim \frac{1}{2\pi} 10^{9},} \qquad (85)$$

In the mantle the main forces balance is:

$$\boxed{Pressure + Viscous + Buoyancy = 0} \qquad (86)$$

.

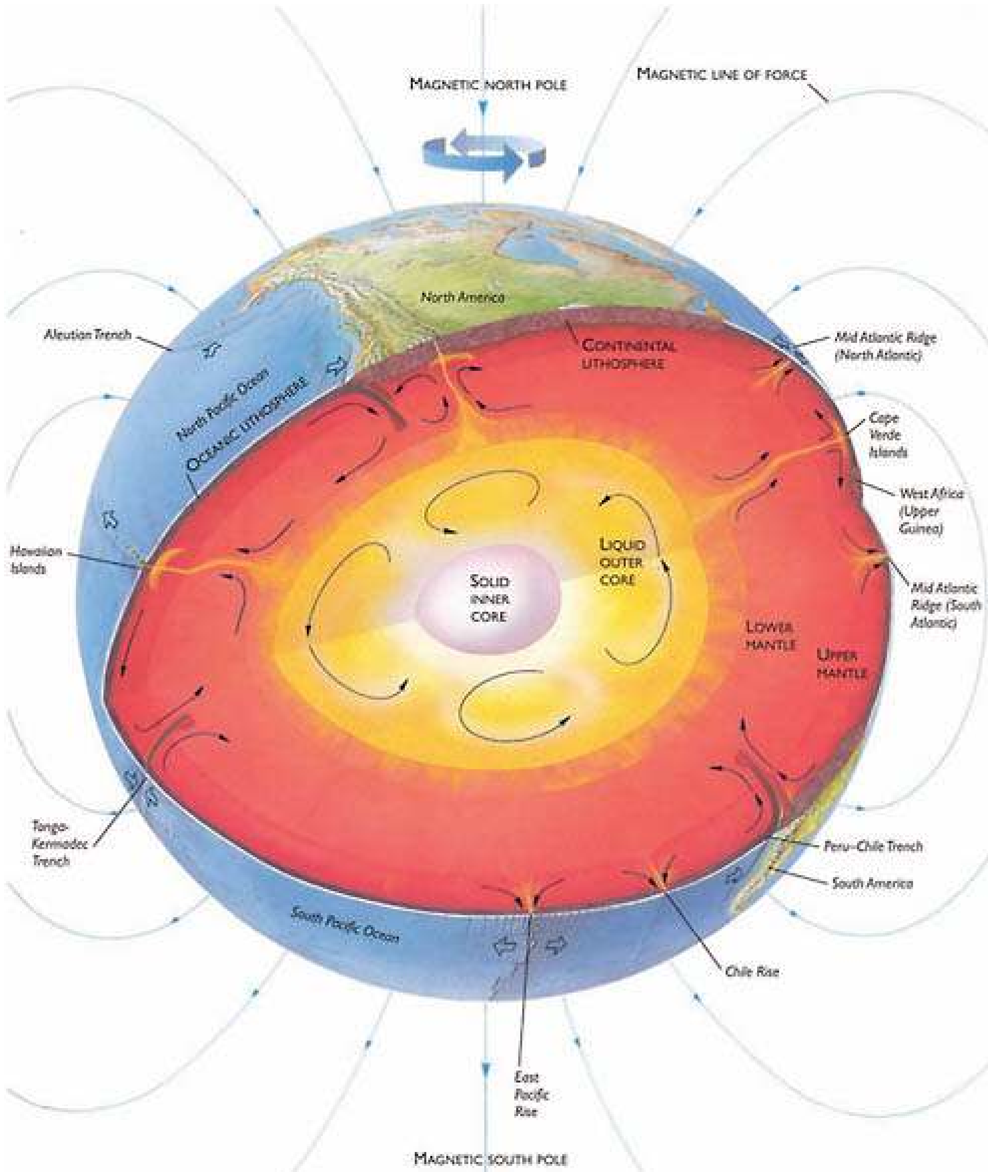
Earth Mantle convection



Byrd Polar Research Center at Ohio State University

Is the Earth's core a rotating fluid ?

It is now weel accepted that the Earth's core is subject to convection due to the crystallization of the inner core that release latent heat and light elements.



When investigating Earth's core convection from the point of view of an observer rotating at the surface of the Earth, shall we take into account the effect associated to the rotation, i.e. Coriolis, centrifuge...?

Is the Earth's core a rotating fluid ?

- a typical velocity scale based on the westward drift of the magnetic field,

$$U \sim 1cm/s\,(10^{-2}m/s), \qquad (87)$$

- the inertia is negligible compare to the Coriolis acceleration,

$$\boxed{Ro \sim 5 \times 10^{-5},} \qquad (90)$$

Variation Séculaire: T≈400ans, la derive vers l’ouest.

