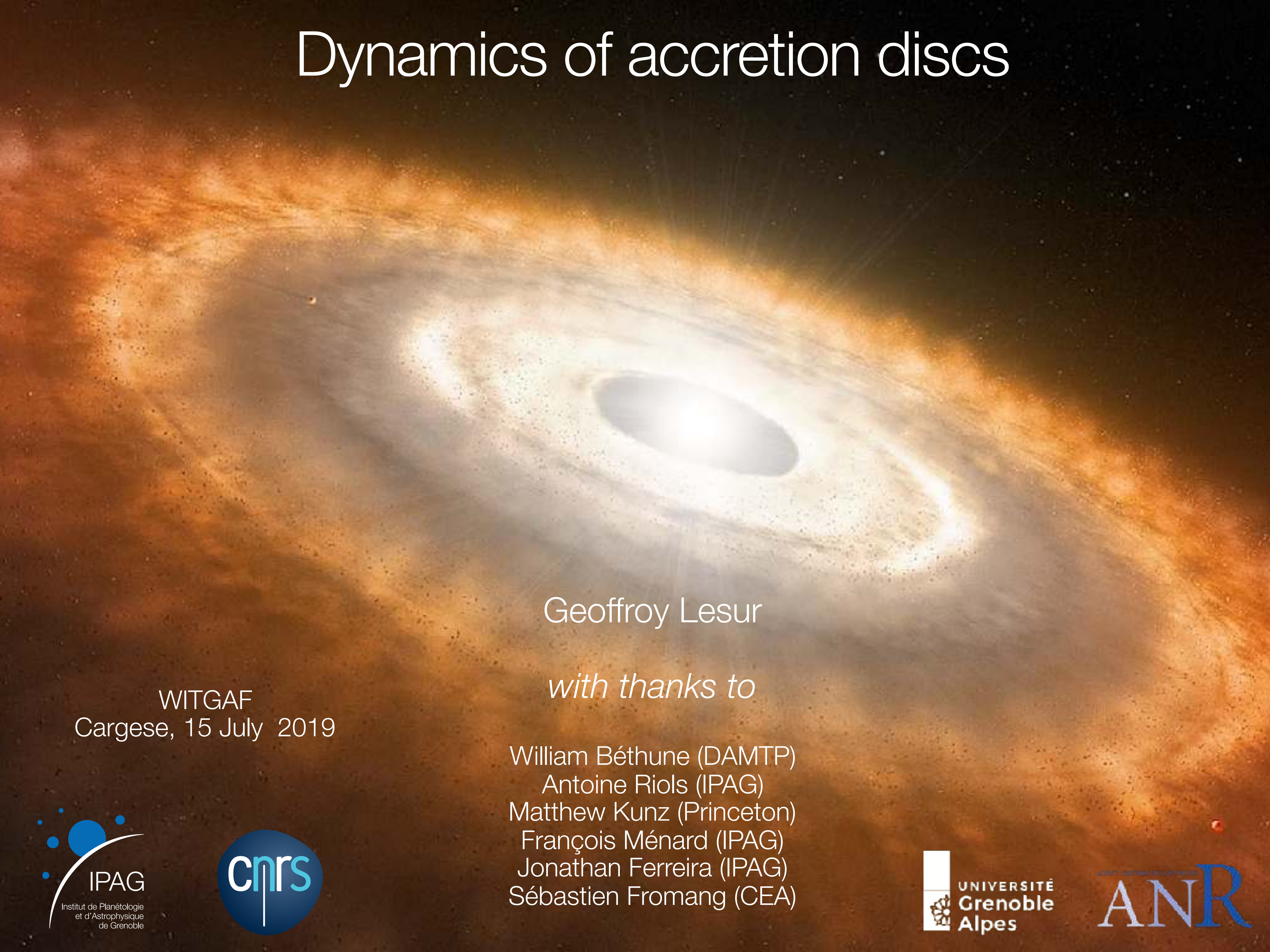
Dynamics of accretion discs



Geoffroy Lesur

with thanks to

William Béthune (DAMTP)
Antoine Riols (IPAG)
Matthew Kunz (Princeton)
François Ménard (IPAG)
Jonathan Ferreira (IPAG)
Sébastien Fromang (CEA)

WITGAF
Cargese, 15 July 2019

IPAG
Institut de Planétologie
et d'Astrophysique
de Grenoble

Université
Grenoble
Alpes

ANR

What is the most efficient way to convert the rest mass energy of matter into heat?

A- Burning fuel 4×10^{-8} %

B- Nuclear fission 0.09 %

C- Nuclear fusion 0.09 %

D- ?

Accretion disc around a black hole:
up to 40% efficiency

Overview

- A few fun facts about astrophysical discs (10’)
- How to drive accretion (30’)
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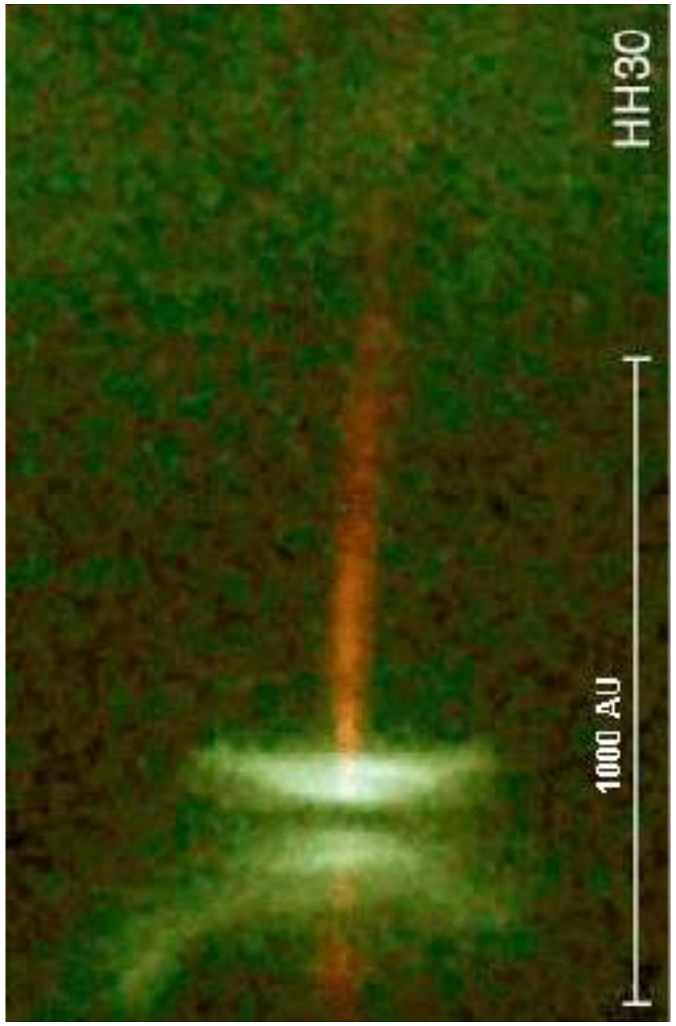
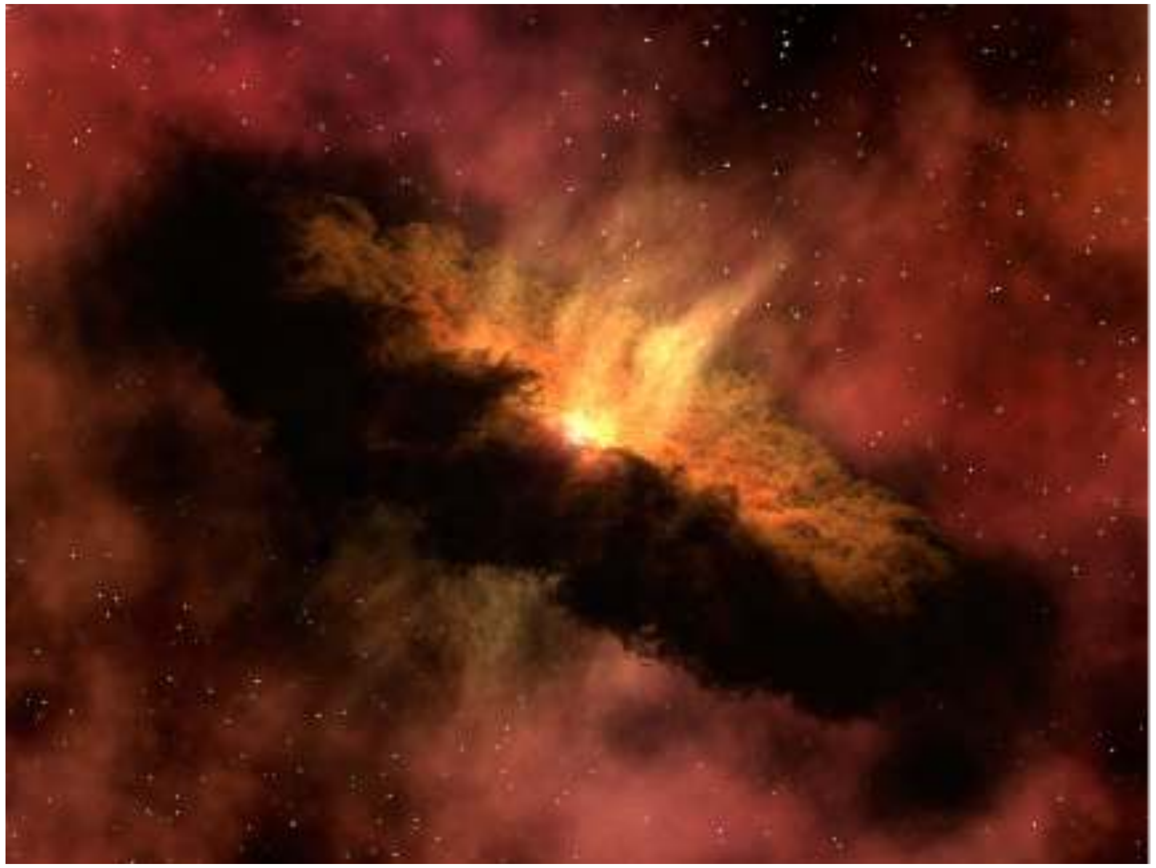
- Application to protoplanetary discs (20’)

Protoplanetary discs



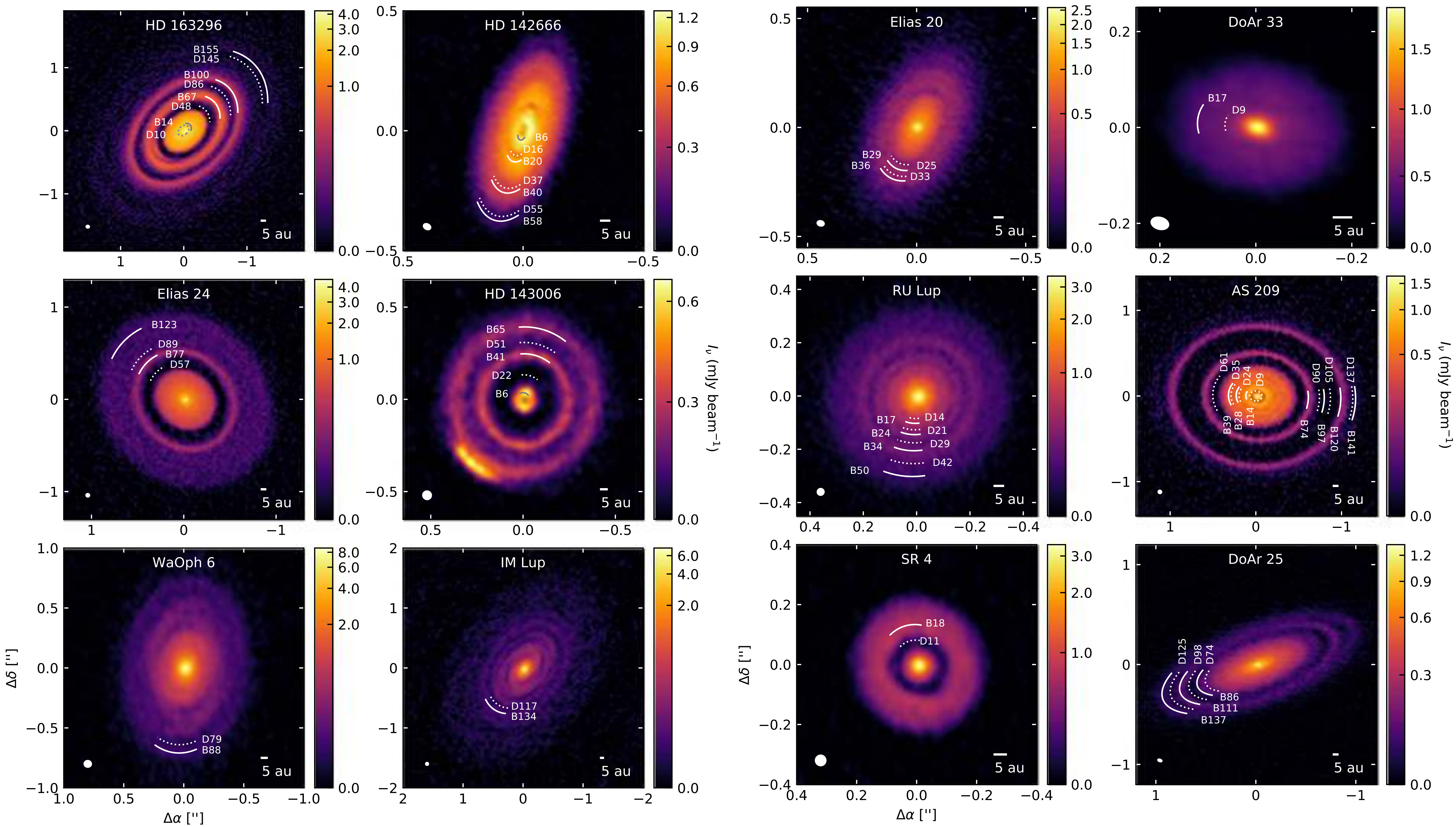
Credit: C. Burrows and J. Krist (STScI),
K. Stapelfeldt (JPL) and NASA



Artist view

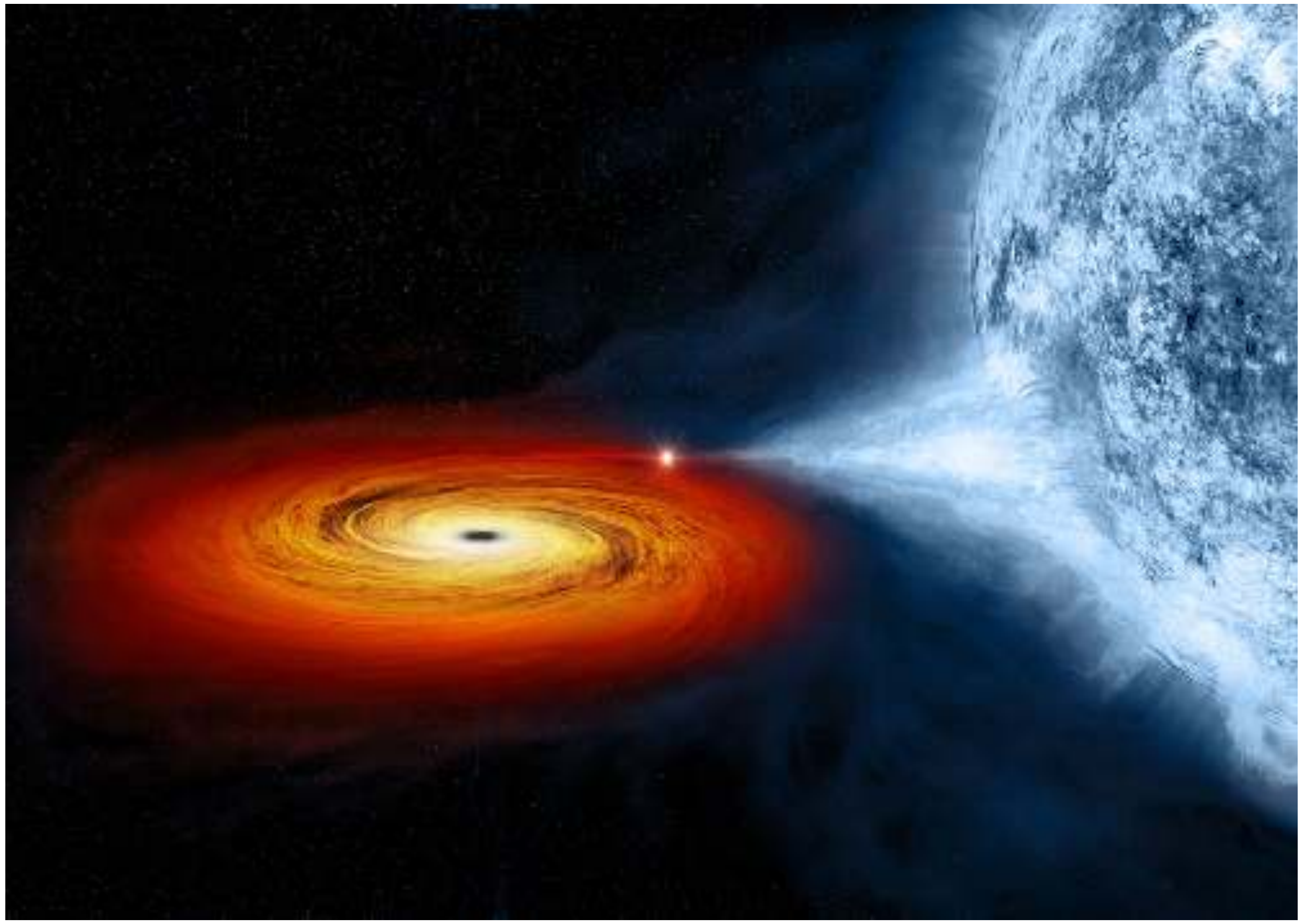
- Size 10^9-10^{13} m
- Central object: young star (10^{30} kg)
- Temperature 10^3-10 K

