Dynamics of accretion discs

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with thanks to

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What is the most efficient way to convert the rest mass energy of matter into heat?

A- Burning fuel       4x10^{-8} \%  
B- Nuclear fission    0.09 \%  
C- Nuclear fusion     0.09 \%  
D- ?

Accretion disc around a black hole: up to 40% efficiency
Overview

- A few fun facts about astrophysical discs (10’)
- How to drive accretion (30’)
- On the difficulty of driving hydrodynamic turbulence (30’)
- A short introduction to magnetised wind flows (60’)
- Application to protoplanetary discs (20’)

Protoplanetary discs

- Size $10^9$-$10^{13}$ m
- Central object: young star ($10^{30}$ kg)
- Temperature $10^3$-$10$ K

Credit: C. Burrows and J. Krist (STScI), K. Stapelfeldt (JPL) and NASA

Artist view
Structures in protoplanetary discs

[Huang+ 2018]
Compact binaries

- Size $10^4$-$10^8$ m
- Central object: white dwarf, neutron star, black hole ($10^{30}$ kg)
- Temperature $10^5$-$10^3$ K
Active galactic nuclei (blazars, quasars...)

- Size $10^{10} - 10^{15}$ m
- Central object: black hole ($10^{36} - 10^{39}$ kg = $10^6 - 10^9$ M$_{\text{Sun}}$)
- Temperature $10^5 - 10^2$ K
M87: staring at a supermassive black hole

Model of M87

Because rotation velocities are relativistic, one expects Doppler beaming in the blue shifted region.

General relativistic magneto-hydrodynamic model (GRMHD)

M87 April 6

Blurred GRMHD

Brightness Temperature ($10^9$ K)
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**Equations of motion**

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla \left( P + \frac{B^2}{8\pi} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} \]

- Magnetic pressure
- Magnetic tension

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \]

\[ \frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} - \mathbf{B} \nabla \cdot \mathbf{u} \]

- Transport
- Warping
- Compression
Assume a thin, weakly magnetised disc

\[ u_A \ll u_\phi \quad c_s \ll u_\phi \quad (u_r, u_z) \ll u_\phi \]

\[ u_\phi = R \Omega(R) \quad \text{with} \quad \Omega(R) = (GM_\odot)^{1/2} R^{-3/2} \]

Disc temporal evolution dictated by small deviations from the Keplerian profile:

\[ \mathbf{u} = \mathbf{v} + R \Omega(R) \mathbf{e}_\phi \]
Disc Dynamics

Mass conservation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]

Introduce: \( \overline{Q} = \int d\phi \int_{z=-h}^{z=h} dz \, Q \) and \( \Sigma = \overline{\rho} \)

\[ \frac{\partial \Sigma}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} R \overline{\rho v_r} + \left[ \rho u_z \right]_{z=-h}^{+h} = 0 \]
Angular momentum conservation:

\[
\frac{\partial (\rho R u_\phi)}{\partial t} + \nabla \cdot \left[ \rho R u_\phi \mathbf{u} - R \frac{B_\phi B}{4\pi} + R \left( P + \frac{B^2}{8\pi} \right) e_\phi \right] = 0
\]

Combine it with mass conservation, squeeze it, stretch it:

\[
\rho \nu_r \frac{\partial}{\partial R} \Omega R^2 + \frac{1}{R} \frac{\partial}{\partial R} R^2 \left[ \rho \nu_\phi \nu_r - \frac{B_\phi B_r}{4\pi} \right] + R \left[ \rho \nu_\phi \nu_z - \frac{B_\phi B_z}{4\pi} \right]^{+h}_{z=-h} = 0
\]

accretion  radial stress  vertical stress (aka wind stress)
Disc Dynamics

alpha disc model

Introduce the dimensionless number

\[ \alpha = \frac{\rho \nu_\phi \nu_r - B_\phi B_r/4\pi}{\Sigma \Omega^2 H} \]

Estimated accretion rate

\[ \rho \nu_r \sim -\alpha c_s \Sigma \frac{H}{R} \]

Compare to observations: \(10^{-4} < \alpha < 10^{-1}\)

New questions!

What is responsible for anomalous viscosity?

How large is \(\alpha\) ?