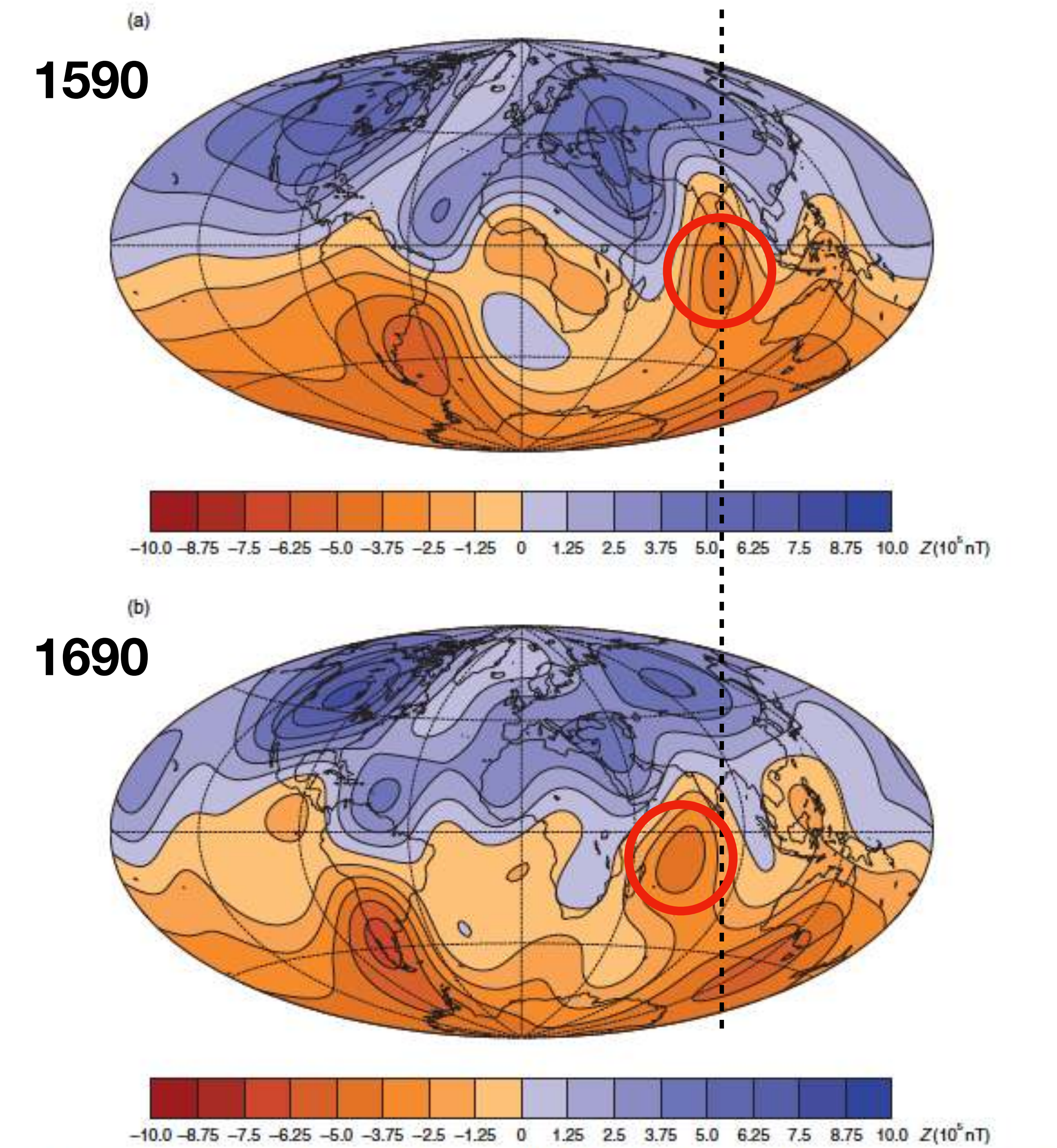


Figure 29 Vertical magnetic field B_Z at the core surface in (a) AD 1590 and (b) AD 1690 from the model *gufm1* of Jackson *et al.* (2000). Plots are Molleweide projection, units are nanotesla.

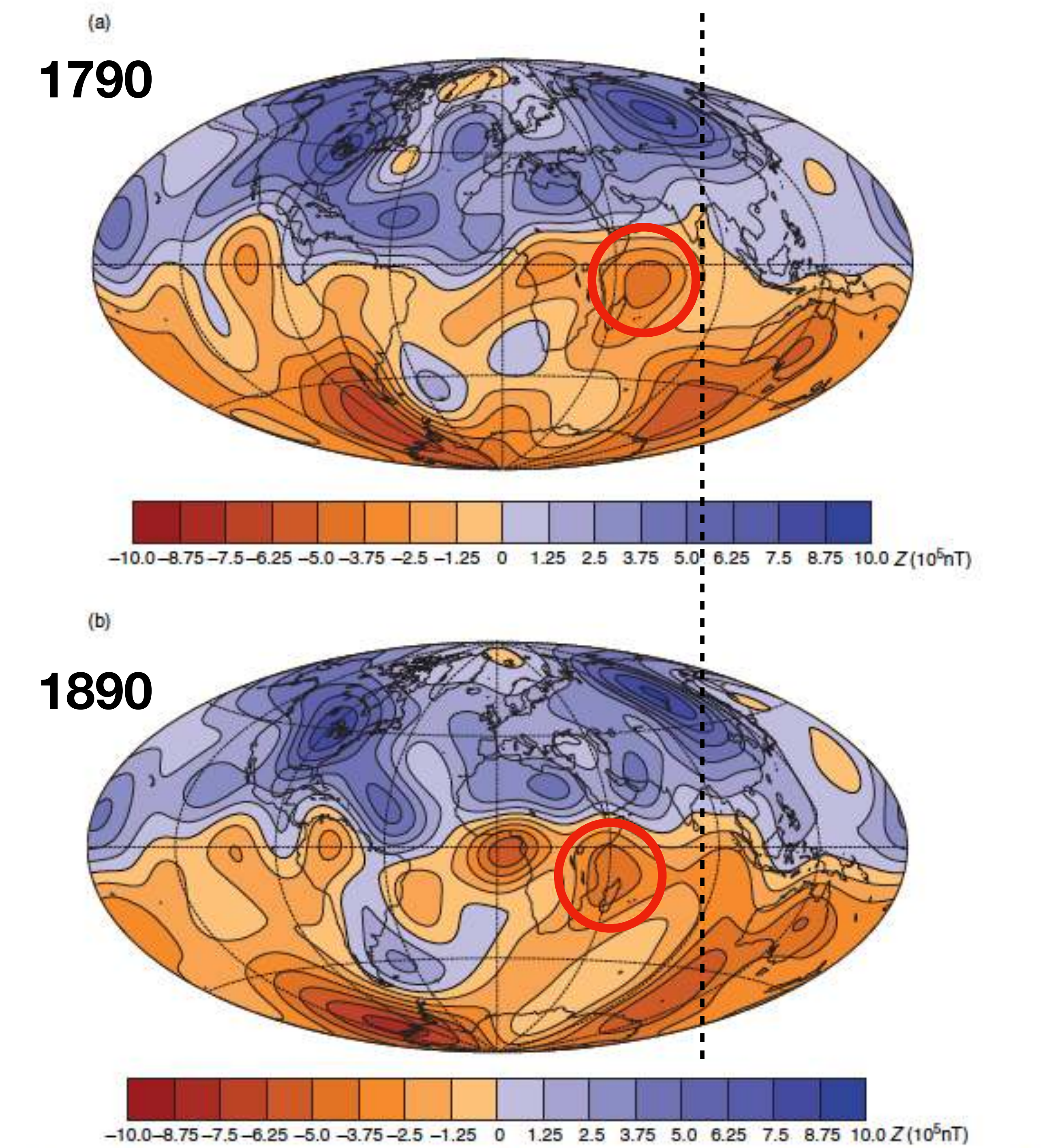


Figure 30 Vertical magnetic field B_Z in (a) AD 1790 and (b) AD 1890 from the model *gufm1* of Jackson *et al.* (2000). Plots are Molleweide projection, units are nanotesla.