Structures in protoplanetary discs



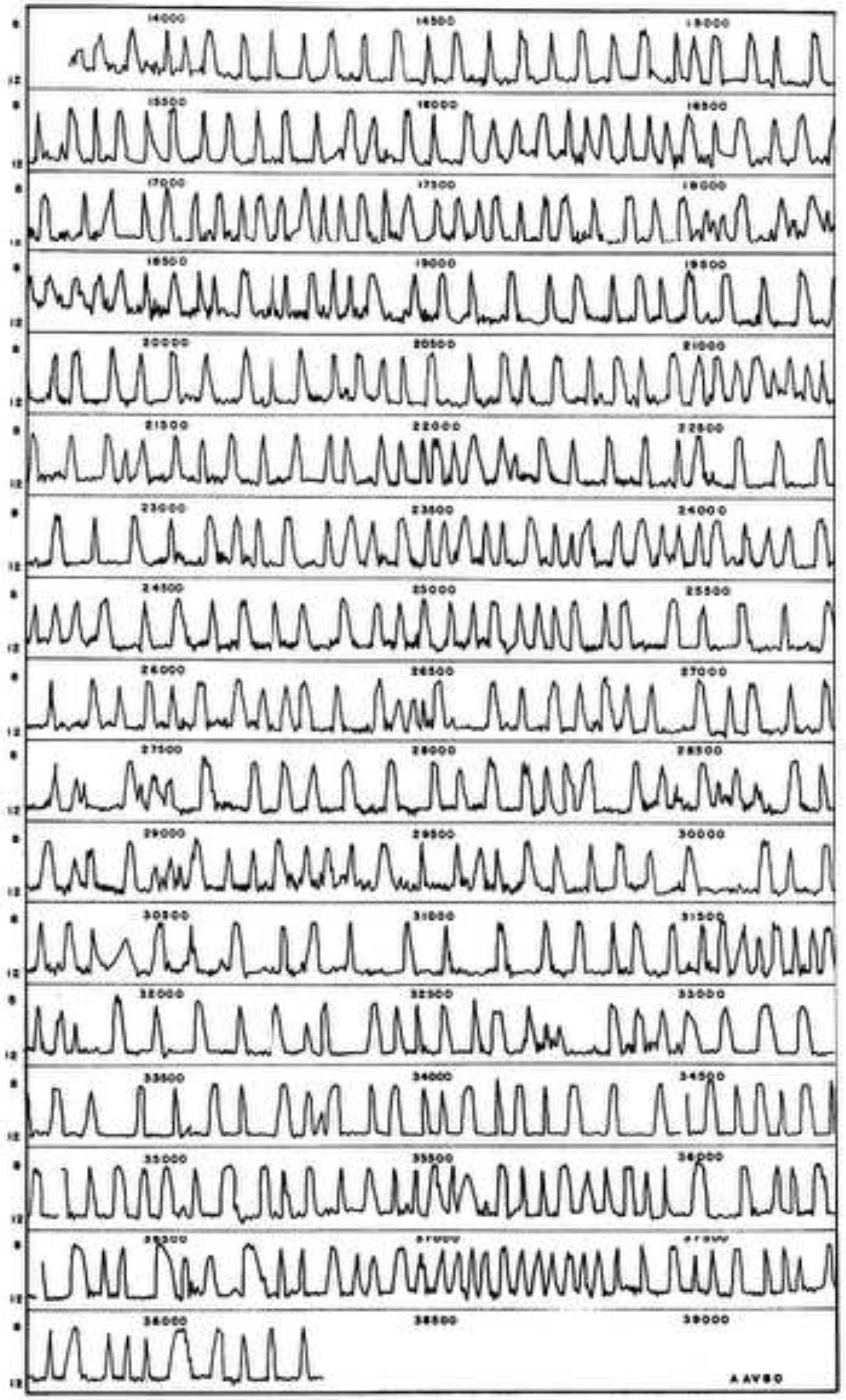
[Huang+ 2018]

Compact binaries



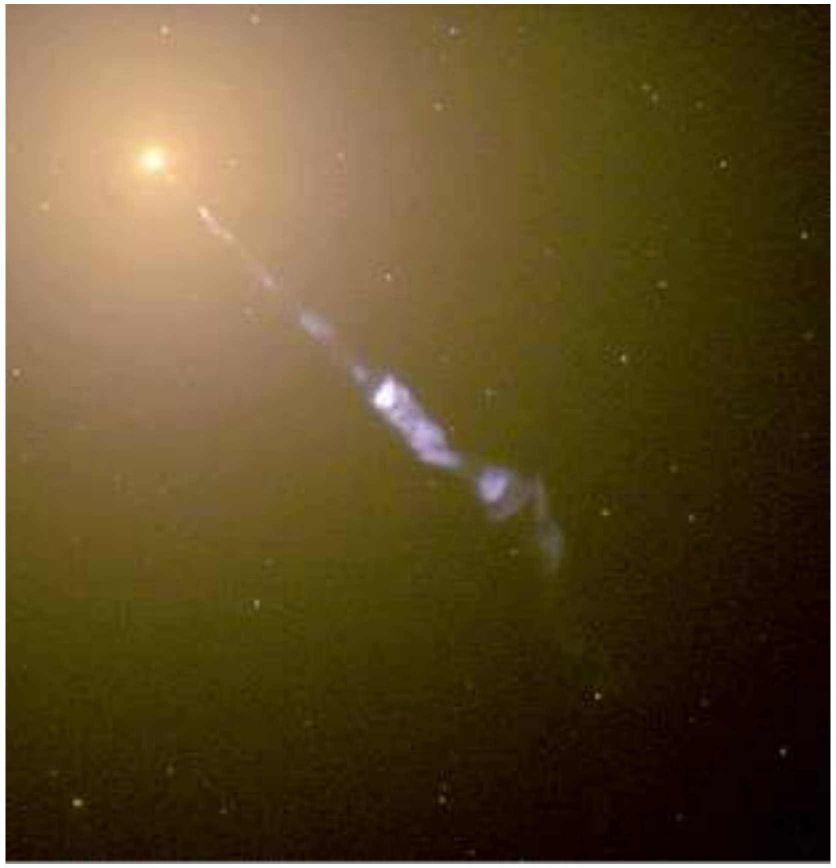
Artist view

- Size 10^4-10^8 m
- Central object: white dwarf, neutron star, black hole (10^{30} kg)
- Temperature 10^5-10^3 K

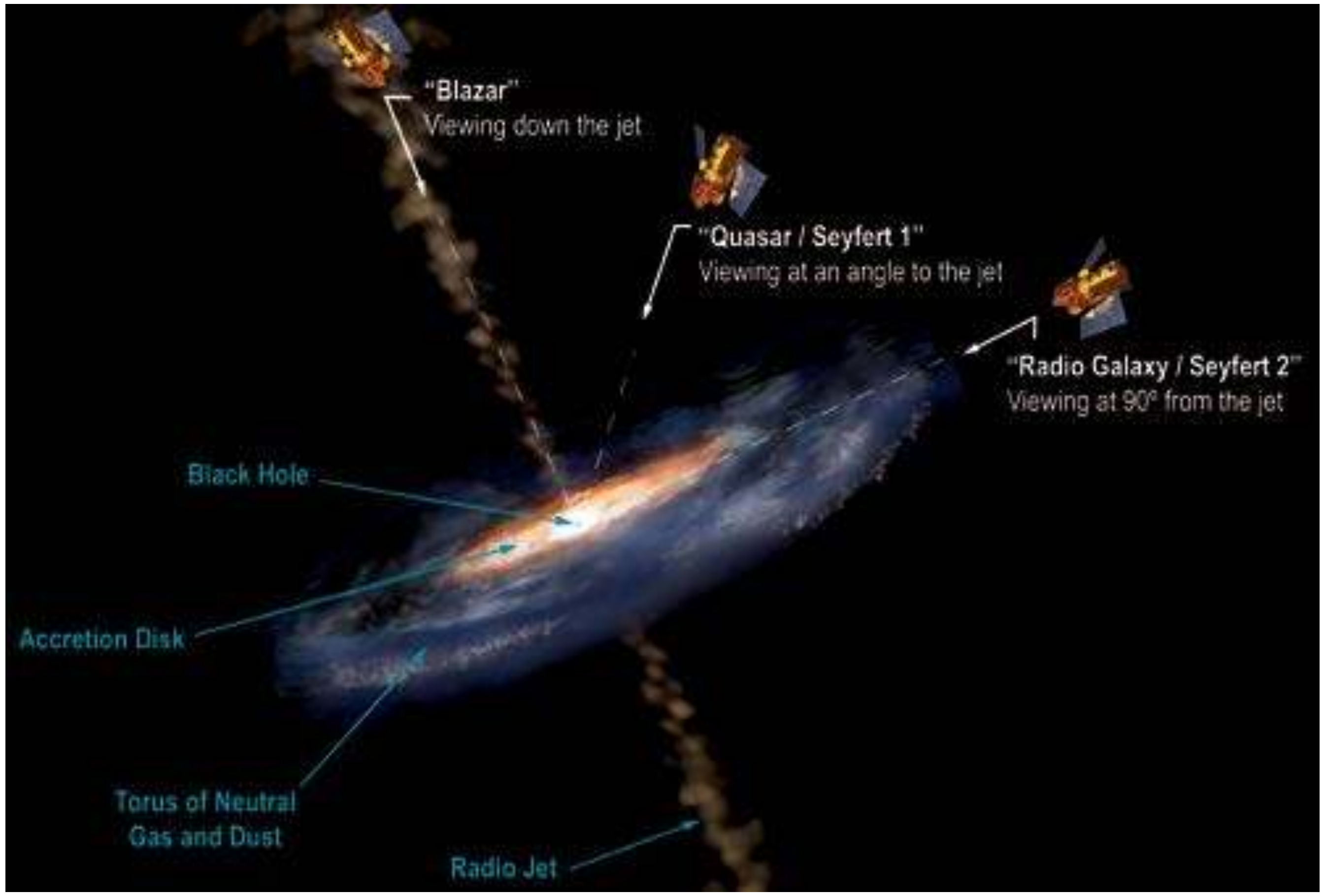


LIGHT CURVE OF SS CYGNI
1896 — 1963

Active galactic nuclei (blazars, quasars…)



M87

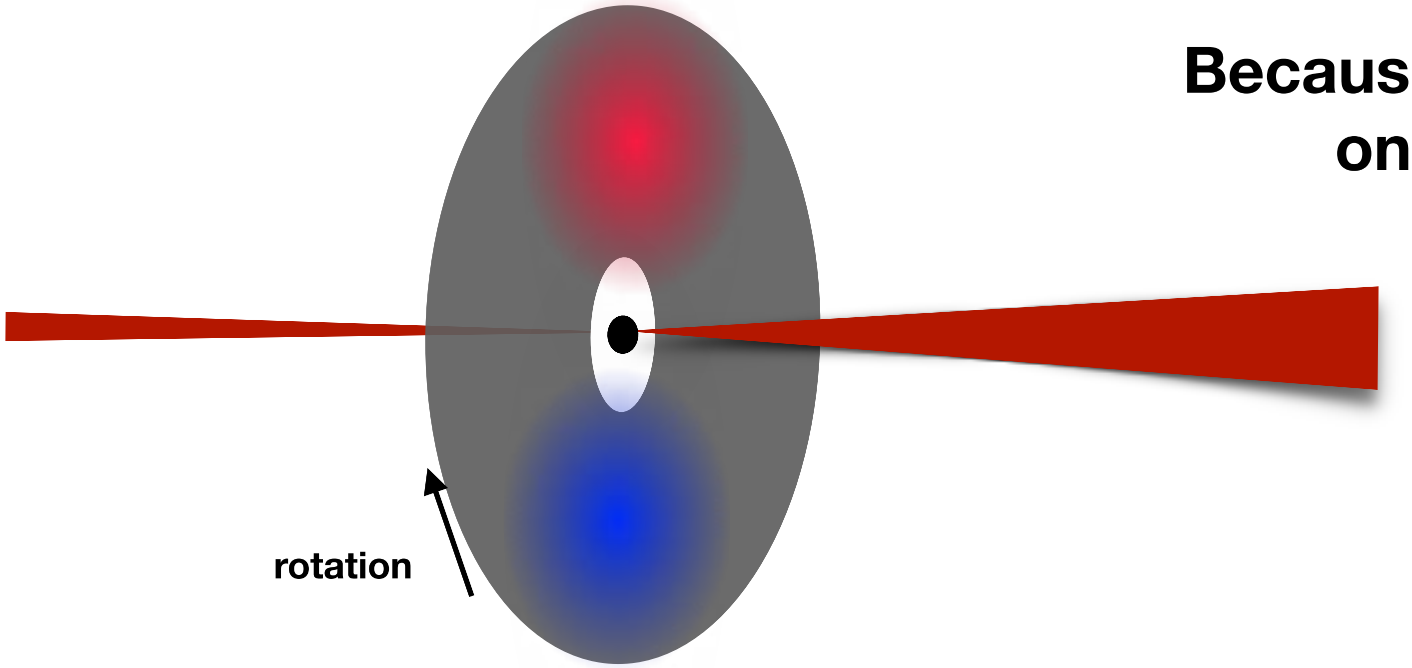


- Size 10^{10}-10^{15} m
- Central object: black hole (10^{36}-10^{39} kg=10^{6}-10^{9} M_{sun})
- Temperature 10^{5}-10^{2} K

M87: staring at a supermassive black hole

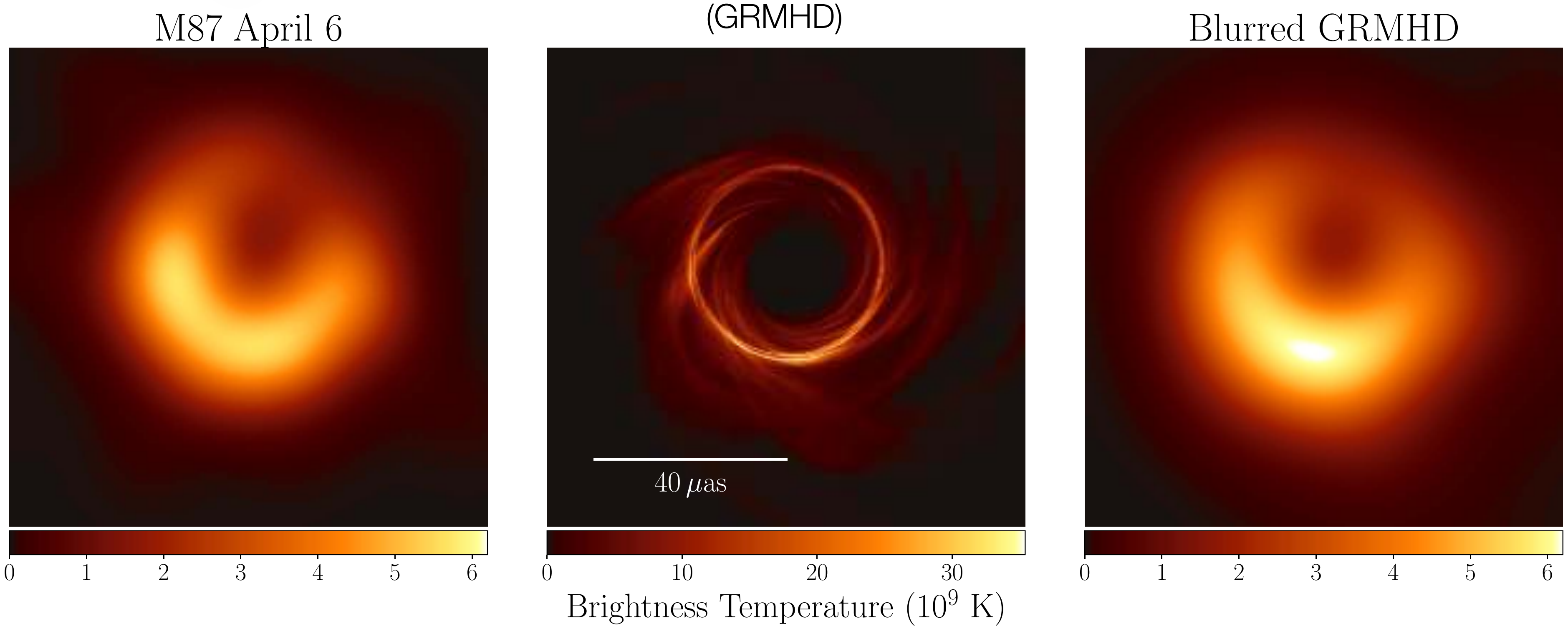
Event Horizon Telescope collaboration, ApJL (2019)

Model of M87



Because rotation velocities are relativistic, one expects Doppler beaming in the blue shifted region

General relativistic magneto-hydrodynamic model (GRMHD)



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Equations of motion

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\boldsymbol{u} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}\right) = -\boldsymbol{\nabla}\left(P + \frac{B^2}{8\pi}\right) + \frac{\boldsymbol{B}\cdot\nabla\boldsymbol{B}}{4\pi}$$

Magnetic pressure

Magnetic tension

$$\frac{\partial\boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{u}\times\boldsymbol{B}) \Longleftrightarrow \frac{\partial\boldsymbol{B}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{B} = \boldsymbol{B}\cdot\nabla\boldsymbol{u} - \boldsymbol{B}\nabla\cdot\boldsymbol{u}$$

Transport

Warping

Compression

Disc Dynamics

Radial equilibrium

Radial equilibrium

$$\cancel{\frac{\partial u_r}{\partial t}} + \cancel{\boldsymbol{u}\cdot\boldsymbol{\nabla} u_r} - \frac{u_\phi^2}{R} = \cancel{\frac{\boldsymbol{B}\cdot\boldsymbol{\nabla} B_r}{4\pi\rho}} - \cancel{\frac{B_\phi^2}{4\pi\rho R}} - \frac{1}{\rho}\frac{\partial\left(\cancel{P}+\cancel{B^2/8\pi}\right)}{\partial R} - \frac{GM_\odot}{R^2}$$

- Assume a thin, weakly magnetised disc

$$v_A \ll u_\phi \qquad c_s \ll u_\phi \qquad (u_r, u_z) \ll u_\phi$$

→ $u_\phi = R\Omega(R)$ with $\Omega(R) = (GM_\odot)^{1/2} R^{-3/2}$

- Disc temporal evolution dictated by small deviations from the Keplerian profile:

$$\boldsymbol{u} = \boldsymbol{v} + R\Omega(R)\boldsymbol{e_\phi}$$

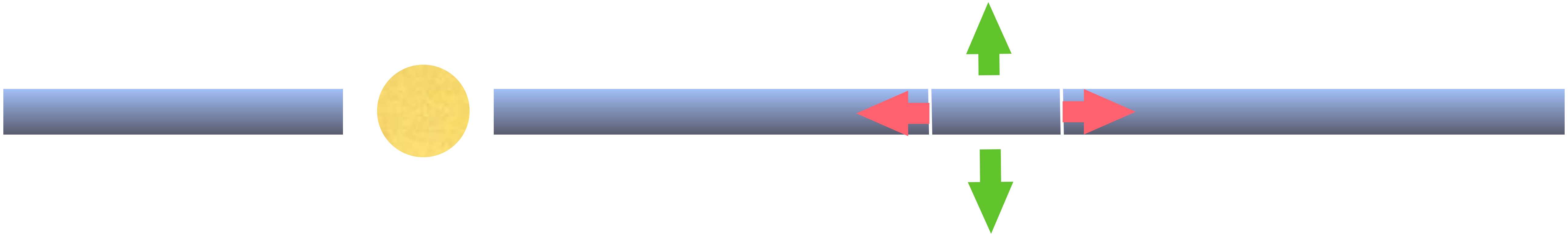
Disc Dynamics

Mass conservation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \rho \boldsymbol{u} = 0$$

Introduce: $\overline{Q} = \int d\phi \int_{z=-h}^{z=+h} dz\, Q$ and $\Sigma = \overline{\rho}$

$$\longrightarrow \quad \frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R} R\overline{\rho v_r} + \Big[\rho v_z\Big]_{z=-h}^{+h} = 0$$



Disc Dynamics

Angular momentum conservation

Angular momentum conservation:

$$\frac{\partial(\rho R u_\phi)}{\partial t} + \boldsymbol{\nabla}\cdot\left[\rho R u_\phi \boldsymbol{u} - R\frac{B_\phi \boldsymbol{B}}{4\pi} + R\left(P + \frac{B^2}{8\pi}\right)\boldsymbol{e_\phi}\right] = 0$$

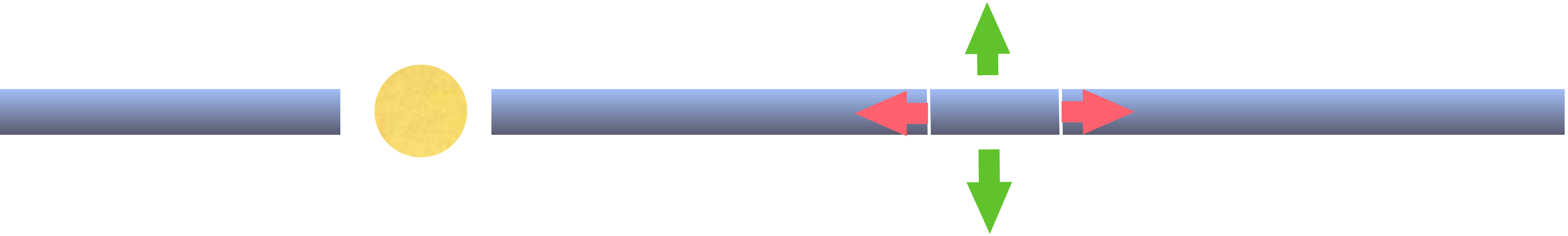
Combine it with mass conservation, squeeze it, stretch it:

$$\overline{\rho v_r}\frac{\partial}{\partial R}\Omega R^2 + \frac{1}{R}\frac{\partial}{\partial R}R^2\left[\overline{\rho v_\phi v_r} - \frac{\overline{B_\phi B_r}}{4\pi}\right] + R\left[\rho v_\phi v_z - \frac{B_\phi B_z}{4\pi}\right]_{z=-h}^{+h} = 0$$

accretion

radial stress

vertical stress
(aka wind stress)



Disc Dynamics

alpha disc model

[Shakura-Sunyaev 1973]

- Introduce the dimensionless number

$$\alpha = \frac{\overline{\rho v_\phi v_r} - \overline{B_\phi B_r}/4\pi}{\Sigma \Omega^2 H}$$

- Estimated accretion rate

$$\overline{\rho v_r} \sim -\alpha c_s \Sigma \frac{H}{R}$$

- Compare to observations: $10^{-4} < \alpha < 10^{-1}$

New questions!