What about winds?
The zoo of disc instabilities

Local instabilities:

- Magnetorotational instability (MRI): shear driven instability but requires an ionised plasma (Velikhov 1959, Chandrasekhar 1960, Balbus & Hawley 1991)

- Gravitational instabilities: only for massive & cold enough disc (Gammie 2001, Paardekooper 2012)

- Subcritical shear instability: probably not efficient enough, if it exists (see later) (Lesur & Longaretti 2005, Schartman et al. 2012, Edlund & Ji 2014)

- Vertical Shear instability: driven by vertical shear (actually link to the baroclinicity of the disc) (Urpin & Brandenburg (1998), Nelson+ 2013, Barker & Latter 2015)


- Zombie vortex instability: buoyancy critical layer instability (Marcus+ 2013, Marcus+2016, Lesur & Latter 2016)

- Rossby wave instability: requires a local maximum of vortensity (equivalent to Kelvin-Helmholtz) (Lovelace et. al 1999)

- Vertical convective instability: Requires a heat source in the midplane (Cabot 1996, Lesur & Ogilvie 2010, Held & Latter 2018)

Global instabilities:

- Papaloizou & Pringle instability: density wave reflection on the inner edge (Papaloizou & Pringle 1985)
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Subcritical shear instabilities
Origins

The Facts:
- Keplerian shear flows are linearly stable
- Huge Reynolds numbers ($10^{15}$) nonlinear instability? (same thing as pipe flows or Couette flows)

A nonlinear instability in accretion discs?
- Experimental approach: hard to «do» a disc in a lab. Boundary conditions?
- Numerical approach: high Reynolds numbers unreachable: $Re \lesssim 10^4$

Ideal Taylor-Couette

Real life Couette-Taylor (Schartman et al. 2012)
A boundary problem

Turbulence excited by Ekman layers

\[ R_s = 9655 \quad R_s = 19310 \quad R_s = 32180 \]

Lopez & Avila (2017)
Can non-linear, shear-driven, instabilities, if they exist, transport angular momentum efficiently in Keplerian flows?

**A contentious debate…**

**Theory and simulations:**
- Zeldovich (1981): maybe yes
- Durbulle (1993): maybe yes
- Balbus, Hawley & Stone (1996): no
- Richard & Zahn (1999): maybe yes
- Lesur & Longaretti (2005): no
- Oticilla-Monico+ (2014): maybe no
- Bhatia & Mukhopadhyay (2016): maybe yes
- Lopez & Avila (2017), Shi+ 2017: no

**Laboratory experiments**
- Beckley & Colgate (2002): maybe no
- Kageyama+ (2004): maybe no
- Ji+ (2006), Schartman+ (2012): no
- Edlund & Ji (2014): no
- Nordsleek + (2015): maybe no
- Edlund & Ji (2015): no

Finally converging to a « no » (but no formal proof)
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Magnetised winds: a MRI mode becoming non-linear

Particle A is displaced inwards, Particle B is displaced outwards

From angular momentum conservation, particle A gets a faster angular velocity while particle B gets slower

The accumulated toroidal field create a vertical magnetic pressure gradient, pushing B upwards and A downwards

As particles A and B drift, an azimuthal magnetic field builds up between the particles
We assume stationary, axisymmetric, ideal MHD.

Field strength controlled by the plasma parameter $\beta_p = \frac{8\pi P_{\text{midplane}}}{B_z^2}$.
Stationary equations

The need for a magnetically diffusive disc

\[ \nabla \cdot B_p = 0 \]
(solenoidal condition)

\[ \nabla \cdot \rho u_p = 0 \]
(mass & solenoidal condition)

\[ \rho u_p \cdot \nabla u_R = \rho \Omega^2 r - \partial_R P + \frac{J_\phi B_z}{c} - \frac{J_z B_\phi}{c} - \rho \partial_R \psi \]
(R-momentum)

\[ \nabla \cdot \left( \rho u_p \Omega^2 R^2 - R \frac{B_p B_\phi}{4\pi} \right) = 0 \]
(\( \phi \)-momentum)

\[ \rho u_p \cdot \nabla u_z = -\partial_z P - \partial_z \left( \frac{B_\phi^2 + B_R^2}{8\pi} \right) + \frac{B_R \partial_R B_z}{4\pi} - \rho \partial_z \psi \]
(z-momentum)

\[ \nabla \times (u_p \times B_p) = 0, \]
(poloidal induction)

\[ \nabla \cdot \frac{1}{R} (\Omega R B_p - B_\phi u_p) = 0. \]
(\( \phi \) induction)

\[ B_p = \frac{1}{R} \nabla a \times e_\phi. \]