From Jackson and Finlay in *Treatise of Geophysics 5, Geomagnetism*.

Is the Earth's core a rotating fluid ?

- a typical velocity scale based on the westward drift of the magnetic field,

$$U \sim 1cm/s\,(10^{-2}m/s), \quad (87)$$

- a typical length scale based on the depth of the liquid outer core,

$$L \sim 2 \times 10^{6} m, \quad (88)$$

- a typical viscosity in the core,

$$\nu \sim 10^{-7}. \quad (89)$$

- the inertia is negligible compare to the Coriolis acceleration,

$$\boxed{Ro \sim 5 \times 10^{-5},} \quad (90)$$

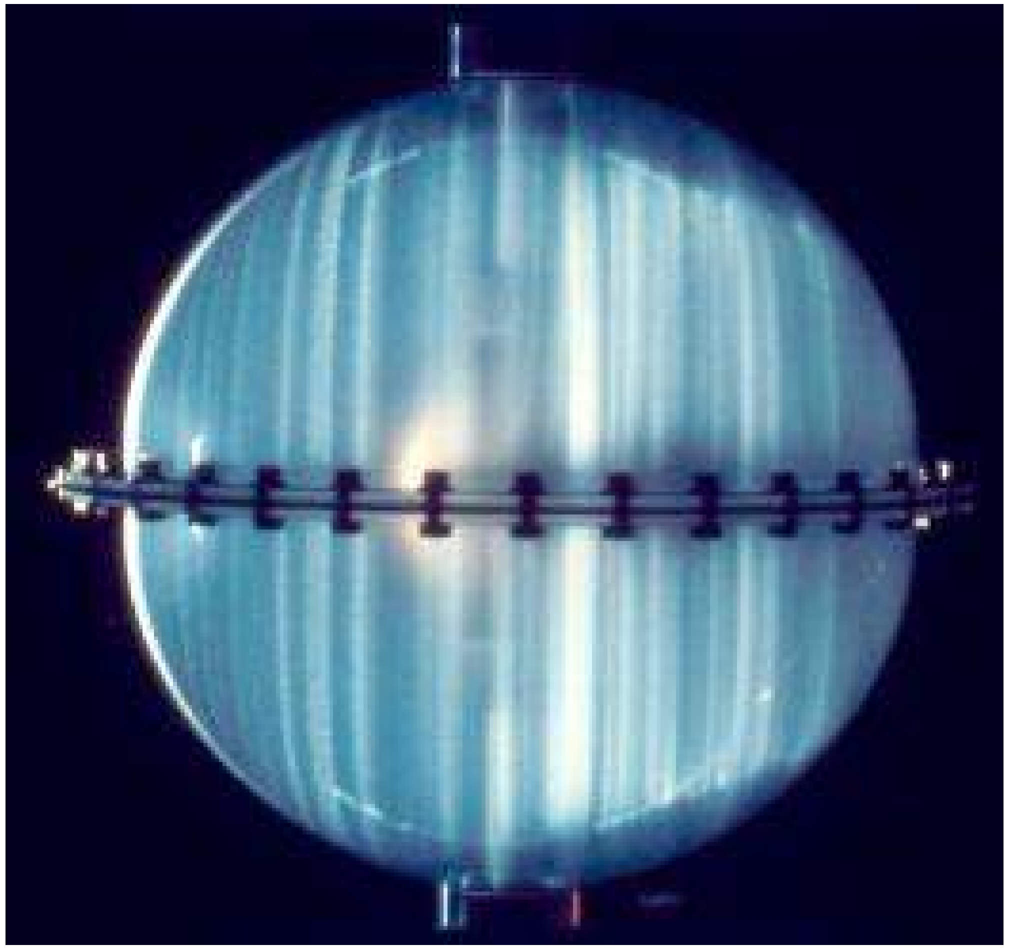
- the viscous force is negligible compare to the Coriolis acceleration,

$$\boxed{E \sim 10^{-15},} \quad (91)$$

In the Earth's core, the flow is strongly influenced by the rotation and essentially inviscid. The dominant force balance is:

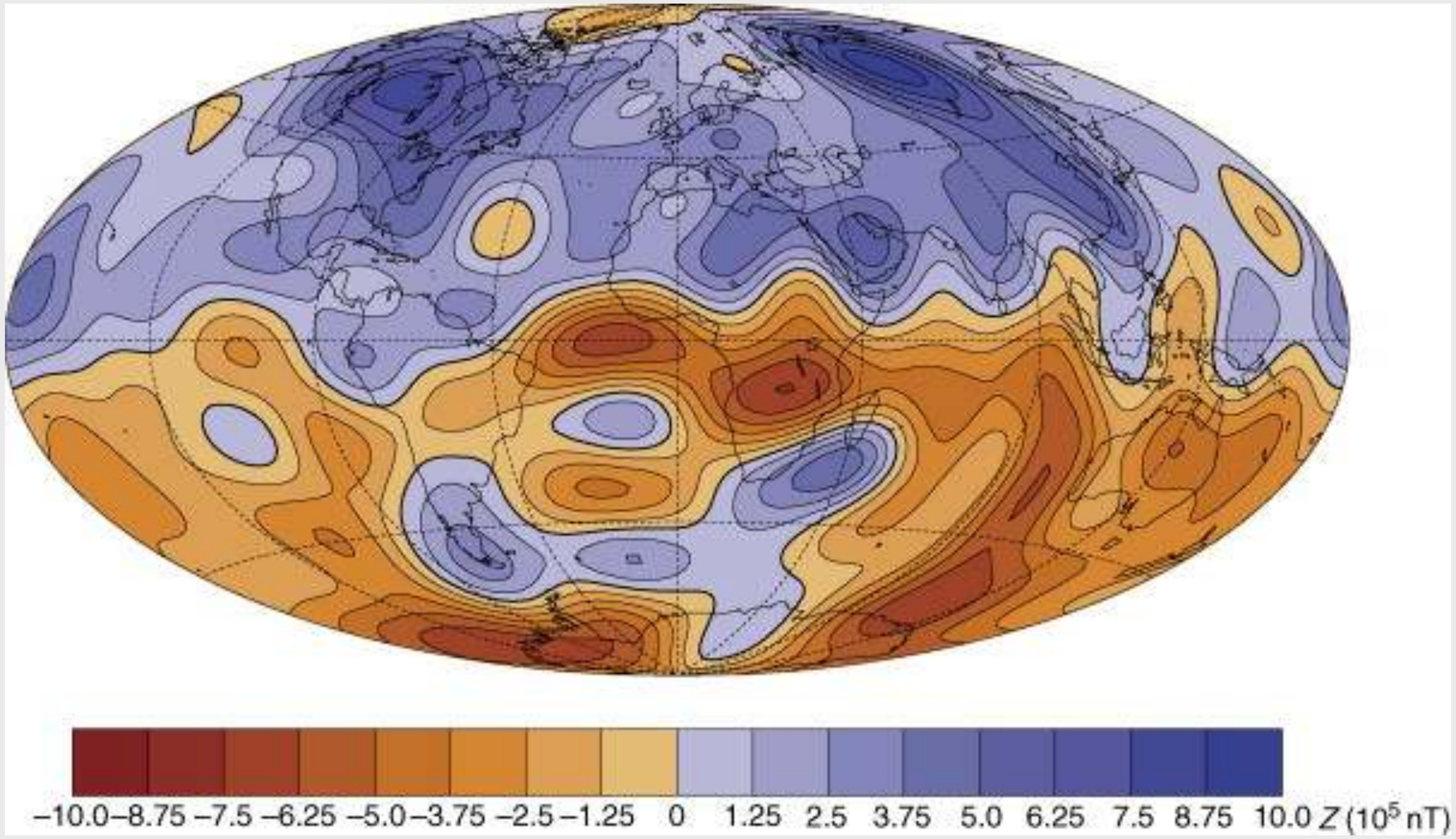
$$\boxed{\frac{\partial \mathbf{u}}{\partial t} + Coriolis + Pressure + (Buoyancy, Lorentz) = 0} \quad (92)$$