- What is responsible for anomalous viscosity?
- How large is α ?
- What about winds ?

The zoo of disc instabilities

[see Fromang & Lesur 2017
for a complete review]

Local instabilities:

- Magnetorotational instability (MRI): shear driven instability but requires an ionised plasma
  (Velikhov 1959, Chandrasekhar 1960, Balbus & Hawley 1991)
  COVERED BY C. BARUTEAU
- Gravitational instabilities: only for massive & cold enough disc
  (Gammie 2001, Paardekooper 2012)
  COVERED BY C. BARUTEAU
- Subcritical shear instability: probably not efficient enough, if it exists (see later)
  (Lesur & Longaretti 2005, Schartman et al. 2012, Edlund & Ji 2014)
- Vertical Shear instability: driven by vertical shear (actually link to the baroclinicity of the disc)
  (Urpin & Brandenburg (1998), Nelson+ 2013, Barker & Latter 2015)
  COVERED BY C. BARUTEAU
- « Baroclinic » instabilities (SBI, convective overstability): requires a radially unstable entropy profile
  (Petersen+ 2007, Lesur & Papaloizou 2010, Klahr & Hubbard 2014)
  COVERED BY C. BARUTEAU
- Zombie vortex instability: buoyancy critical layer instability
  (Marcus+ 2013, Marcus+2016, Lesur & Latter 2016)
- Rossby wave instability: requires a local maximum of vortensity (equivalent to Kelvin-Helmholtz)
  (Lovelace et. al 1999)
  COVERED BY C. BARUTEAU
- Vertical convective instability: Requires a heat source in the midplane
  (Cabot 1996, Lesur & Ogilvie 2010, Held & Latter 2018)

Global instabilities:

- Papaloizou & Pringle instability: density wave reflection on the inner edge
  (Papaloizou & Pringle 1985)

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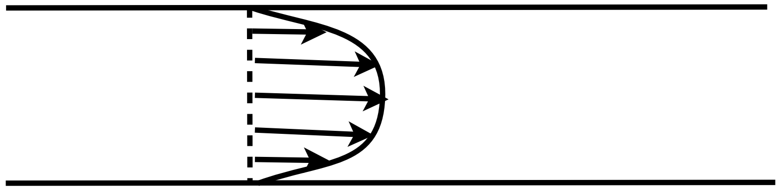
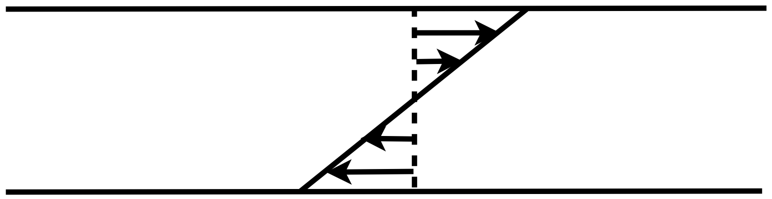
Subcritical shear instabilities
Origins

The Facts:

- Keplerian shear flows are linearly stable
- Huge Reynolds numbers (10^{15}) → nonlinear instability? (same thing as pipe flows or Couette flows)



pipe flow

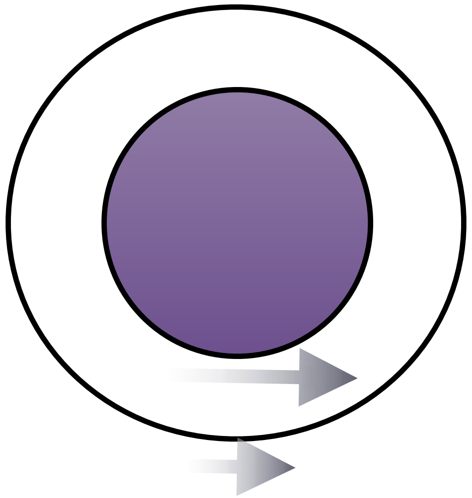


couette flow

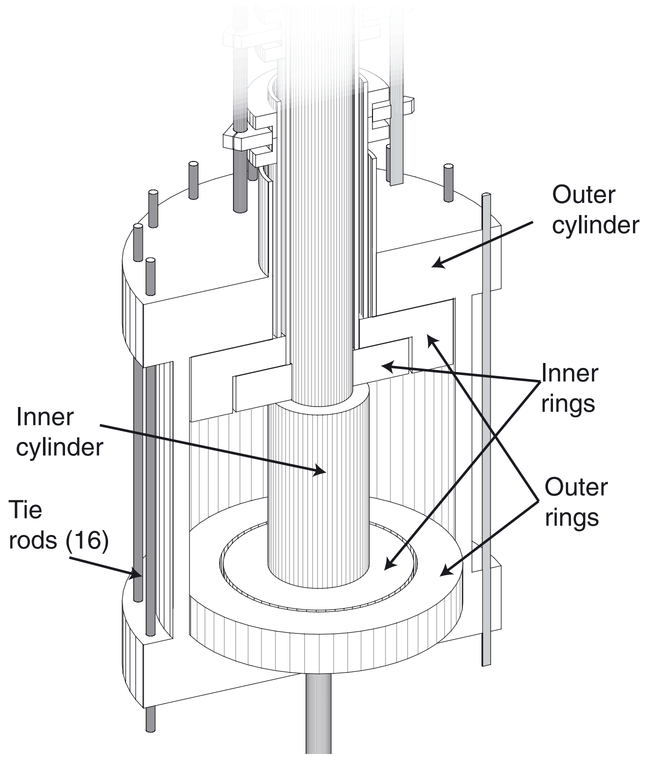
A nonlinear instability in accretion discs?

- Experimental approach: hard to «do» a disc in a lab. Boundary conditions?
- Numerical approach: high Reynolds numbers unreachable: $Re \lesssim 10^4$

Ideal Taylor-Couette

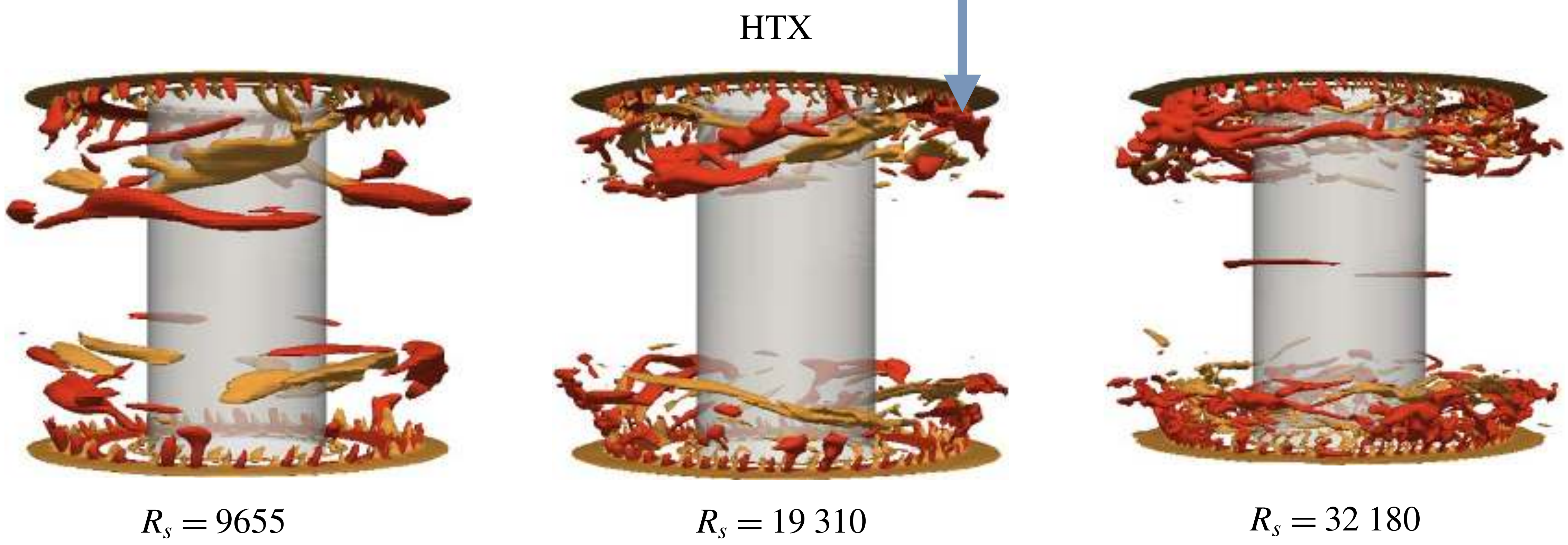


Real life Couette-Taylor (Schartman et al. 2012)



A boundary problem

Turbulence excited by Ekman layers



$R_s = 9655$ $R_s = 19\,310$ $R_s = 32\,180$

Lopez & Avila (2017)

Can non-linear, shear-driven, instabilities, if they exist, transport angular momentum efficiently in Keplerian flows?

[H. Ji]

A contentious debate…

Theory and simulations:

- Zeldovich (1981): maybe yes
- Durbulle (1993): maybe yes
- Balbus, Hawley & Stone (1996): no
- Richard & Zahn (1999): maybe yes
- Longaretti (2002), Chagelishvilli+ (2003), Tevzadze+ (2003), Yecko (2004), Umurhan & Regev (2004), Mukhopadhyay+ (2005), Afshordi+ (2005), Dubrulle+ (2005), Ogilvie & Garaud (2005): maybe yes
- Lesur & Longaretti (2005): no
- Rincon+ 2007, Lithwick (2007, 2009), Mukhopadhyay+ (2011), Avila (2012), Mukhopadhyay & Chattopadhyay (2013): maybe yes
- Osticlla-Monico+ (2014): maybe no
- Bhatia & Mukhopadhyay (2016): maybe yes
- Lopez & Avila (2017), Shi+ 2017: no

Laboratory experiments

- Richard & Zahn (2001): yes
- Beckley & Colgate (2002): maybe no
- Kageyama+ (2004): maybe no

- Ji+(2006), Schartman+ (2012): no
- Paoletti & Lathrop (2011), Paoletti (2012): yes

- Edlund & Ji (2014): no
- Nordslek + (2015): maybe no
- Edlund & Ji (2015): no

Finally converging to a « no » (but no formal proof)

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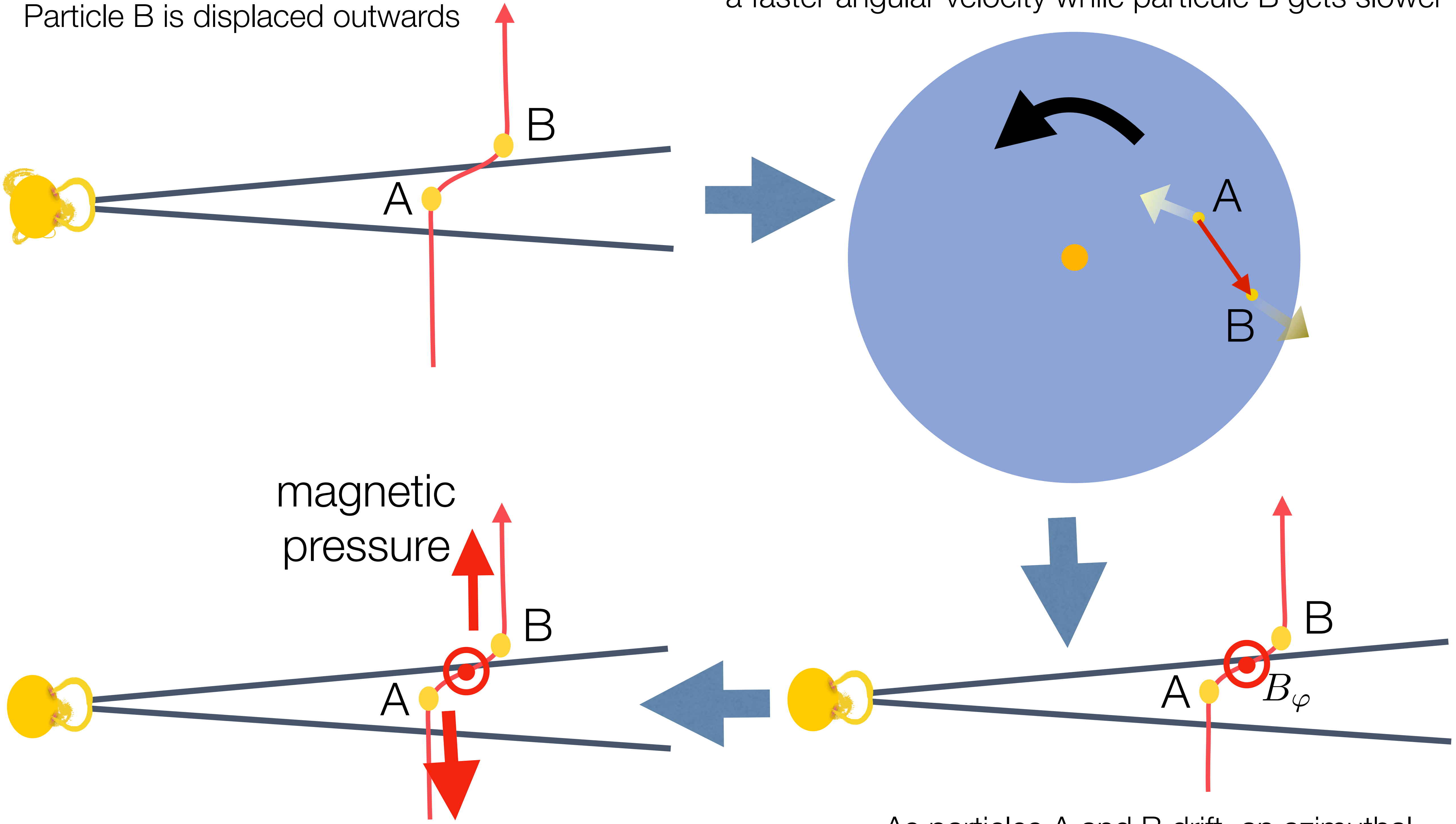
Magnetised winds: a MRI mode becoming non-linear



Particle A is displaced inwards,
Particle B is displaced outwards

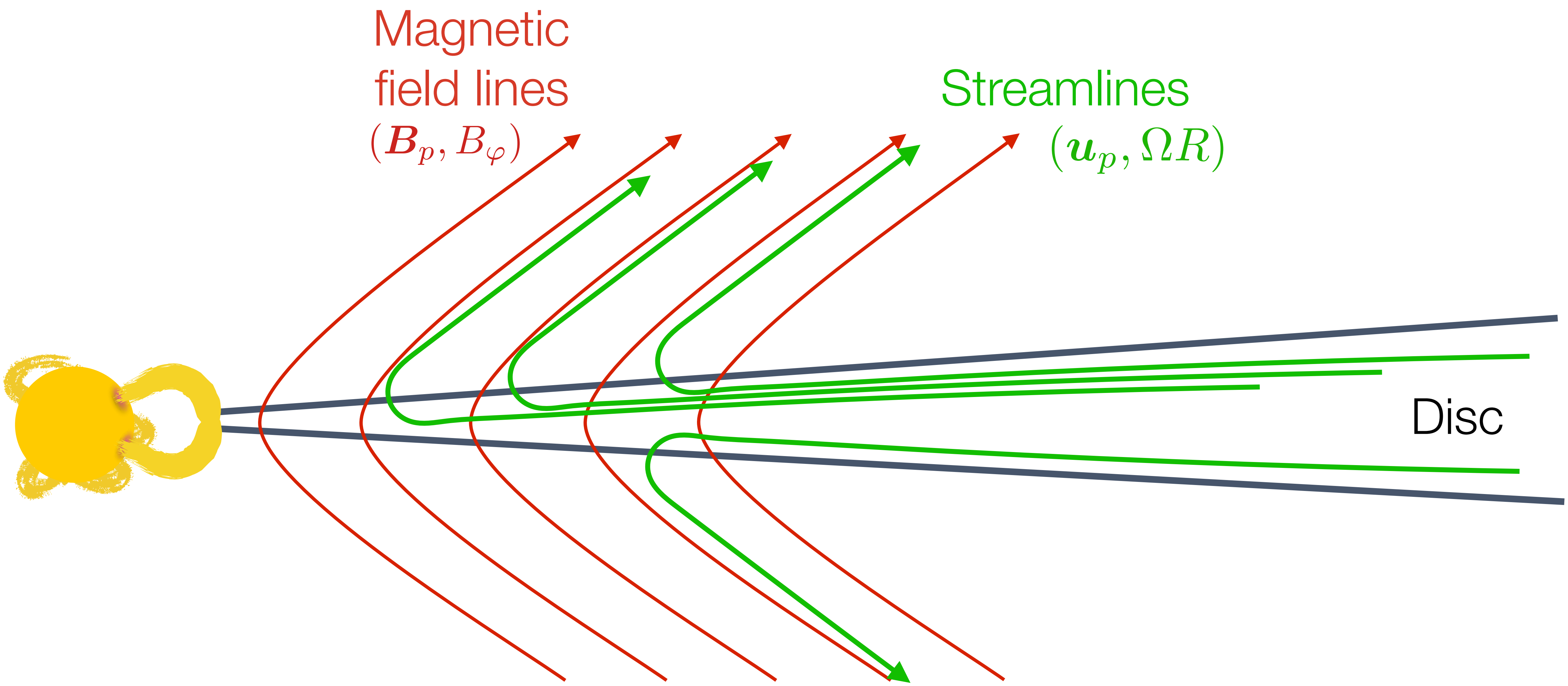
From angular momentum conservation, particle A gets a faster angular velocity while particule B gets slower

As particles A and B drift, an azimuthal magnetic field builds up between the particles

The accumulated toroidal field create a vertical magnetic pressure gradient, pushing B upwards and A downwards

Outflows

Framework



We assume stationary, axisymmetric, ideal MHD

Field strength controlled by the plasma $\beta_p = \dfrac{8\pi P_{\rm midplane}}{B_z^2}$ parameter .