\( u_p // B_p \) not possible in the disc

non-ideal MHD region
Stationary equations

Critical points

The system of equations has 3 critical points (= critical layers for hydro people)

\[ u_p = V_{\text{slow}} \approx c_s \]

\[ u_p = V_A = \frac{B_p}{\sqrt{4\pi \rho}} \]

\[ u_p = V_{\text{fast}} \]  
Fast magnetosonic point

Alfvén point

Slow magnetosonic point

\[ R_0 \text{ « Launching radius »} \]

\[ R_A \text{ « Alfvén radius »} \]

An outflow is causally « disconnected » from its launching point once it has crossed all three critical points
Stationary equations
Invariants along the streamlines

\[ \nabla \cdot B_p = 0 \]
\[ \nabla \cdot \rho u_p = 0 \]
\[ \rho u_p \cdot \nabla u_R = \rho \Omega^2 r - \partial_R P + \frac{J_\phi B_z}{c} - \frac{J_z B_\phi}{c} - \rho \partial_R \psi \]
\[ \nabla \cdot \left( \rho u_p \Omega R^2 - R \frac{B_p B_\phi}{4\pi} \right) = 0 \]
\[ \rho u_p \cdot \nabla u_z = -\partial_z P - \partial_z \left( \frac{B_z^2 + B_R^2}{8\pi} \right) + \frac{B_R \partial_R B_z}{4\pi} - \rho \partial_z \psi \]
\[ \nabla \times (u_p \times B_p) = 0, \]
\[ \nabla \cdot \left( \frac{1}{R} \Omega R B_p - B_\phi u_p \right) = 0. \]
\[ \Omega^*(a) R_A^2 = \ell(a) \equiv \Omega R^2 - \frac{R B_\phi}{4\pi \kappa(a)} \]
\[ \kappa(a) \equiv \frac{\rho u_p}{B_p} \quad \text{« mass loading parameter »} \]
\[ \Omega^*(a) \equiv \Omega - \frac{\kappa(a)}{\rho R} B_\phi \quad \text{« rotation speed of magnetic surfaces »} \]

In addition, one can create an energy invariant:

\[ \mathcal{B} \equiv \frac{u^2}{2} + \psi_G + \mathcal{H} - \frac{R \Omega^*(a) B_\phi}{4\pi \kappa(a)} \quad \text{« Benoulli invariant »} \]
Back to the accretion problem

Angular momentum conservation:

\[
\rho v_r \frac{\partial}{\partial R} \Omega R^2 + \frac{1}{R} \frac{\partial}{\partial R} R^2 \left[ \rho v_\phi v_r - \frac{B_\phi B_r}{4\pi} \right] + R \left[ \rho v_\phi v_z - \frac{B_\phi B_z}{4\pi} \right] \bigg|_{z=-h}^{+h} = 0
\]

accretion \hspace{1cm} \text{radial stress} \hspace{1cm} \text{vertical stress (aka wind stress)}

Using MHD invariants:

\[
R \left[ - \frac{B_\phi B_z}{4\pi} \right] \bigg|_{-h}^{+h} = R \frac{B_{z0}^2}{4\pi} \kappa (\lambda - 1)
\]

Once the MHD invariants are known for a given solution, one can predict the accretion rate, and mass loss rate
Typical solutions
Self-similar solutions

Typical « cold wind » solution

[Casse & Ferreira 2000]
Typical solutions
Self-similar solutions

\[ \kappa \]
\[ \lambda \]

\[ \varepsilon = 0.1 \]
\[ \alpha_m = 1 \]

\[ \omega_A = 1.44 \]
\[ \rho_A = 1.12 \]
\[ \omega_A = 1.16 \]

Casse & Ferreira (2000)
Numerical simulations

time = 0

Zanni+ 2007
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Accretion rate onto the stellar surface

Variable extinction

[Venuti+2014]
Ionisation sources in protoplanetary discs

Thermal ionisation

X-rays

Far-UV

~1AU

~30AU

Cosmic rays

« non ideal » MHD effects

- Ohmic diffusion (electron-neutral collisions)
- Ambipolar Diffusion (ion-neutral collisions)
- Hall Effect (electron-ion drift)

Amplitude of these effects depends strongly on location & composition
Some technical « details » are intentionally hidden...
Ambipolar diffusion

Am < 100  « damped » MRI

[Thi+2018]
What do observers say?
Line broadening

- Emission lines from the gas are broaden by:
  - Keplerian rotation $V_k$
  - Thermal velocity $v_{th} \approx c_s \ll V_k$
  - Turbulence $v_{turb} \approx \sqrt{\alpha c_s}$

Measuring line broadening due to turbulence requires very precise measures/estimates of $V_k$ and $c_s$