Core convection

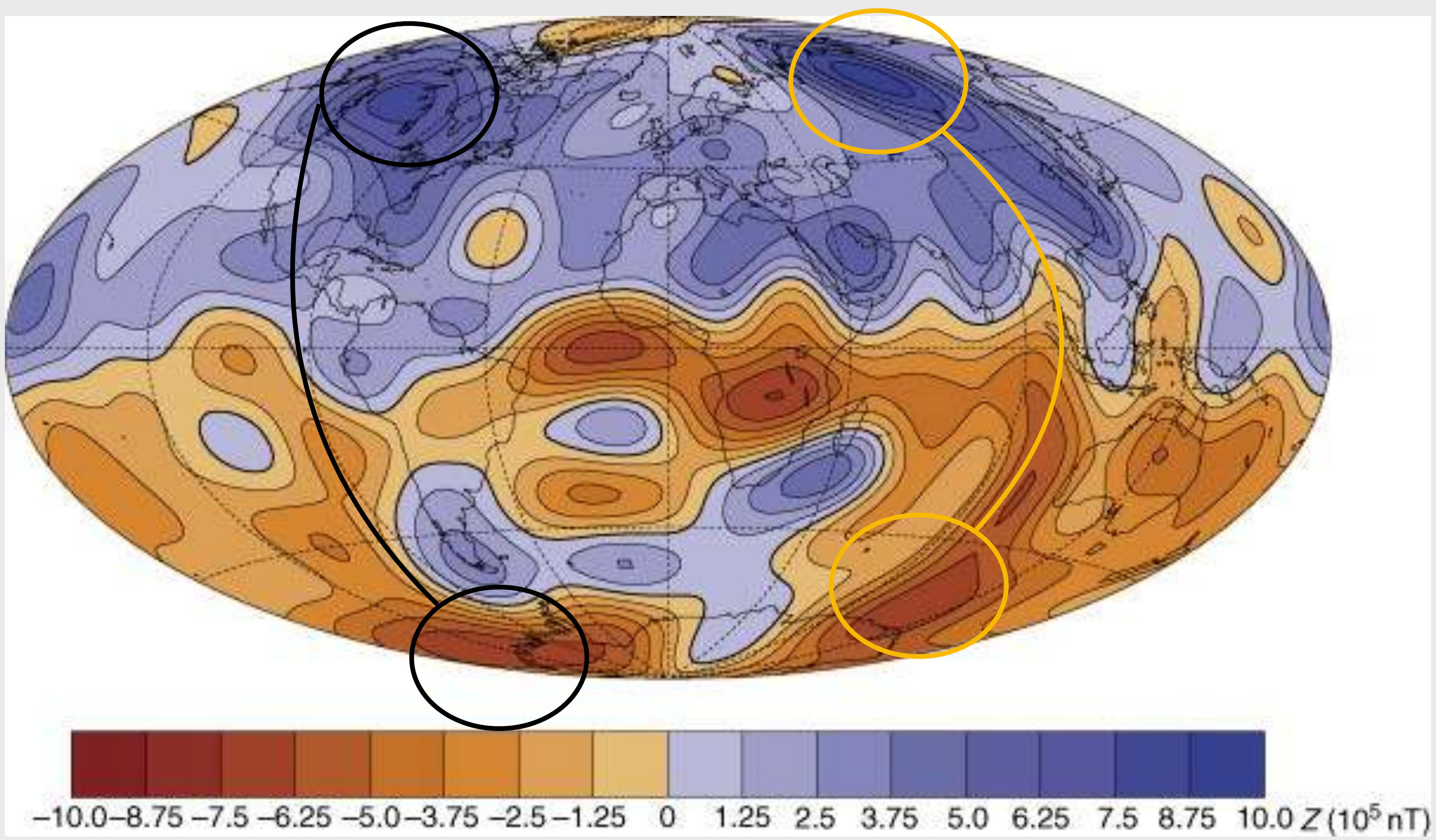


Laboratory experiments (by P. Olson, Johns Hopkins University, Baltimore, USA)