Stationary equations

The need for a magnetically diffusive disc

$\nabla \cdot \boldsymbol{B}_p = 0$ (solenoidal condition) → $\boldsymbol{B}_p = \frac{1}{R}\nabla a \times \boldsymbol{e}_\phi.$

$\nabla \cdot \rho \boldsymbol{u}_p = 0$ (mass & solenoidal condition)

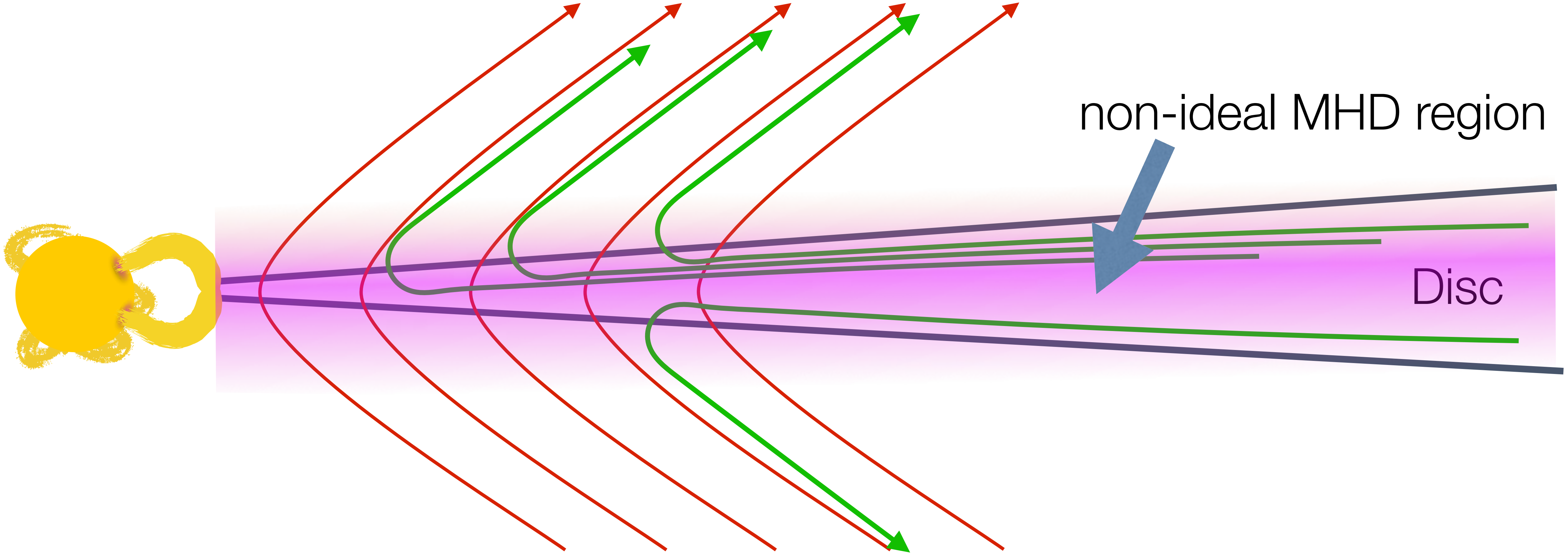
$\rho \boldsymbol{u}_p \cdot \nabla u_R = \rho \Omega^2 r - \partial_R P + \frac{J_\phi B_z}{c} - \frac{J_z B_\phi}{c} - \rho \partial_R \psi$ (R-momentum)

$\nabla \cdot \left(\rho \boldsymbol{u}_p \Omega R^2 - R \frac{\boldsymbol{B}_p B_\phi}{4\pi} \right) = 0$ (ϕ-momentum)

$\rho \boldsymbol{u}_p \cdot \nabla u_z = -\partial_z P - \partial_z \left(\frac{B_\phi^2 + B_R^2}{8\pi} \right) + \frac{B_R \partial_R B_z}{4\pi} - \rho \partial_z \psi$ (z-momentum)

$\nabla \times \left(\boldsymbol{u}_p \times \boldsymbol{B}_p \right) = 0,$ (poloidal induction) → $\boldsymbol{u}_p // \boldsymbol{B}_p$ → not possible in the disc

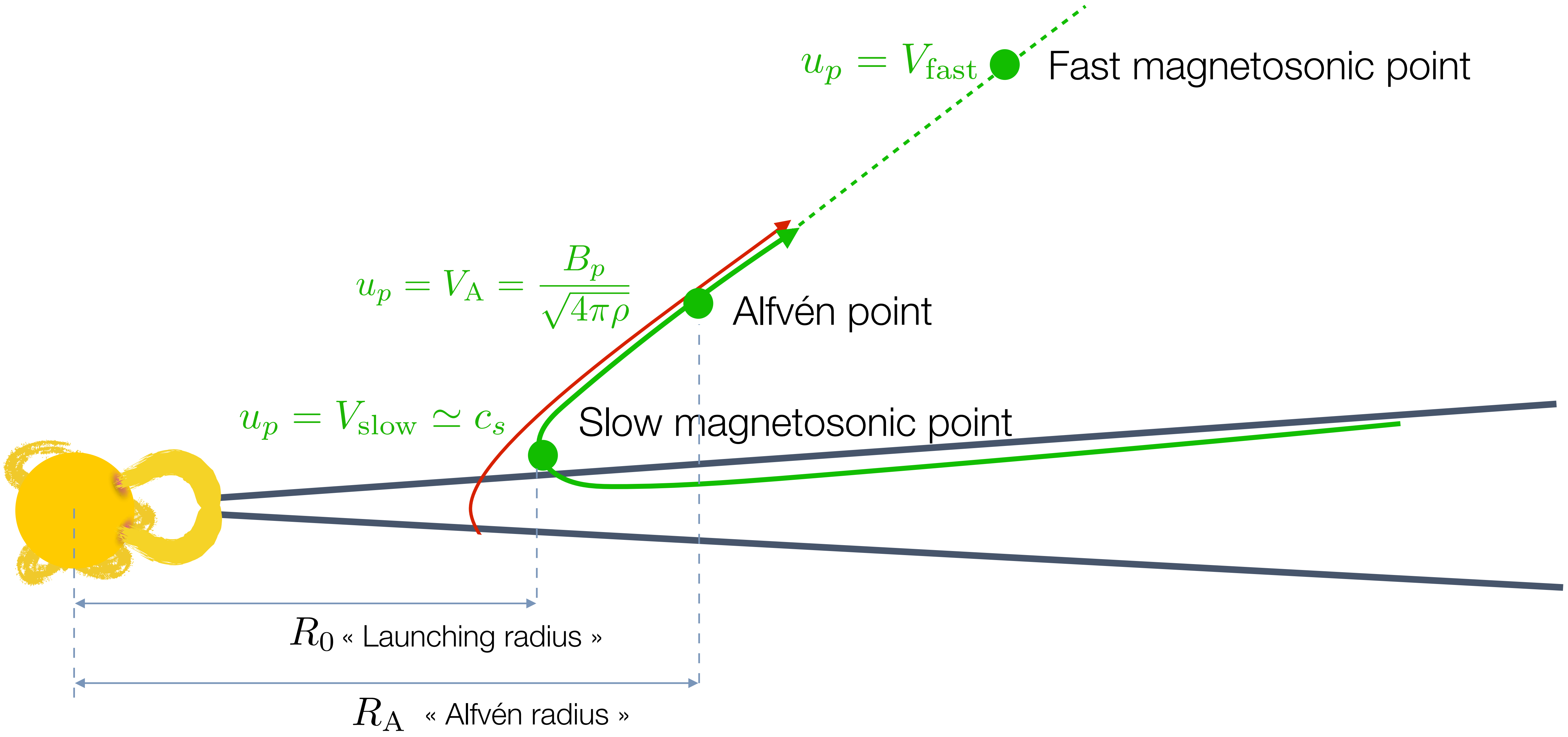
$\nabla \cdot \frac{1}{R}\left(\Omega R \boldsymbol{B}_p - B_\phi \boldsymbol{u}_p \right) = 0.$ (ϕ induction)



Stationary equations

Critical points

The system of equations has 3 critical points (= critical layers for hydro people)



An outflow is causally « disconnected » from its launching point once it has crossed all three critical points

Stationary equations

Invariants along the streamlines

[Blandford & Payne 1982]

$$\left.\begin{array}{r}\boldsymbol{\nabla}\cdot\boldsymbol{B}_p = 0\\ \boldsymbol{\nabla}\cdot\rho\boldsymbol{u}_p = 0\end{array}\right) \Longrightarrow \kappa(a) \equiv \frac{\rho u_p}{B_p}$$

« mass loading parameter »

$$\rho\boldsymbol{u}_p\cdot\boldsymbol{\nabla}u_R = \rho\Omega^2 r - \partial_R P + \frac{J_\phi B_z}{c} - \frac{J_z B_\phi}{c} - \rho\partial_R\psi$$

$$\boldsymbol{\nabla}\cdot\left(\rho\boldsymbol{u}_p\Omega R^2 - R\frac{\boldsymbol{B}_p B_\phi}{4\pi}\right) = 0 \Longrightarrow \Omega^*(a)R_A^2 = \ell(a) \equiv \Omega R^2 - \frac{RB_\phi}{4\pi\kappa(a)}$$

« angular momentum parameter »

$$\rho\boldsymbol{u}_p\cdot\boldsymbol{\nabla}u_z = -\partial_z P - \partial_z\left(\frac{B_\phi^2 + B_R^2}{8\pi}\right) + \frac{B_R\partial_R B_z}{4\pi} - \rho\partial_z\psi$$

$$\boldsymbol{\nabla}\times\left(\boldsymbol{u}_p\times\boldsymbol{B}_p\right) = 0,$$

$$\boldsymbol{\nabla}\cdot\frac{1}{R}\left(\Omega R\boldsymbol{B}_p - B_\phi\boldsymbol{u}_p\right) = 0. \Longrightarrow \Omega^*(a) \equiv \Omega - \frac{\kappa(a)}{\rho R}B_\phi$$

« rotation speed of magnetic surfaces »

In addition, one can create an energy invariant :

$$\mathcal{B} \equiv \frac{u^2}{2} + \psi_G + \mathcal{H} - \frac{R\Omega^*(a)B_\phi}{4\pi\kappa(a)}$$

« Benoulli invariant »

Back to the accretion problem

Angular momentum conservation:

$$\overline{\rho v_r}\frac{\partial}{\partial R}\Omega R^2 + \frac{1}{R}\frac{\partial}{\partial R}R^2\left[\overline{\rho v_\phi v_r} - \frac{\overline{B_\phi B_r}}{4\pi}\right] + R\left[\rho v_\phi v_z - \frac{B_\phi B_z}{4\pi}\right]_{z=-h}^{+h} = 0$$

accretion

radial stress

vertical stress
(aka wind stress)

Using MHD invariants:

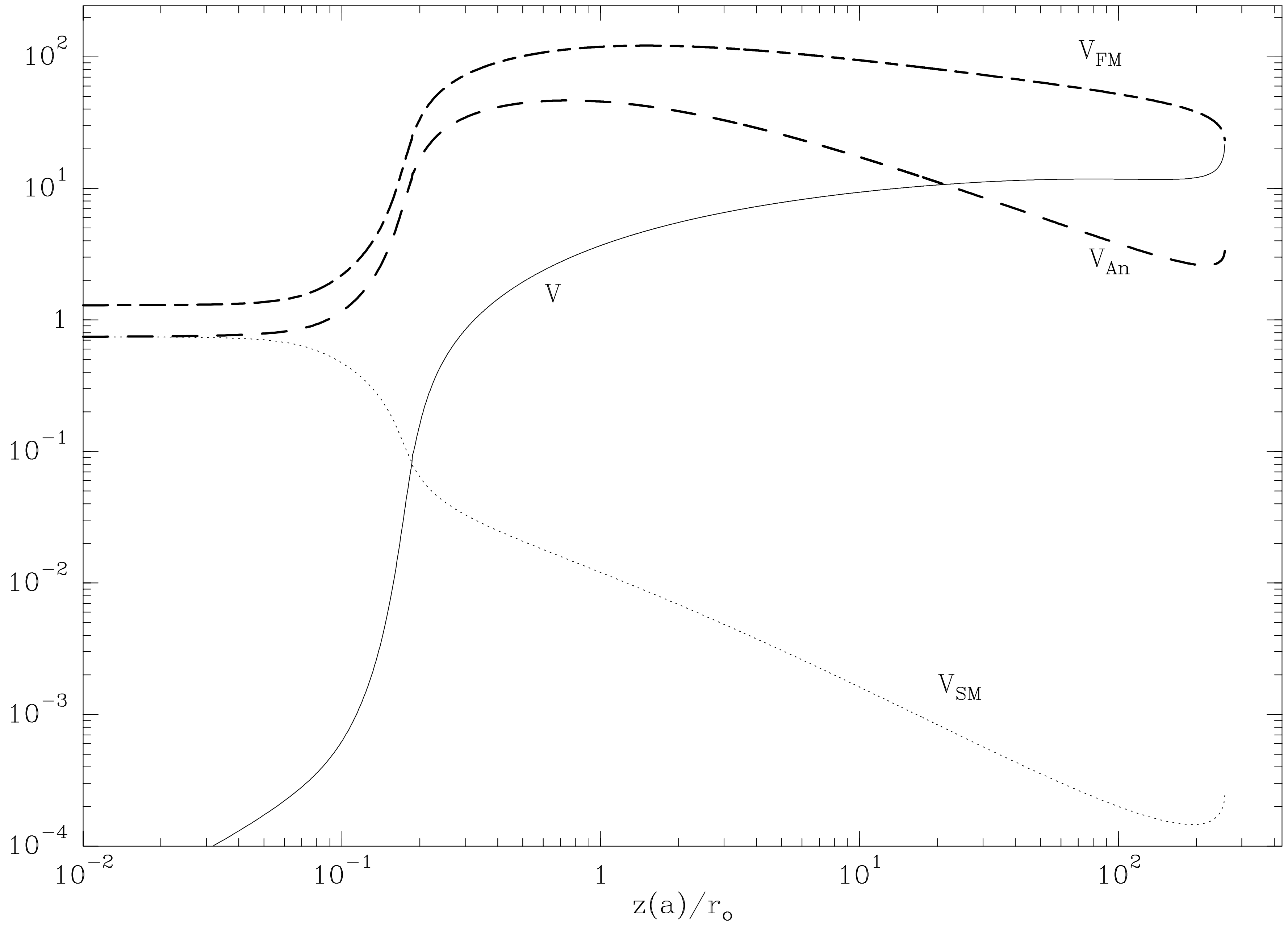
$$R\left[-\frac{B_\phi B_z}{4\pi}\right]_{-h}^{+h} = R\frac{B_{z0}^2}{4\pi}\kappa(\lambda-1)$$

Once the MHD invariants are known for a given solution, one can predict the accretion rate, and mass loss rate

Typical solutions

Self-similar solutions

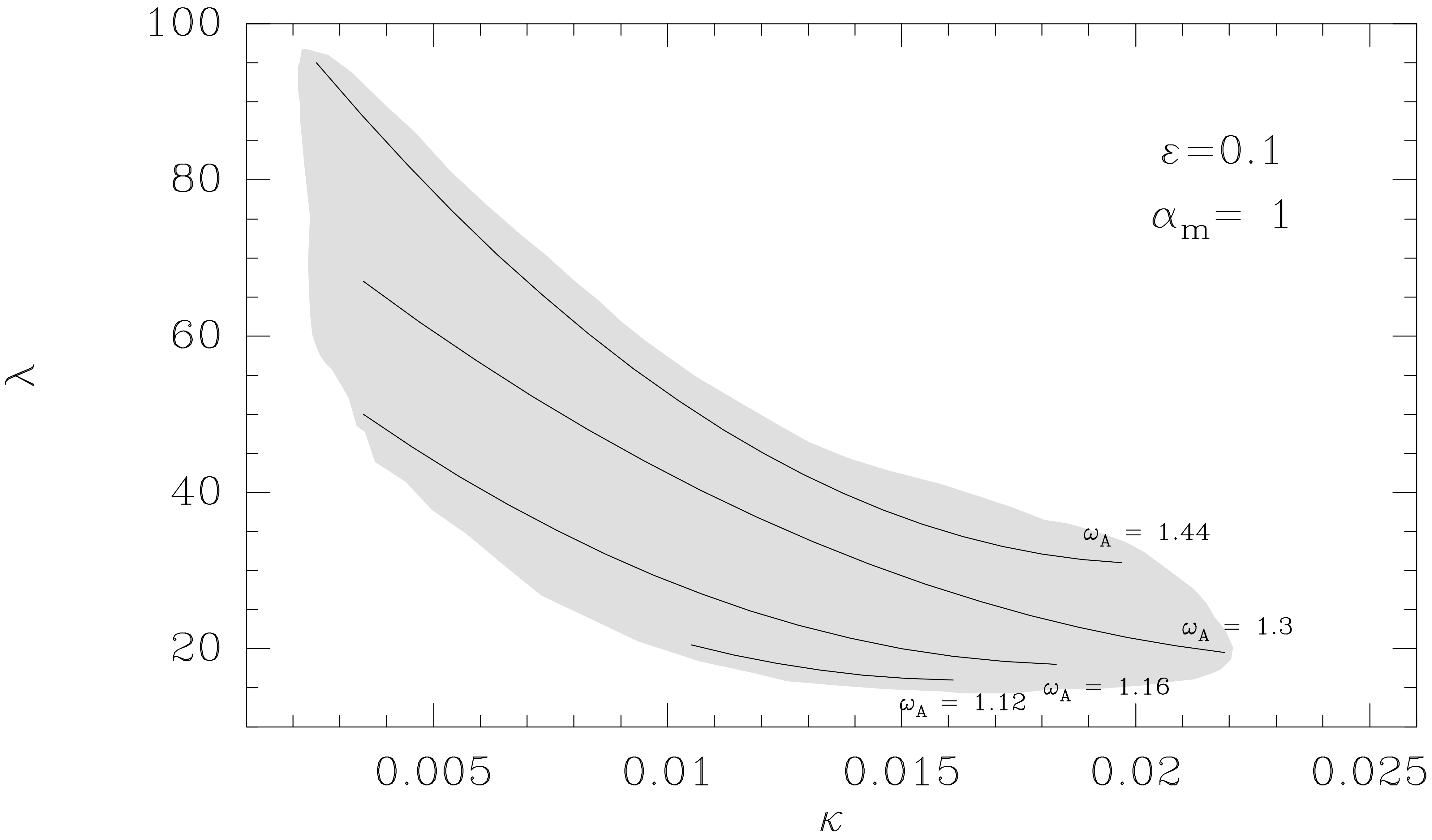
Typical « cold wind » solution



[Casse & Ferreira 2000]