Turbulence velocity smaller than 0.04 $c_s$

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![Figure 6. CO(3-2) high resolution spectra (black line) compared to the median model when turbulence is allowed to move toward very low values (red dotted–dashed lines) or when it is fixed at 0.1 km s$^{-1}$ (blue dashed lines). All spectra have been normalized to their peak flux to better highlight the change in shape. The models with weak turbulence provide a significantly better fit to the data despite the fact that the turbulence is smaller than the spectral resolution of the data. [Flaherty+2015]](http://example.com)
Dust settling (I)

The thickness of the dust layer depends on the competition between settling and turbulent mixing.
Dust settling in edge on discs

HST

ALMA band 6

mm-sized dust grains are strongly settled → low level of turbulence

[Courtesy F. Ménard]
Summary: Failure of the turbulent disc model

**Theoretical**
- Discs are very weakly ionised
- “Non-ideal” MHD effects
- MHD turbulence too weak to explain observed accretion rates [Turner+2014, PPVI]

**Observational**
- Turbulent line broadening (CO, DCO+) smaller than expected from MHD turbulence [Flaherty+2015, 2017]
- Vertical dust settling stronger than expected from MHD turbulence [Pinte+2016]
- Turbulence (if it exists) is much weaker than anticipated in the turbulent disc model

**Key questions**
- What drives accretion in protoplanetary discs?
- Which process is responsible for the large scale structures we observe?
Wind-driven accretion in magnetically «dead» discs

Global simulations

Numerical setup

Locally isothermal model ($T \propto R^{-1}$)

Disc including prescribed ambipolar diffusion profile

Poloidal field threading the disc

$\beta_p = \frac{8\pi P_{\text{midplane}}}{B_z^2}$

Analysis in a zoomed in domain

perfect conductor inner boundary+damping of poloidal velocity

3D grid

Pluto code, static mesh refinement

Disc including prescribed ambipolar diffusion profile
Global picture

$\beta_p = 10^4$, $A_{\text{mid}} = 1$ average from $t=1700$ orbits to $t=2400$ orbits

Field lines (colorbar in log of number density (cm$^{-3}$))

Stream lines (colorbar in log of sonic mach)

Supersonic outflow

Accretion streamers

« Looks » laminar...
Wind invariants

Take 4 representative streamlines and compute ideal MHD invariants
Accretion rate, mass loss rate

- Typical accretion rate~ $10^{-8} - 10^{-6} \, M_\odot/\text{yr}

- Accretion rate mostly controlled by the magnetic flux $\dot{M} \propto \beta^{-(0.5\ldots1)}$

- Wind efficiency defined from

\[
\dot{M}_{\text{wind}} = \int_{R_{\text{in}}}^{R} dR \, R[\rho u_z]_{\text{surface}}
\]

\[
\xi = \frac{1}{\dot{M}} \frac{d\dot{M}_{\text{wind}}}{d \log R}
\]

- Typically have $\xi = 0.2 - 1$

  corona heating leads to larger $\xi$

Turbulence?

$$v_{turb} = \left\langle \left( v - \langle v \rangle \right)^2 \right\rangle^{1/2}$$

Typical velocity fluctuations of the order of 1% of the sound speed

Compatible with observed turbulent broadening of CO lines
Dust Dynamics @ 30 AU

~mm size dust

[ Riols & Lesur 2018 ]
A few take away points

- Astrophysical discs can be accreting thanks to anomalous viscosity (turbulence, waves), or magnetised winds.

- Shear-driven hydrodynamic turbulence is notoriously difficult to trigger in Keplerian flows.

- Winds are full non linear solution to the MHD equations. They require a large scale poloidal field, and some magnetic diffusion in the disc (to allow for accretion).

- In protoplanetary discs:
  - Magnetic diffusion suppresses the MRI, but it provides the diffusion required by wind solutions.
  - These laminar wind solutions naturally reproduce some of the observed features of these discs: accretion rate, low level of turbulence, strong dust settling.
Testing jets kinematics

\[ R \, \lambda \left( \frac{1 \, M_\odot}{M_*} \right)^{1/2} (\text{AU} \times \text{km s}^{-1}) \]

- Extended disc wind
- DG Tau-B (IVC)
- HH 212
- TH 26-R
- RR Aur-B
- TH 28-B

\[ V_p \times (1.0 \, M_\odot/M_*)^{1/2} (\text{km s}^{-1}) \]

[Ferreira+ 2006]
Figure 3: Observation of an atomic jet and a molecular wind observed in CO(2-1) by ALMA in HH30, a protoplanetary disc seen edge-on. Courtesy of C. Dougados (Dougados et al. 2017).