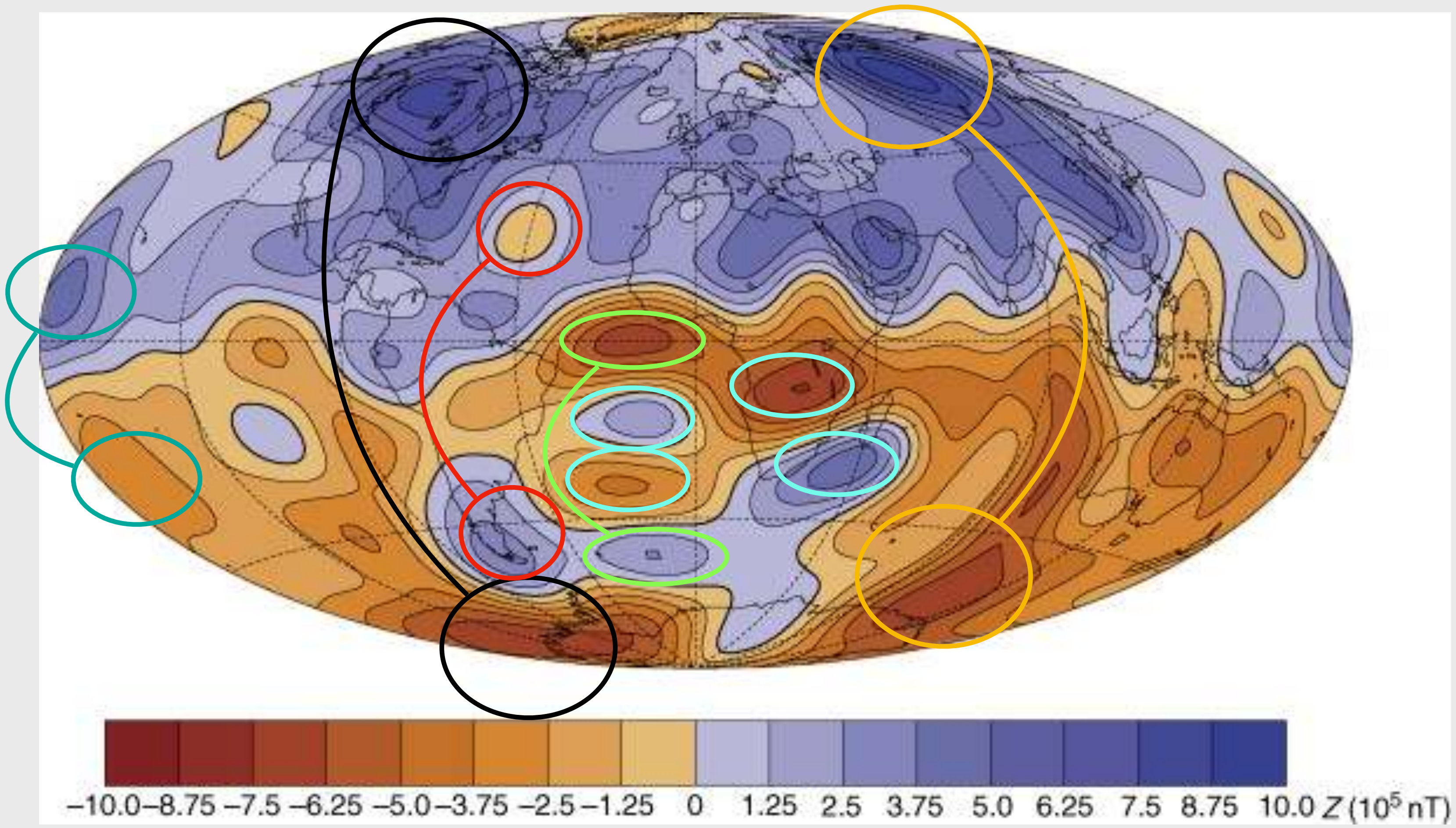
In the Earth's core the convection is strongly influence by the rotation, the convective structures are in the form of elongated columns parallel to the axis of rotation.



From Jackson and Finlay in *Treatise of Geophysics 5, Geomagnetism*.



From Jackson and Finlay in *Treatise of Geophysics 5, Geomagnetism*.