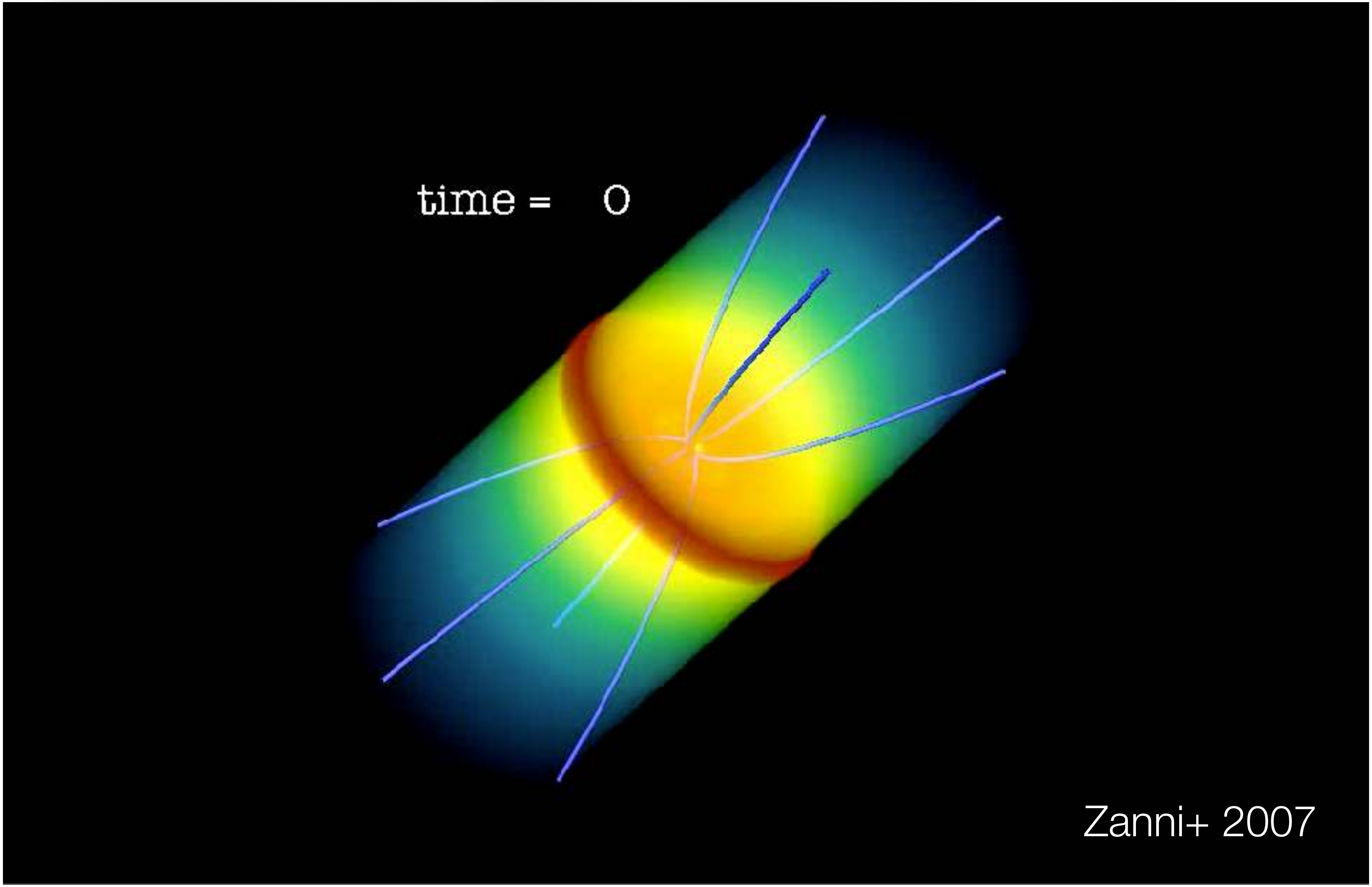
Typical solutions

Self-similar solutions



Casse & Ferreira (2000)

Numerical simulations



Zanni+ 2007

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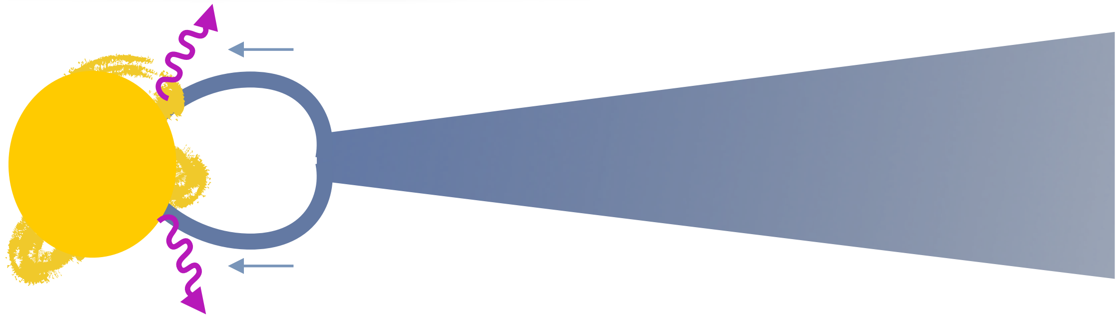
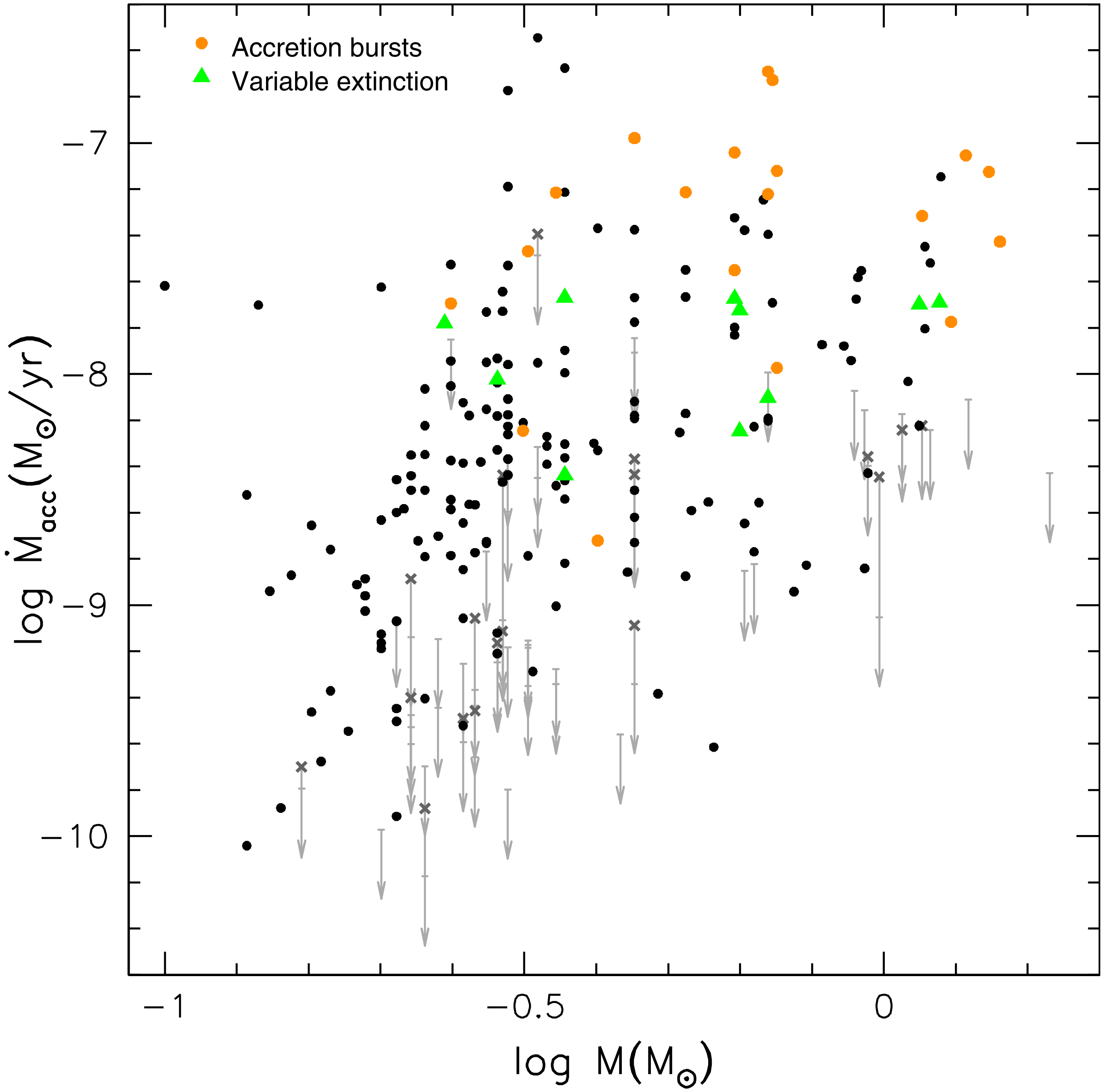
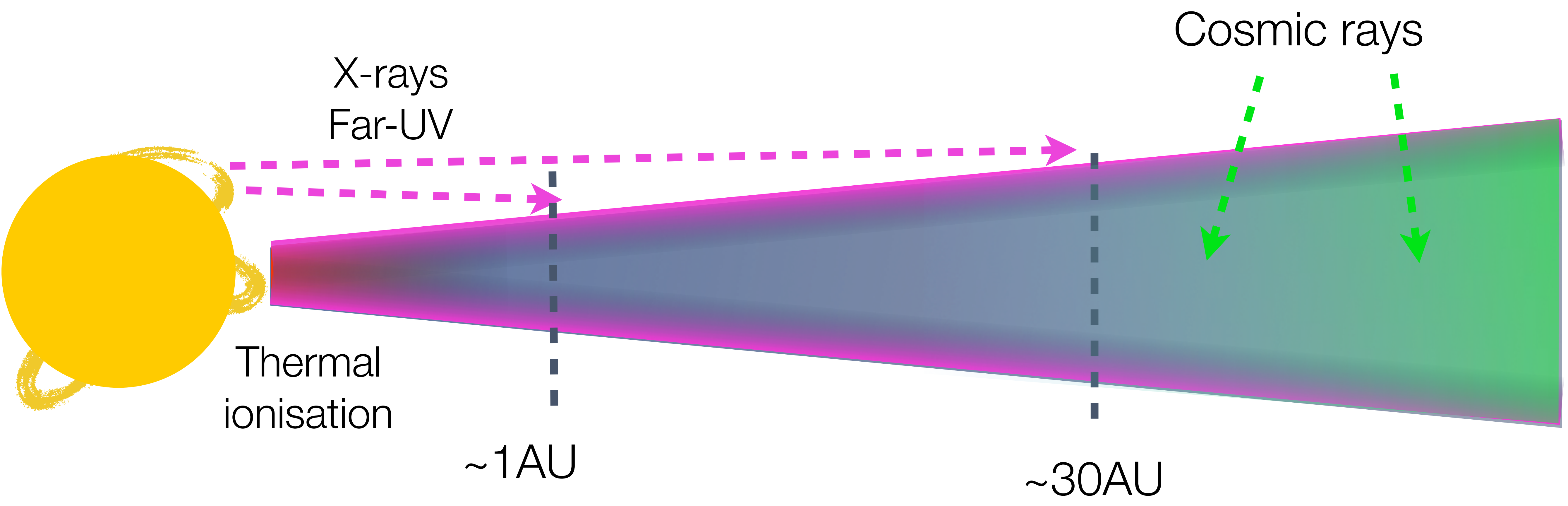
Accretion rate onto the stellar surface





[Venuti+2014]

Ionisation sources in protoplanetary discs

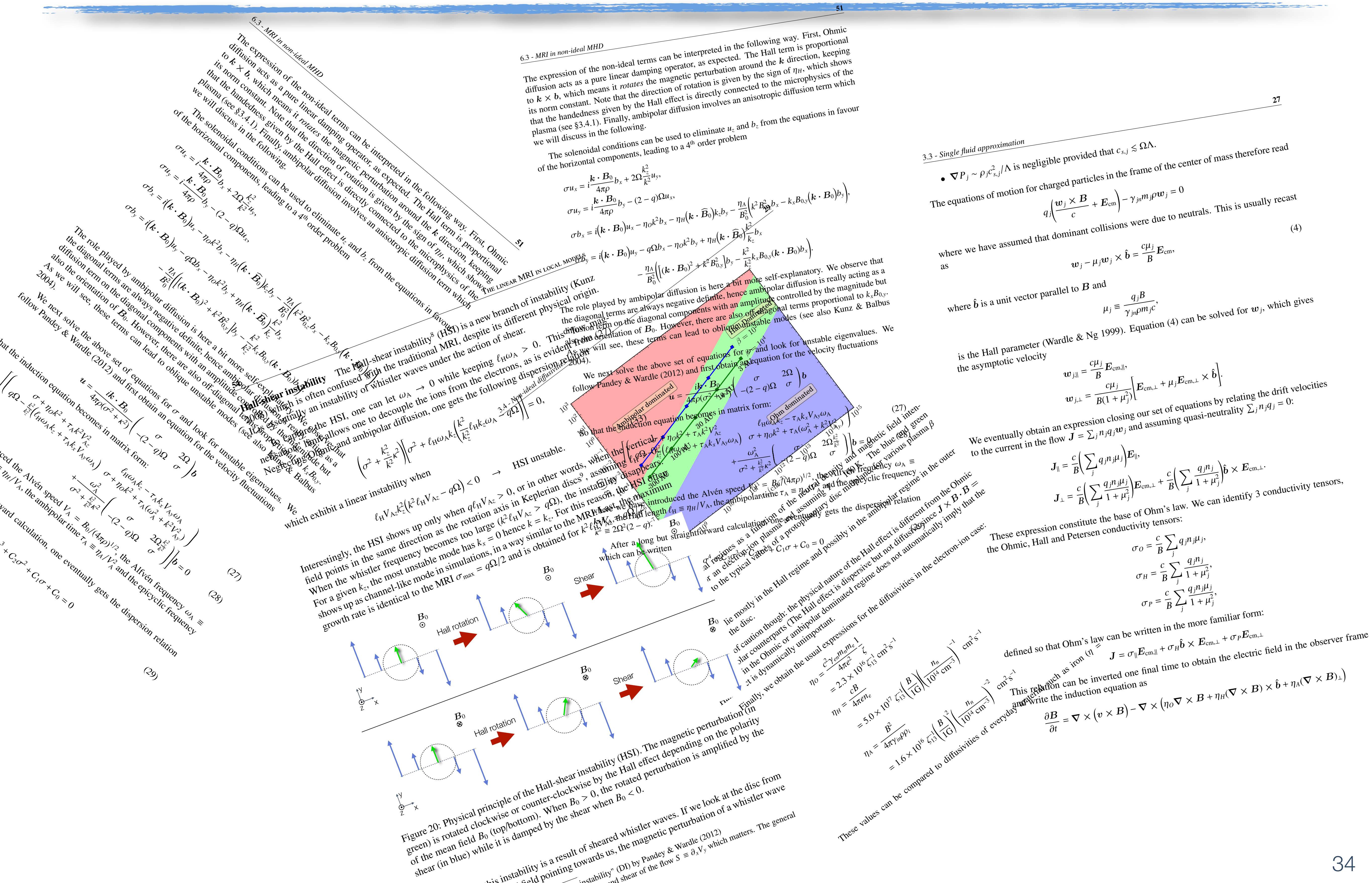


« non ideal » MHD effects

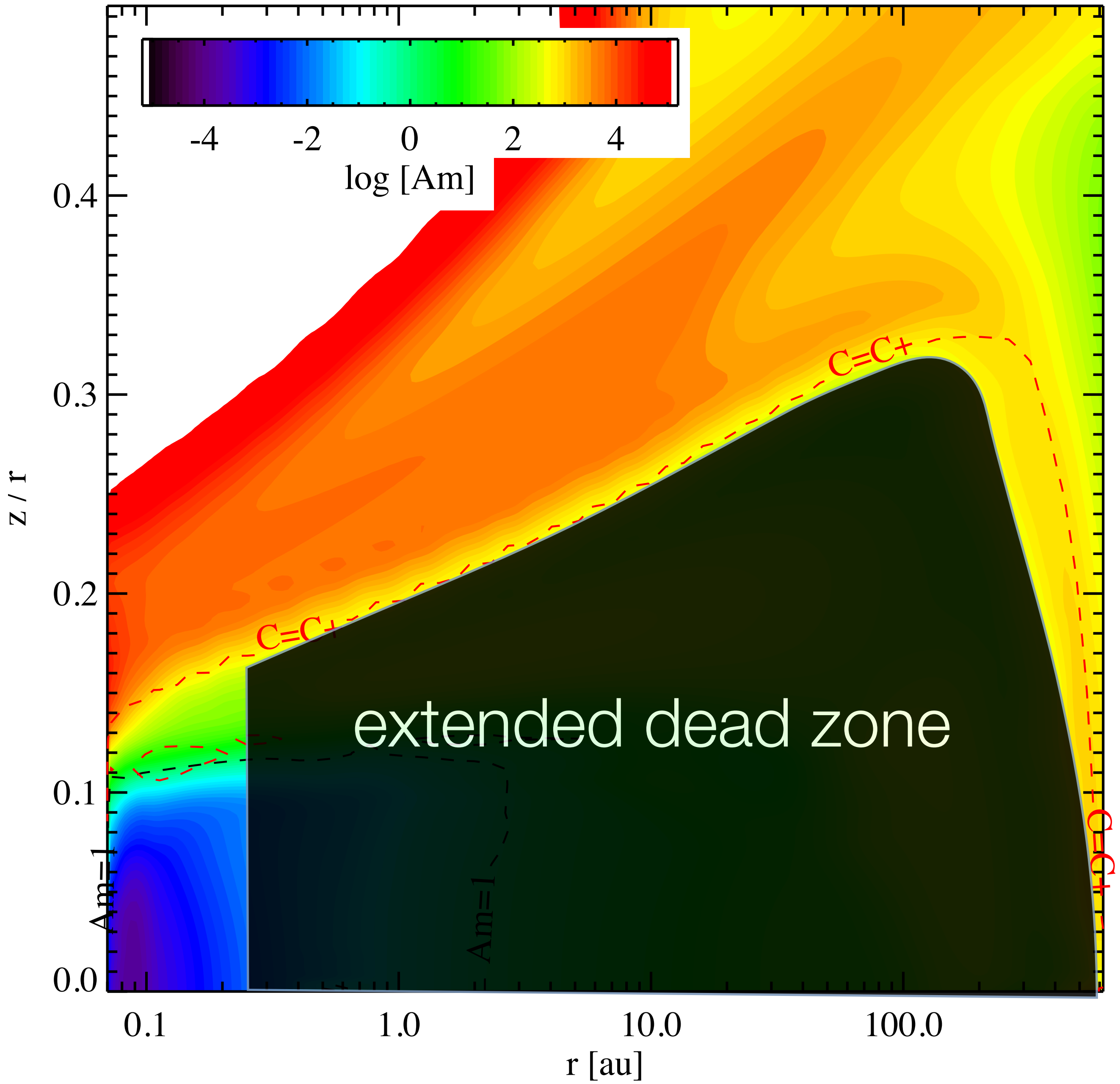
- Ohmic diffusion (electron-neutral collisions)
- Ambipolar Diffusion (ion-neutral collisions)
- Hall Effect (electron-ion drift)

Amplitude of these effects depends strongly on location & composition

Some technical « details » are intentionally hidden…



Ambipolar diffusion



[Thi+2018]

→ Am<100 → « damped » MRI

What do observers say?

Line broadening

- Emission lines from the gas are broaden by:
  - Keplerian rotation V_k
  - Thermal velocity $v_{\rm th} \simeq c_s \ll V_k$
  - Turbulence $v_{\rm turb} \simeq \sqrt{\alpha} c_s$

Measuring line broadening due to turbulence requires very precise measures/estimates of V_k and c_s

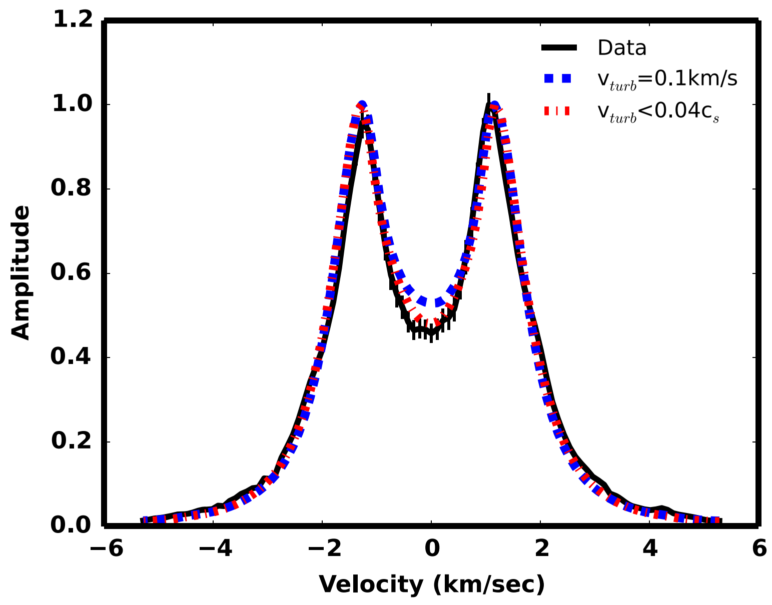
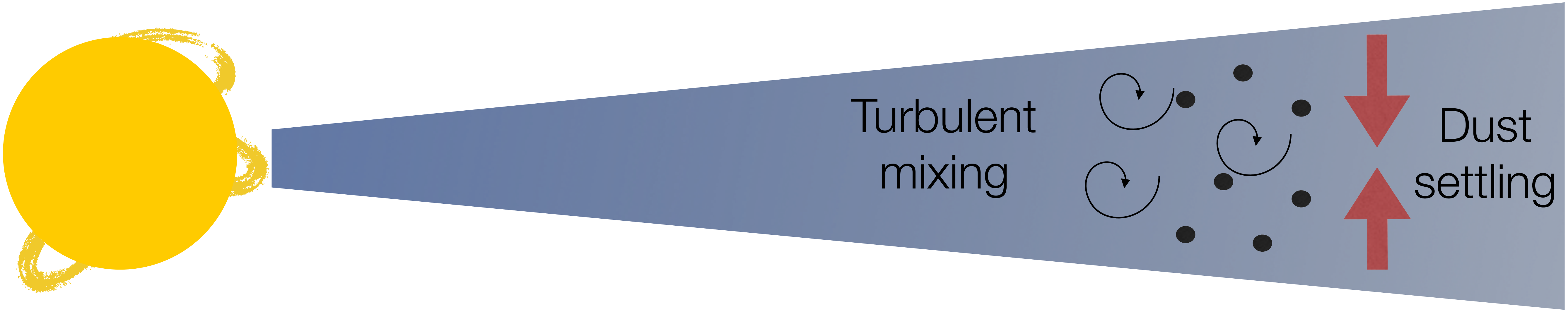


Figure 6. CO(3-2) high resolution spectra (black line) compared to the median model when turbulence is allowed to move toward very low values (red dotted–dashed lines) or when it is fixed at 0.1 km s^{-1} (blue dashed lines). All spectra have been normalized to their peak flux to better highlight the change in shape. The models with weak turbulence provide a significantly better fit to the data despite the fact that the turbulence is smaller than the spectral resolution of the data.

[Flaherty+2015]

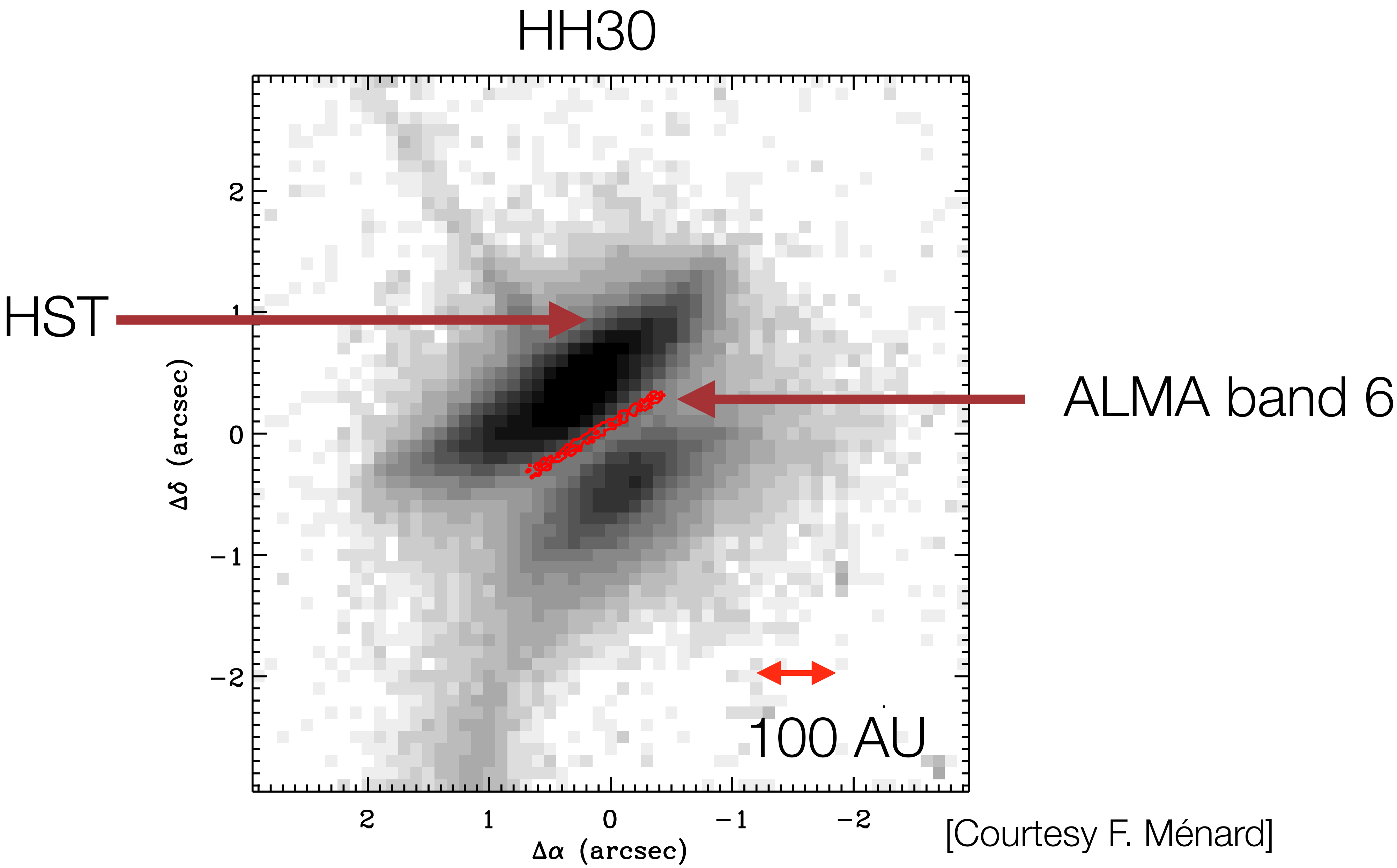
Turbulence velocity smaller than 0.04 c_s

Dust settling (I)



The thickness of the dust layer depends on the competition between settling and turbulent mixing

Dust settling
in edge on discs



[Courtesy F. Ménard]

mm-sized dust grains are strongly settled → low level of turbulence

Summary: Failure of the turbulent disc model

Theoretical

Discs are very weakly ionised

➔ "Non-ideal" MHD effects

➔ MHD turbulence too weak to explain observed accretion rates
[Turner+2014, PPVI]

Observational

- Turbulent line broadening (CO, DCO+) smaller than expected from MHD turbulence
  [Flaherty+2015, 2017]
- Vertical dust settling stronger than expected from MHD turbulence [Pinte+2016]

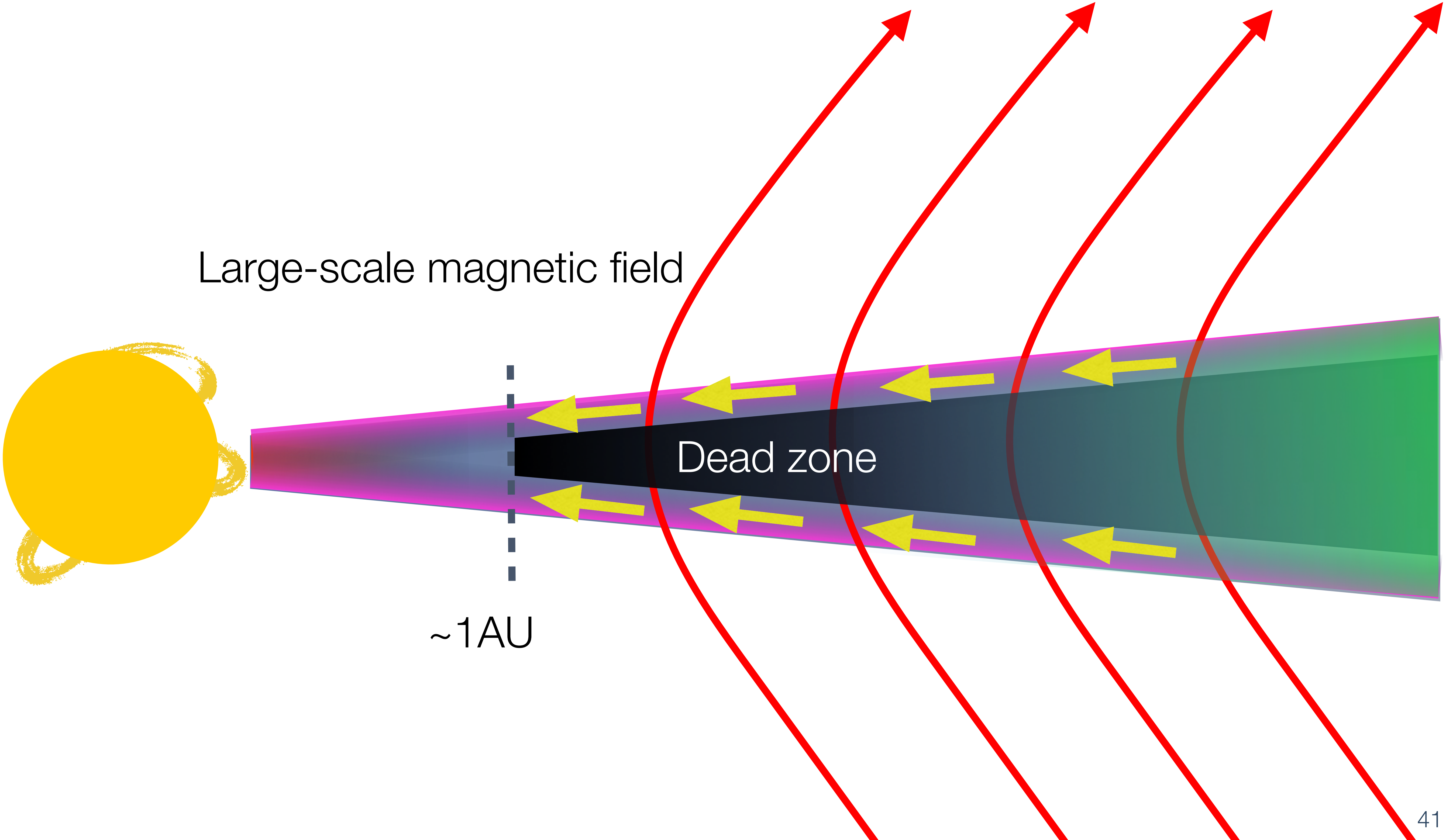
➔ Turbulence (if it exists) is much weaker than anticipated in the turbulent disc model

Key questions

- What drives accretion in protoplanetary discs?
- Which process is responsible for the large scale structures we observe?

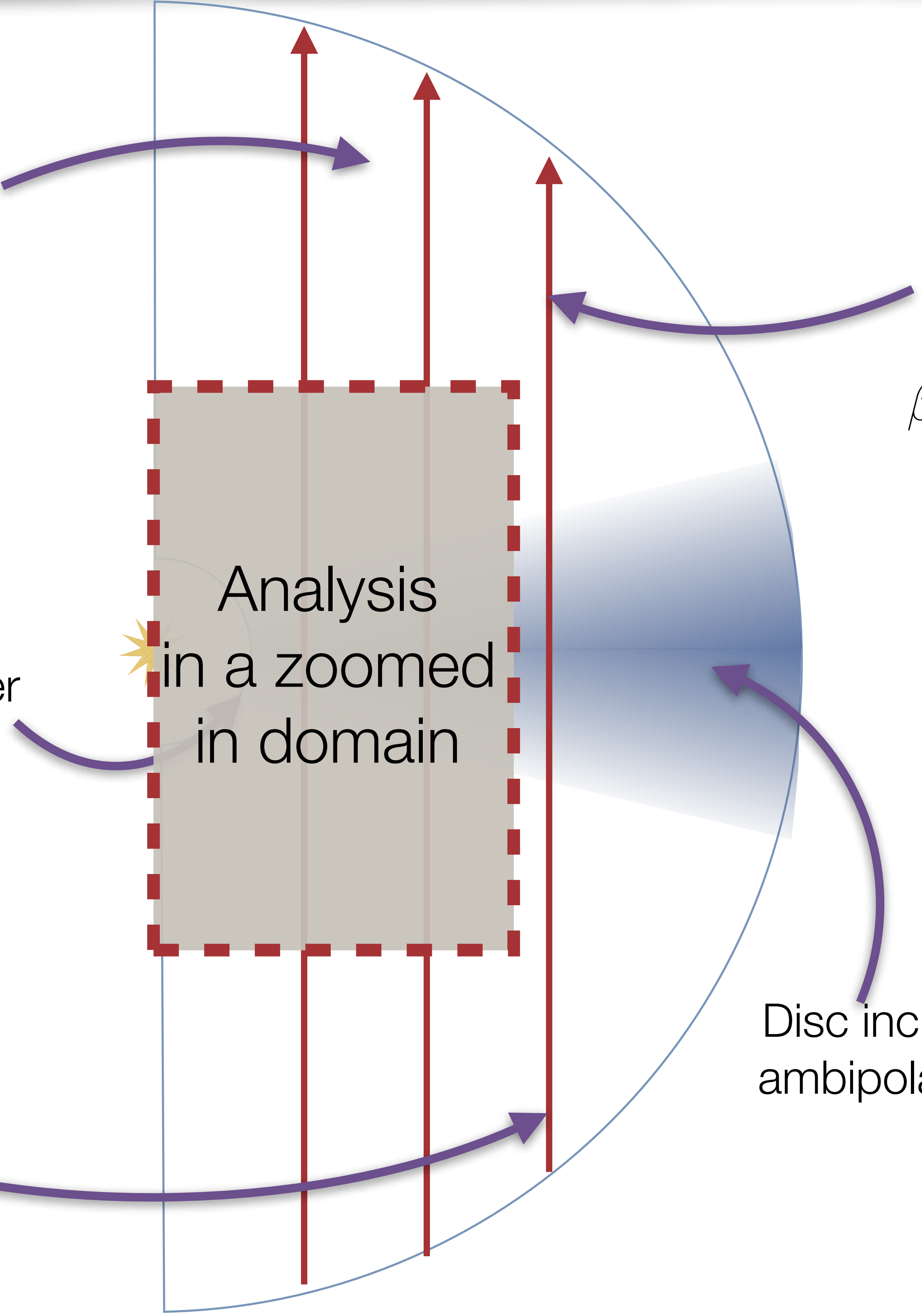
Wind-driven accretion in magnetically « dead » discs

[Wardle & Konigl 1993, Bai+ 2013, Lesur+ 2014, Simon+ 2015 in local models,
Gressel+2015, Béthune+2017, Bai 2017, Wang+ 2018, … in global geometry]



Global simulations

Numerical setup



Locally isothermal model ($T \propto R^{-1}$)

Poloidal field
threading the disc

$$\beta_p = \frac{8\pi P_{\mathrm{midplane}}}{B_z^2}$$

Analysis
in a zoomed
in domain

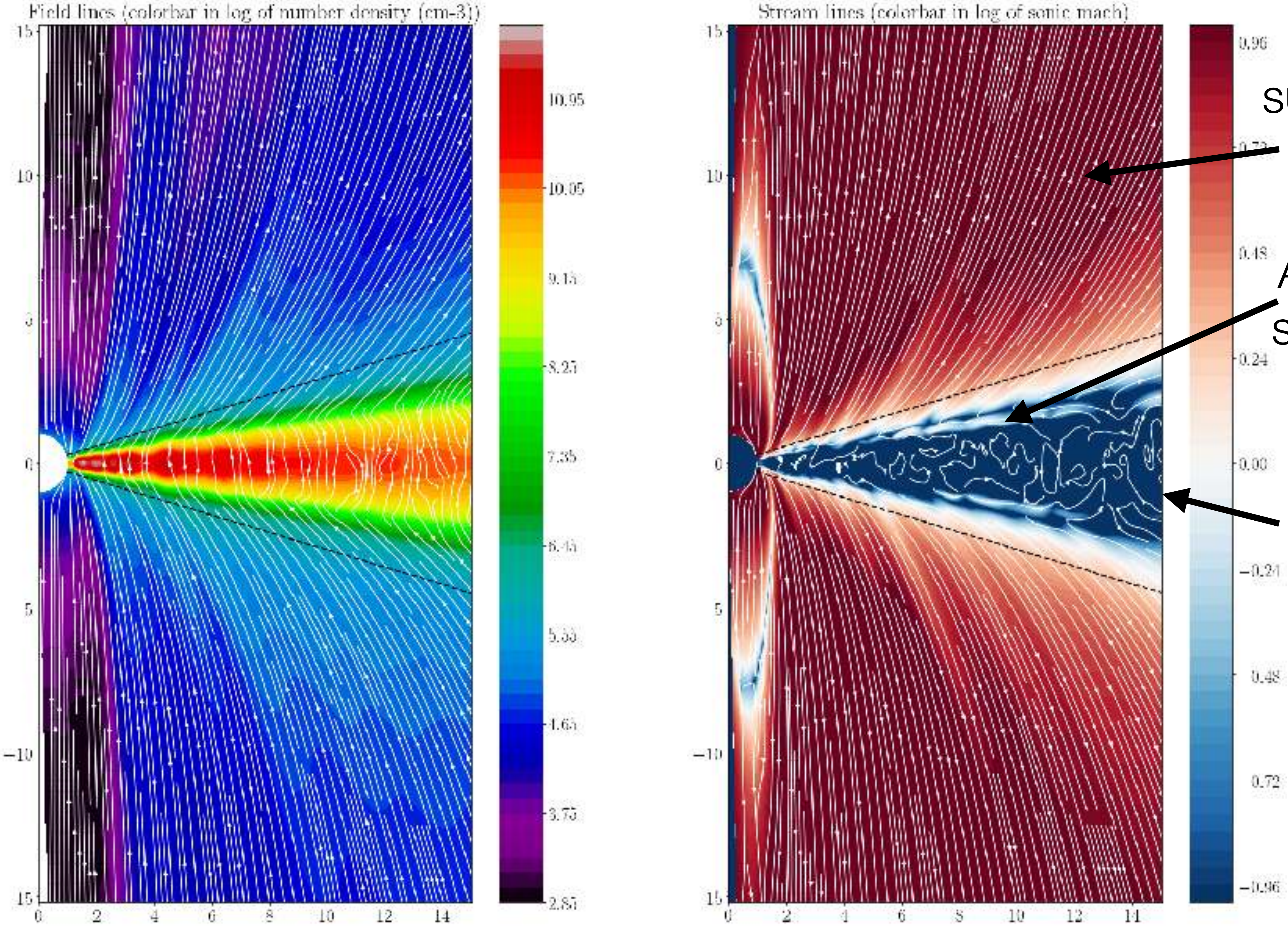
perfect conductor inner
boundary+damping of
poloidal velocity