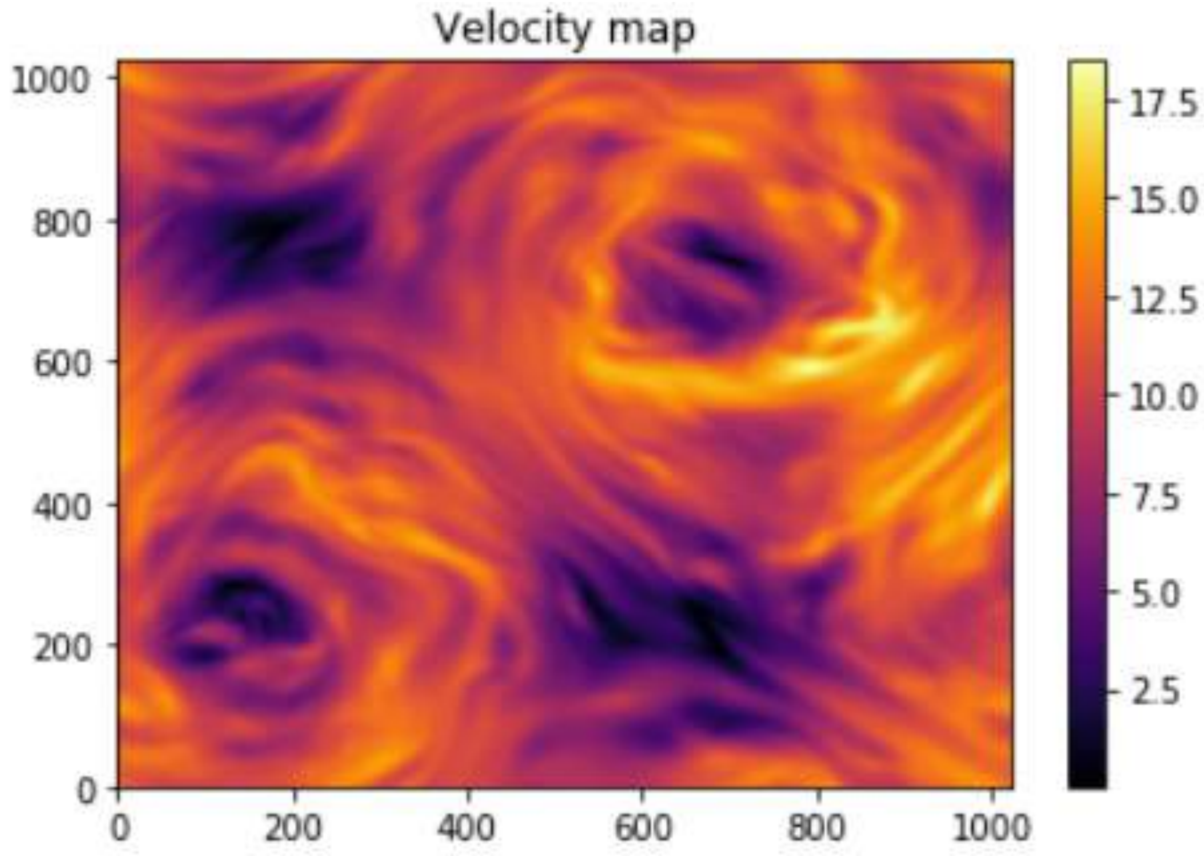


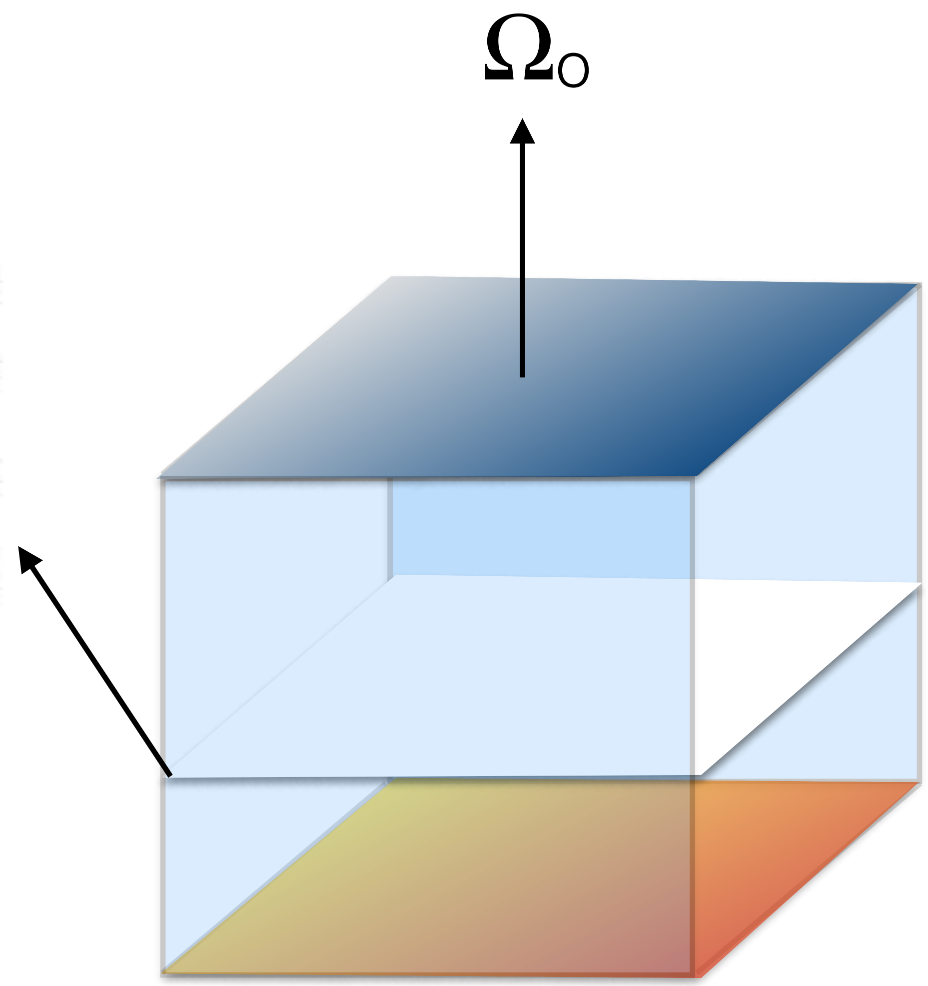
From Jackson and Finlay in *Treatise of Geophysics 5, Geomagnetism*.

To go further

1. Greenspan section 1
2. K.K. Zhang section 1
3. M. Rieutord section 8
4. Talk by Keith Julien on Quasi-geostrophic models