Disc including prescribed
ambipolar diffusion profile

3D grid
Pluto code, static mesh
refinement

Global picture

$\beta_p = 10^4,\ \mathrm{Am}_{\mathrm{mid}} = 1$ average from t=1700 orbits to t=2400 orbits



Field lines (colorbar in log of number density (cm-3))

Stream lines (colorbar in log of sonic mach)

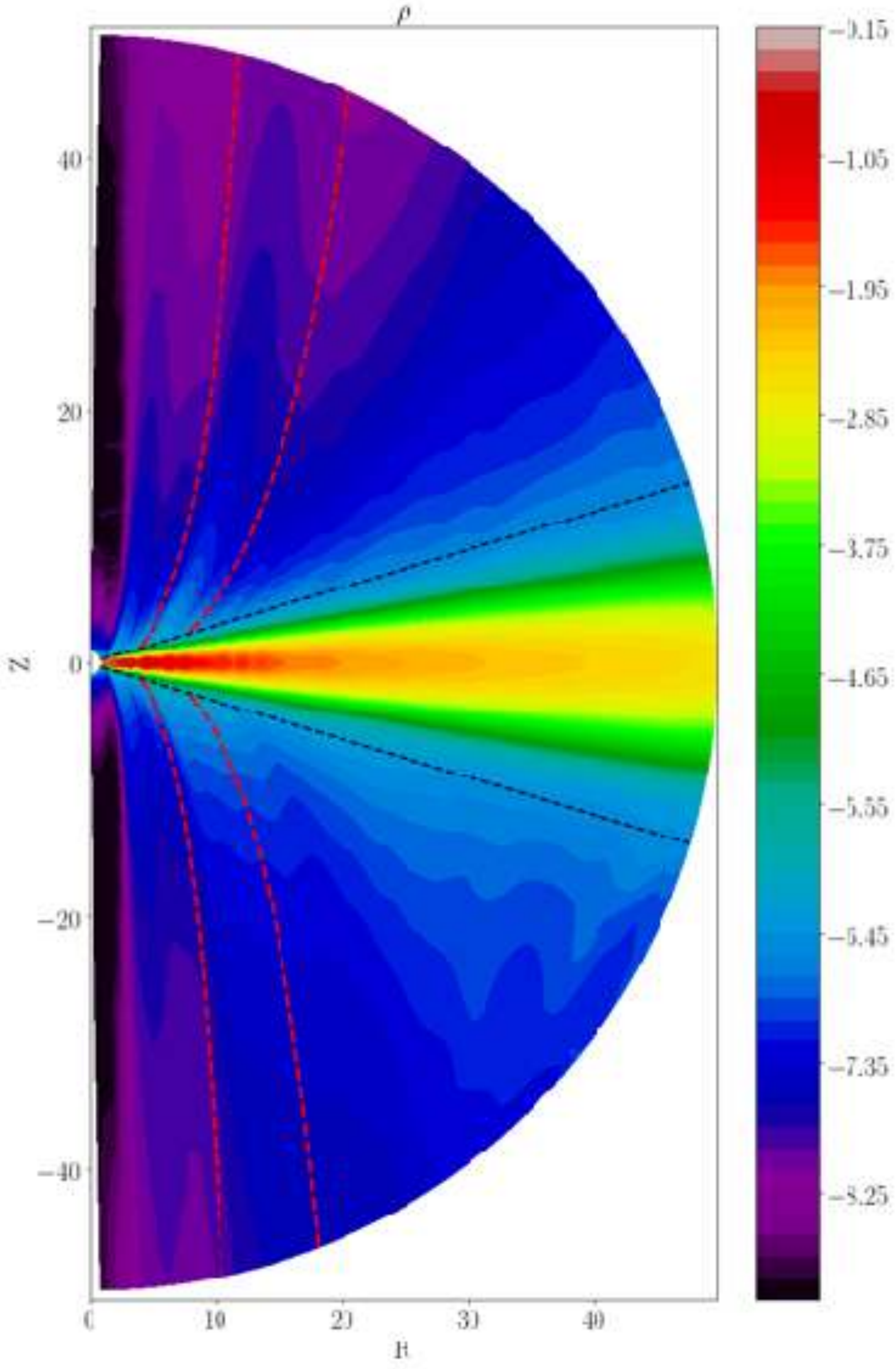
supersonic outflow

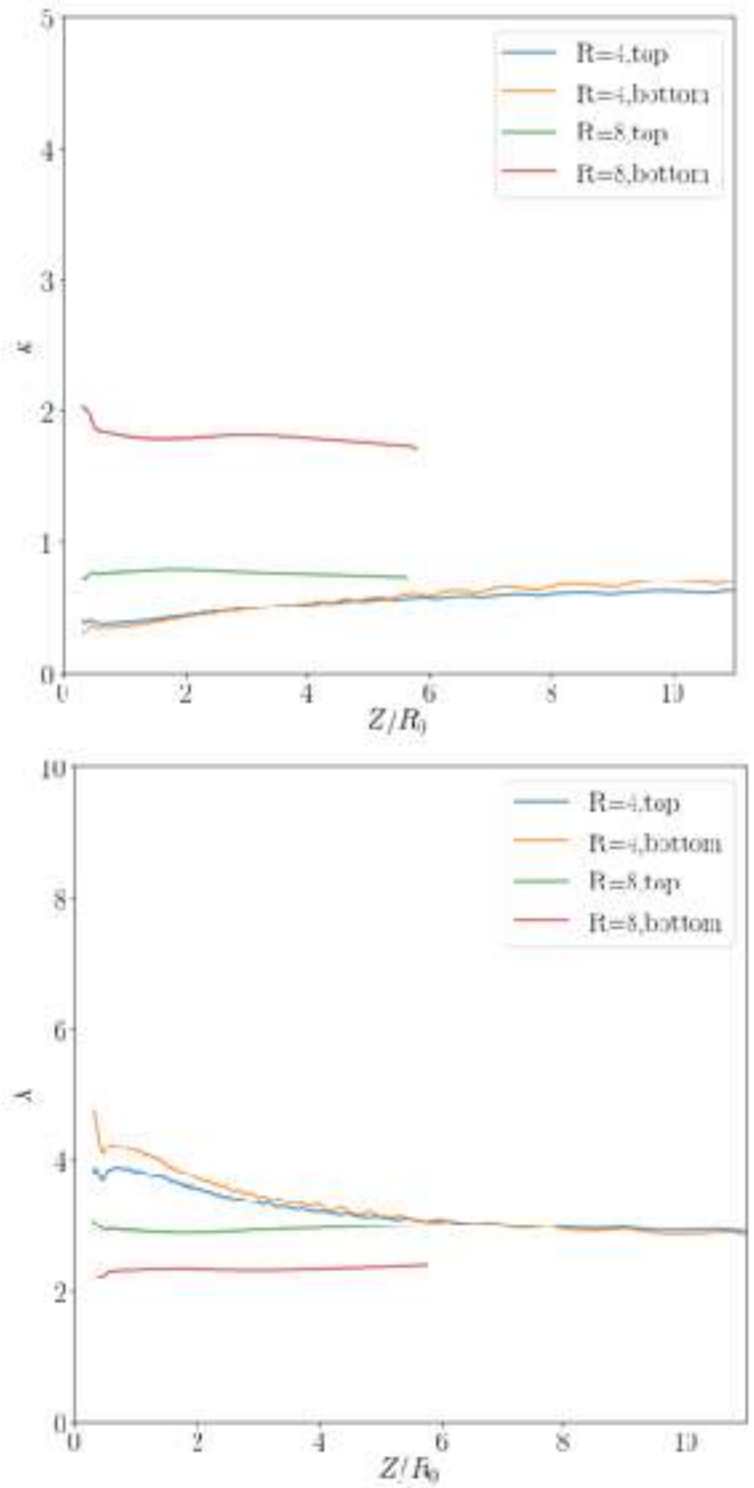
Accretion streamers

« looks » laminar…

Wind invariants

Take 4 representative streamlines and compute ideal MHD invariants





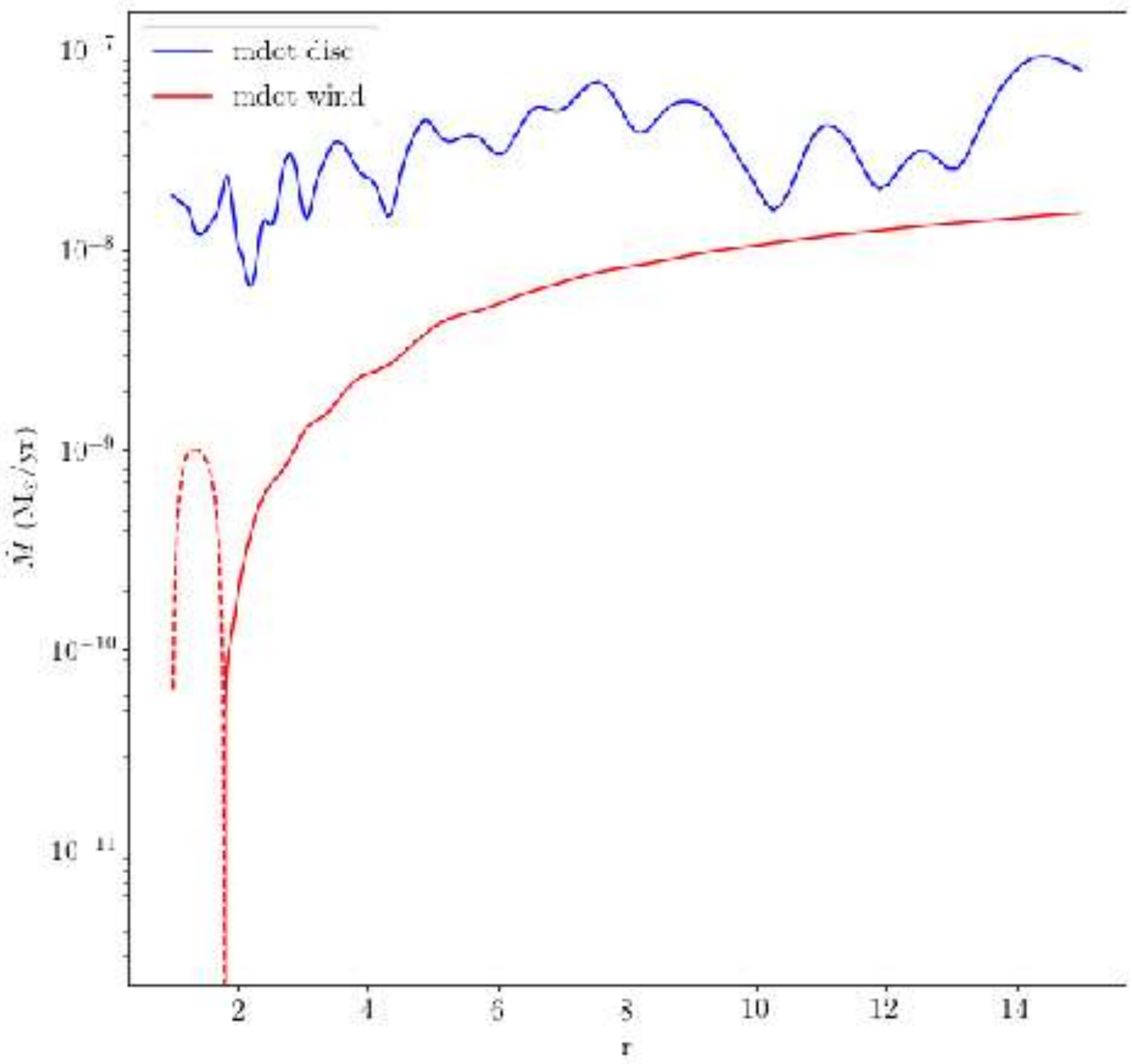
Accretion rate, mass loss rate

- Typical accretion rate~ 10^{-8}—$10^{-6}\ M_\odot/\mathrm{yr}$
- Accretion rate mostly controlled by the magnetic flux $\dot{M} \propto \beta^{-(0.5—1)}$
- Wind efficiency defined from

$$\dot{M}_{\mathrm{wind}} = \int_{R_{\mathrm{in}}}^{R} \mathrm{dR}\, R[\rho u_z]_{\mathrm{surface}}$$

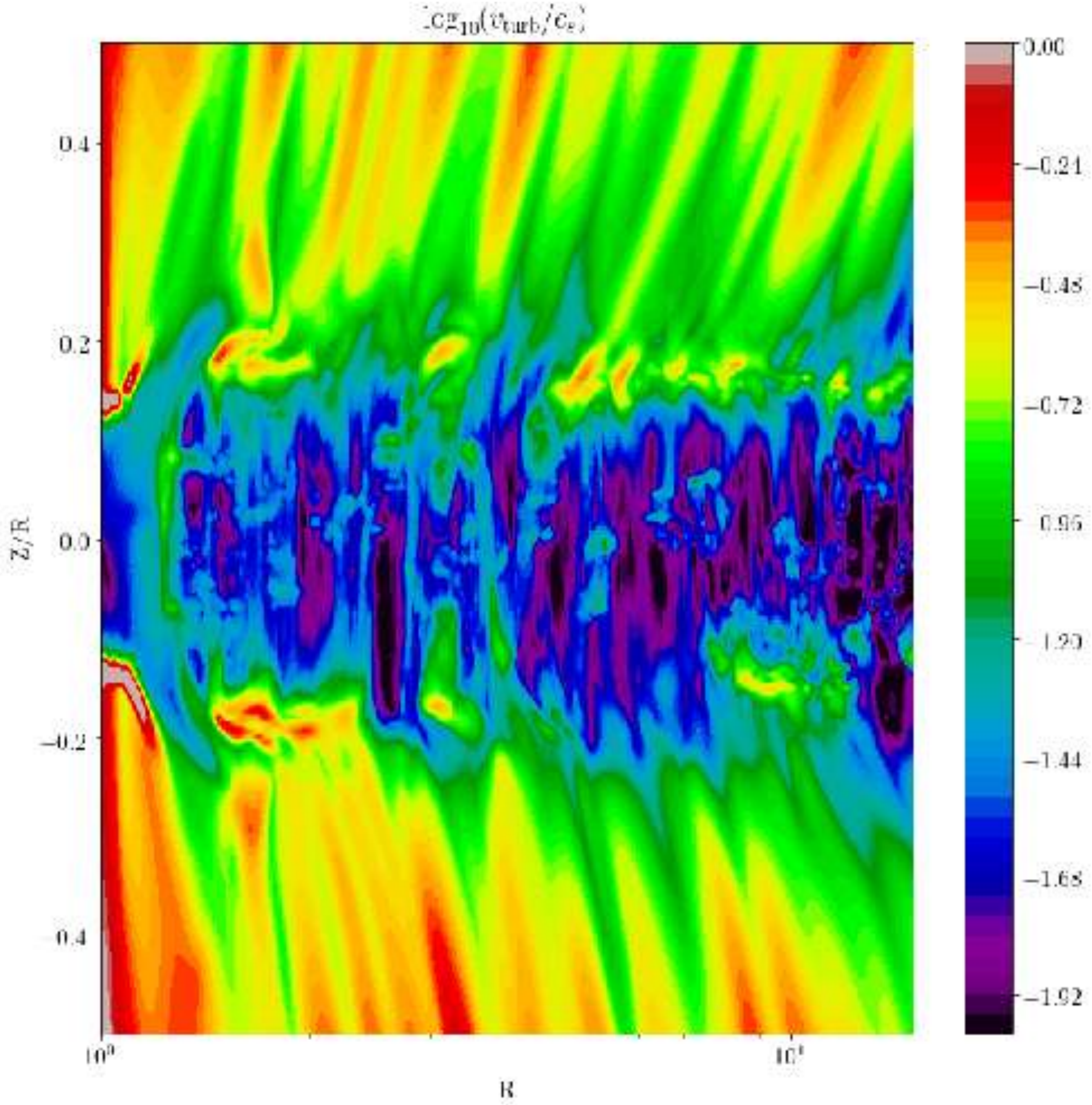
$$\xi = \frac{1}{\dot{M}} \frac{\mathrm{d}\dot{M}_{\mathrm{wind}}}{\mathrm{d}\log R}$$

- Typically have $\xi = 0.2—1$
  corona heating leads to larger ξ
  [Casse & Ferreira 2000, Béthune+2017, Bai 2017, Wang+2018]



Turbulence?