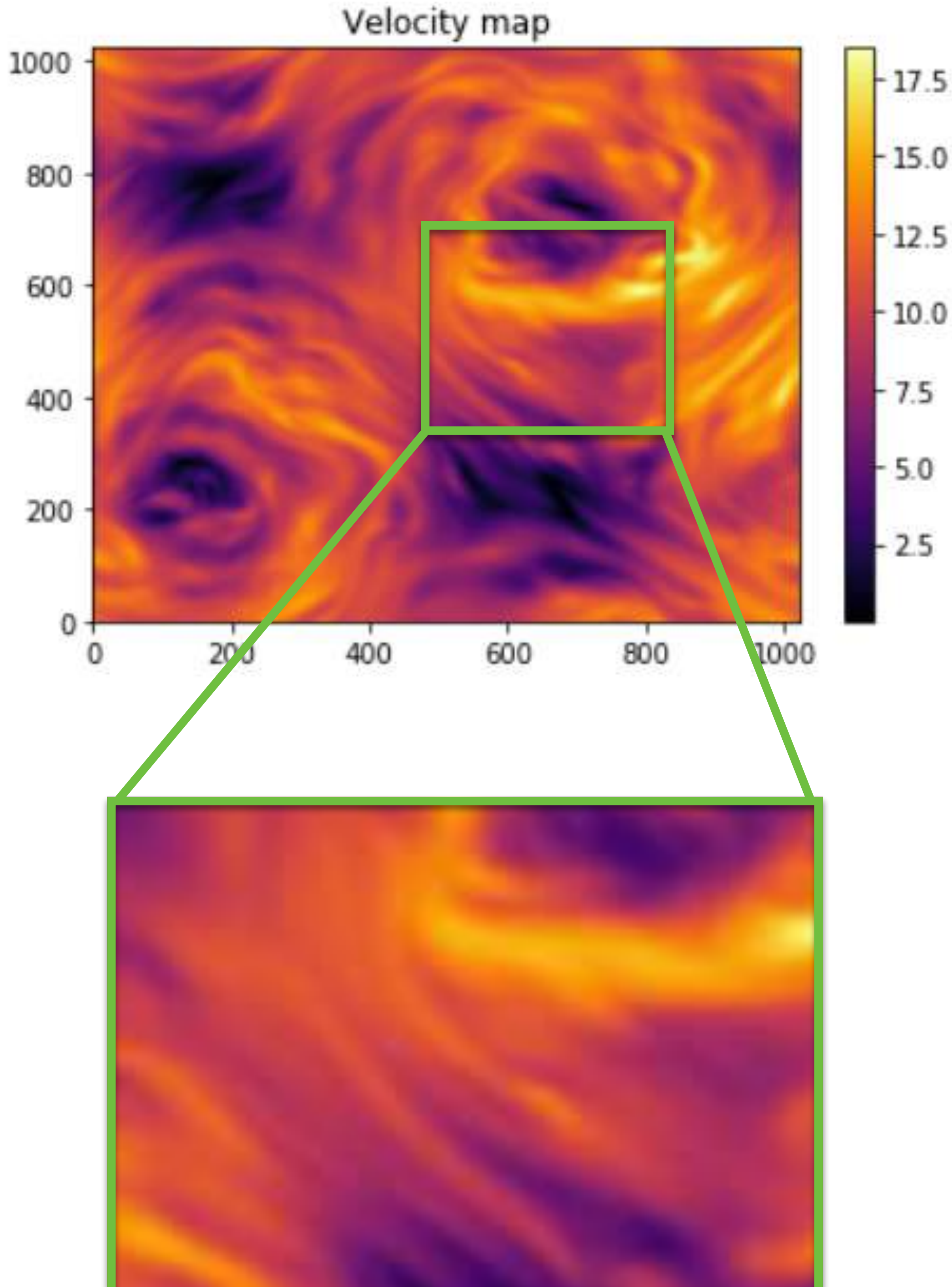
Nothing is white or black

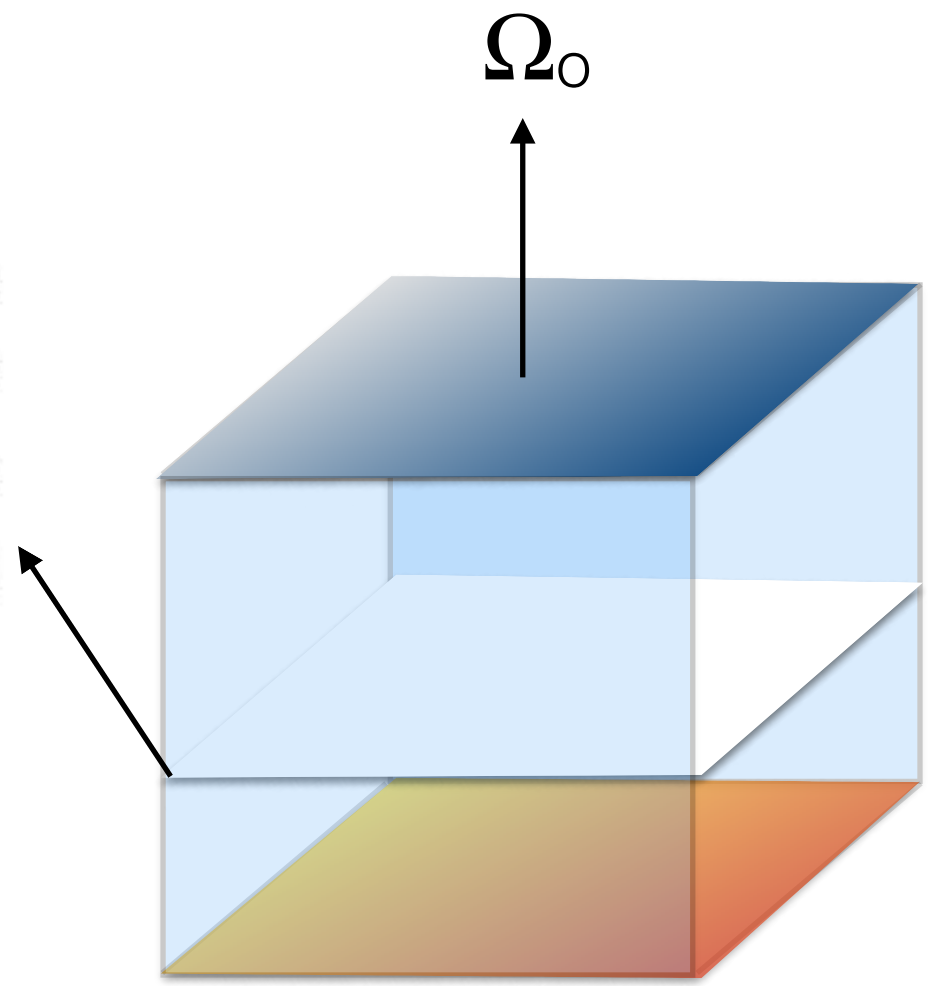




Rapidly rotating thermal convection in a box, courtesy of Meredith Plumley ETHZ

Nothing is white or black





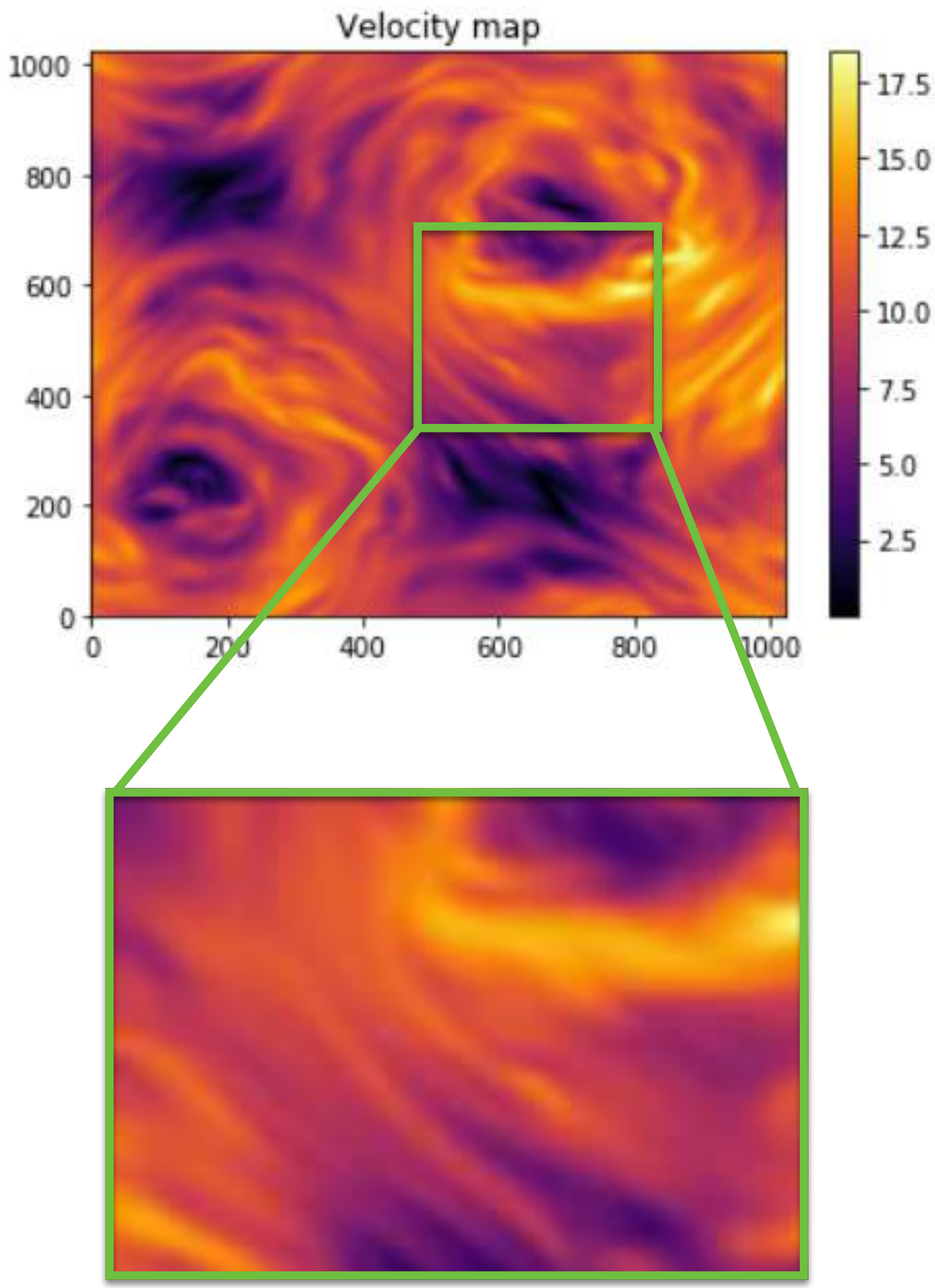
$E \sim 10^{-6}$ Still inviscid at first order