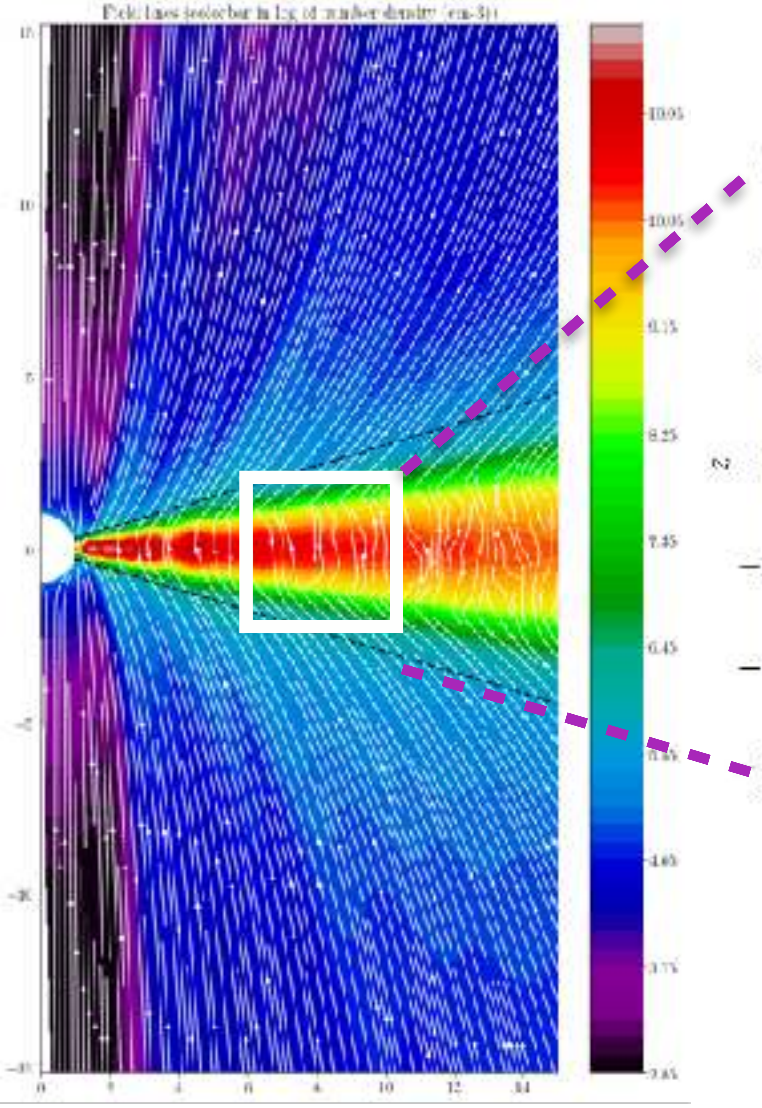
$$v_{\rm turb} = \left\langle \left(v - \langle v \rangle\right)^2 \right\rangle^{1/2}$$



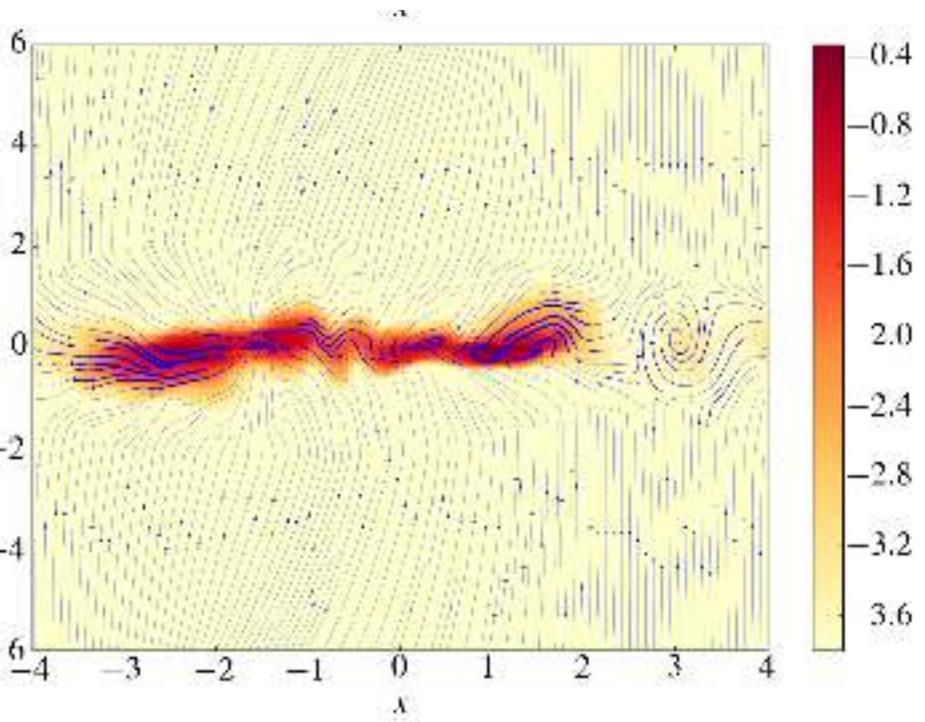
Typical velocity fluctuations of the order of 1% of the sound speed

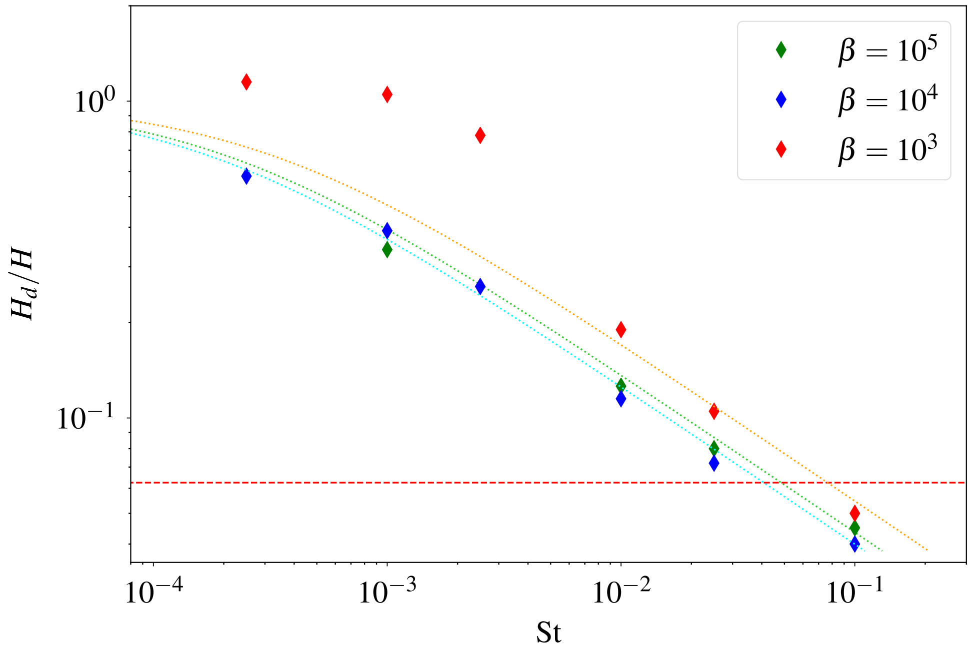
Compatible with observed turbulent broadening of CO lines

Dust Dynamics @ 30 AU

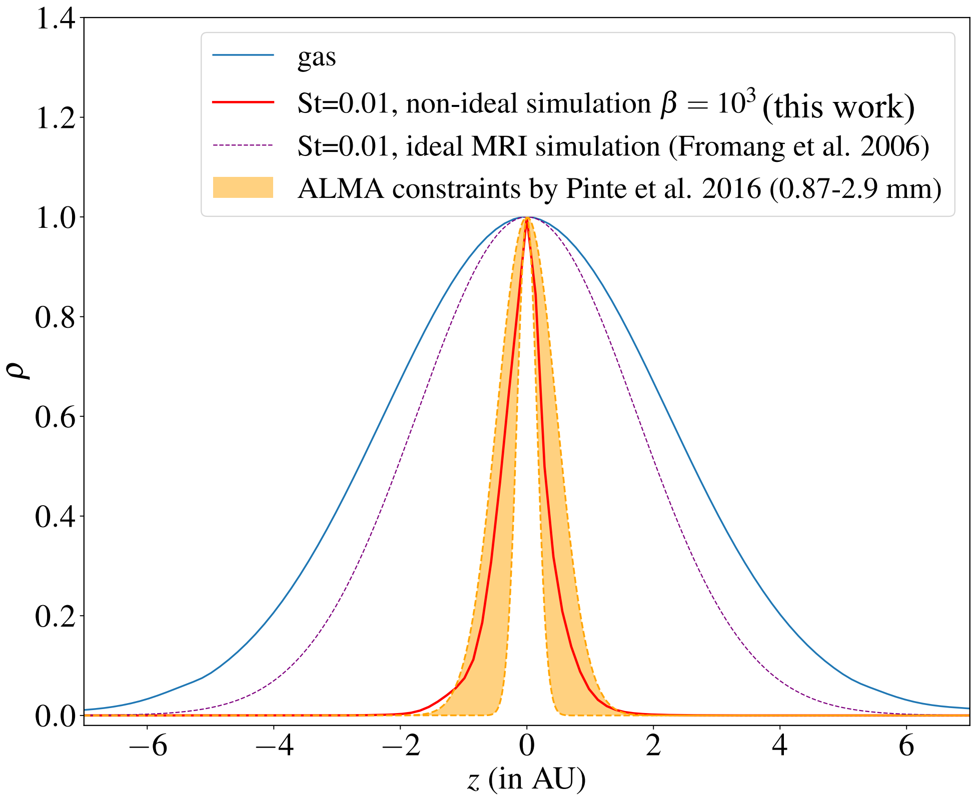


~mm size dust





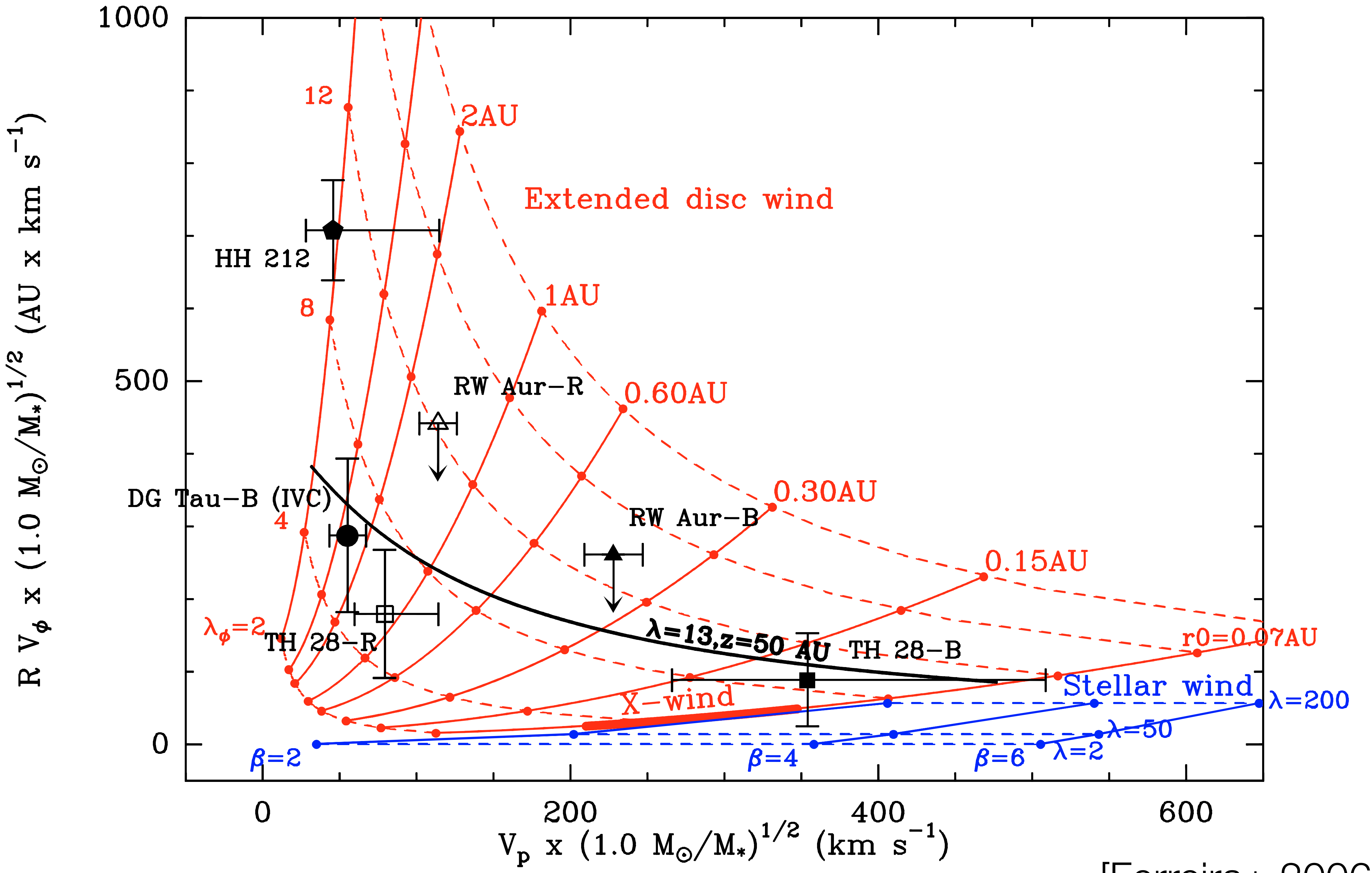
[Riols & Lesur 2018]



A few take away points

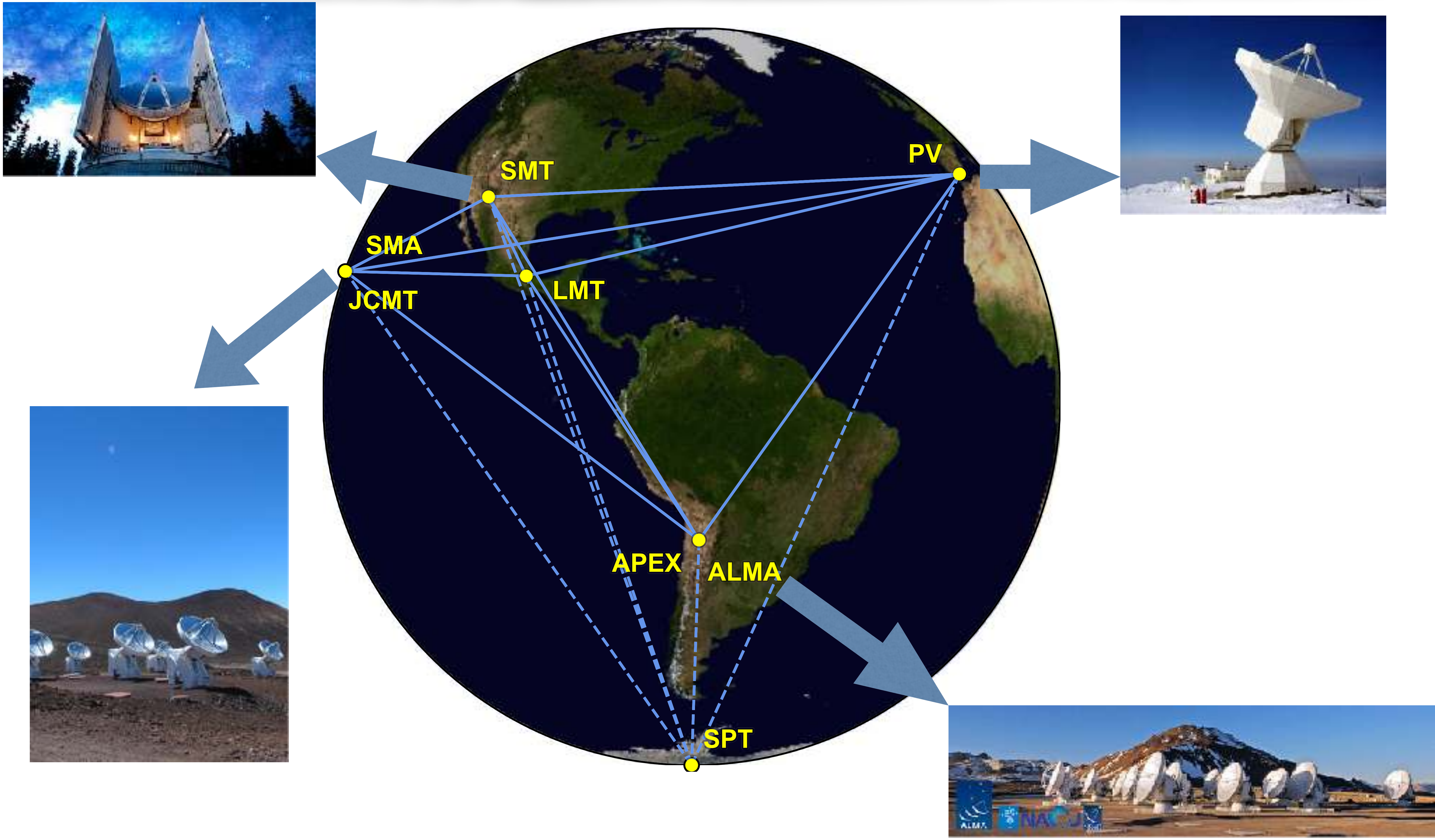
- Astrophysical discs can be accreting thanks to anomalous viscosity (turbulence, waves), or magnetised winds
- shear-driven hydrodynamic turbulence is notoriously difficult to trigger in Keplerian flows
- Winds are full non linear solution to the MHD equations. They require a large scale poloidal field, and some magnetic diffusion in the disc (to allow for accretion)
- In protoplanetary discs:
  - magnetic diffusion suppresses the MRI, but it provides the diffusion required by wind solutions.
  - these laminar wind solutions naturally reproduce some of the observed features of these discs: accretion rate, low level of turbulence, strong dust settling.

Testing jets kinematics

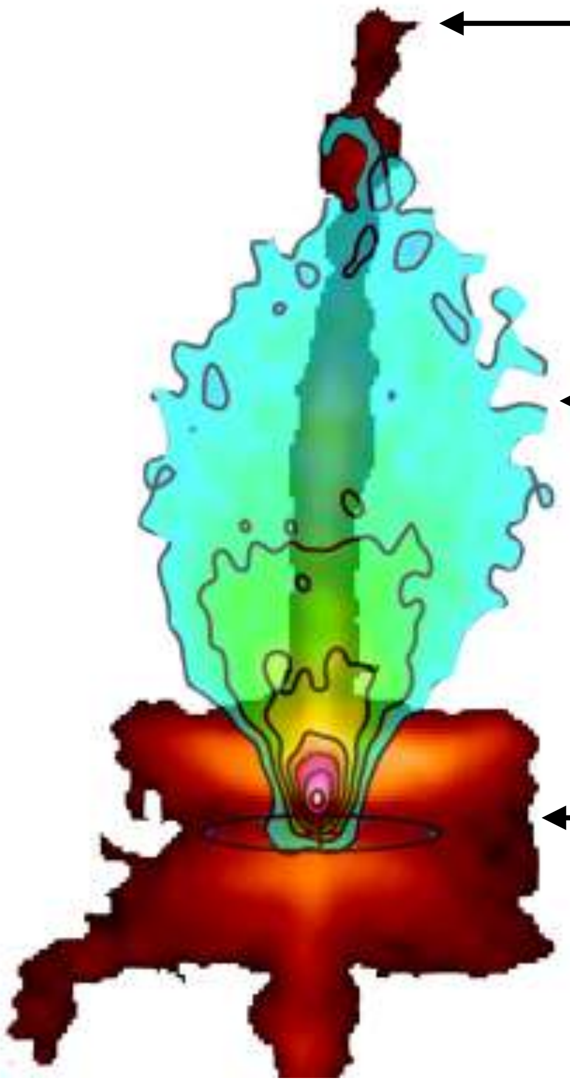


[Ferreira+ 2006]

Observing M87



Ejection evidence in HL tau



Atomic Jet

Molecular outflow

Disc seen edge on

Figure 3: Observation of an atomic jet and a molecular wind observed in CO(2-1) by ALMA in HH30, a protoplanetary disc seen edge-on. Courtesy of C. Dougados (Dougados et al. 2017).