At the smallest scales, some of the assumptions may not be valid anymore

$$\vec{u} \cdot \vec{\nabla} \vec{u} \lesssim 2\vec{\Omega} \times \vec{u}$$

$$\frac{\partial .}{\partial t} \lesssim 1$$

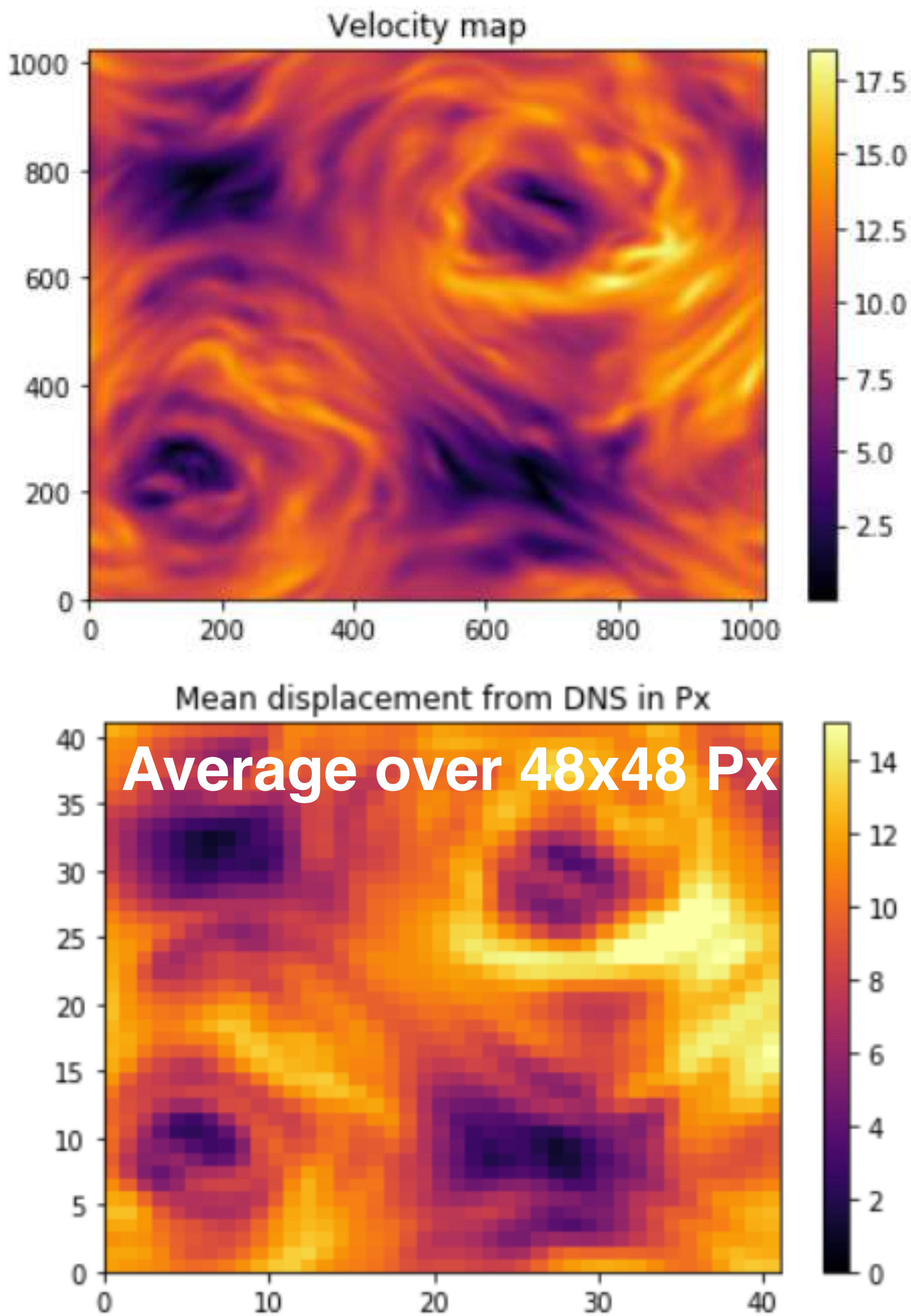
Nothing is white or black



At the smallest scales the flow is not strictly governed by a geostrophic balance, the non linear terms play a role which may lead to local 3D flow structures.

$$\frac{\partial \vec{u}}{\partial t} + 2\vec{\Omega} \times \vec{u} + \vec{u} \cdot \vec{\nabla} \vec{u} = -\vec{\nabla} \Pi$$

Nothing is white or black



In some cases, averaging in time or space to look at the big picture (here we average in space), the large scale flow may satisfy the geostrophic force balance

$E \sim 10^{-6}$

$$< \vec{u} > \cdot \vec{\nabla} < \vec{u} > << \vec{\Omega} \times < \vec{u} >$$

$$\left\langle \frac{\partial .}{\partial t} \right\rangle << 1$$

$$\left\langle \vec{u} \cdot \vec{\nabla} \vec{u} \right\rangle << \vec{\Omega} \times < \vec{u} >$$

Hence, large horizontal structure may be invariant in the direction parallel to $\mathbf{\Omega}$, while at small scale the flow exhibits 3D behaviour.