Lectures 1 and 2
Internal gravity waves in the ocean

- Dispersion relation of inertia-gravity waves revisited

- Internal gravity waves in the ocean
  - The turbulent state of the ocean
  - Mixing processes in the deep ocean
  - The main sources of internal gravity waves in the ocean
  - Breaking processes of oceanic internal gravity waves
  - Focus on parametric subharmonic instability and induced mixing

No stratification → see Jérôme Noir's talk
Internal gravity waves in stars → see Tamara Rogers' talk
For the "internal tide" → see Sonya Legg's talk

Lecture 3: Internal gravity waves in the atmosphere
Lecture 4: Lee waves

Dispersion relation of internal gravity waves

The generation of waves in a fluid medium requires the existence of a restoring force.

In a **stably-stratified** **rotating** medium, two restoring forces exist:

-the **buoyancy force**, due to the stable stratification in density (in the ocean) or in potential temperature (in the atmosphere) along the vertical and

-the **Coriolis force** due to rotation.

The **classical method** to obtain the dispersion relation of internal gravity waves is to **linearize the equations of motions** about the rest state and search for a plane wave solution.

We proceed differently and follow the more direct method used by Lighthill (« Waves in fluids », CUP, 1978) for waves for which **the averaged kinetic and potential energy are equal**.

We first ignore the Coriolis force.

Dispersion relation of internal gravity waves

We proceed differently and follow the more direct method used by Lighthill (1978) for waves for which the averaged kinetic and potential energy are equal.

The principle of the method is the following.

The averaged wave-induced kinetic energy can be writen as $\overline{E}_k = \alpha\,\overline{(\dot{\eta})^2}$ where the overline denotes an average over a wave period and η is the wave-induced vertical displacement.

As well, the averaged wave-induced potential energy can be written as $\overline{E}_p = \beta\,\overline{\eta^2}$.

Using Parseval theorem, for a monochromatic wave, $\overline{E}_k$ can be written as: $\overline{E}_k = \alpha\,\omega^2\,\overline{\eta^2}$ so that the relation $\overline{E}_k = \overline{E}_p$ implies that

$$\boxed{\omega^2 = \frac{\beta}{\alpha} = \frac{\text{generalized stiffness}}{\text{generalized inertia}}}$$

Example of a mass m attached to an oscillating spring of stiffness K, with displacement *x* w.r.t its equilibrium position. Then:

$E_k = \frac{1}{2} m\,\overline{\dot{x}^2} \Rightarrow \alpha = m$ and $E_p = \frac{1}{2} K\,\overline{x^2} \Rightarrow \beta = K$ (ignoring the ½ factor)

and we recover the well-known expression of the oscillating frequency: $\omega^2 = K/m$

Dispersion relation of internal gravity waves

Dispersion relation of internal gravity waves in a non rotating medium

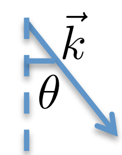
We consider a wave field with wave-induced velocity and buoyancy:

$$(u, w, \rho') = (U, W, R)e^{i(k_x\, x + k_z\, z - \omega\, t)}$$

- **Computation of the averaged kinetic energy**:
  - from the definition of the kinetic energy (per unit volume) : $\overline{E_k} = 0.5\, \rho_0\, (\overline{u^2} + \overline{w^2})$
  - using the incompressibility condition: $u = -(k_z/k_x)\, w$
  - and the link between the wave-induced vertical displacement and w: $w = D\eta/Dt \simeq \dot{\eta}$

  One can show that:

$$\overline{E_k} = 0.5\, \rho_0\, \frac{\overline{\dot{\eta}^2}}{\sin^2\theta} \Rightarrow \alpha = \frac{\rho_0}{\sin^2\theta}$$

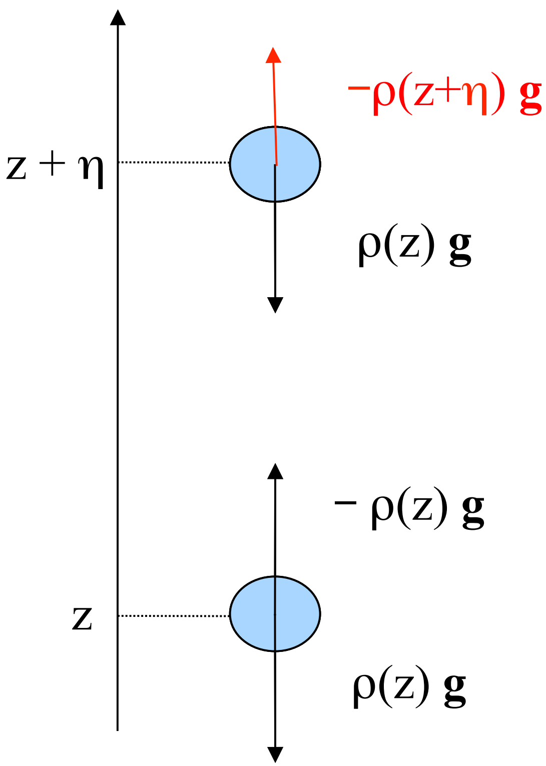


- **Computation of the averaged potential energy**
  - from the definition of the potential energy: $E_p = \int_\eta^0 F\, dz$, where F is the buoyancy force
  - and the definition of the buoyancy force: $F = -\rho_0\, N^2\, \eta$

  One finds $\overline{E_p} = 0.5\, \rho_0\, N^2 \overline{\eta^2} \Rightarrow \beta = \rho_0\, N^2$

which implies that $\boxed{\omega^2 = N^2 \sin^2\theta}$

Oscillations of a fluid particle in a stably stratified fluid



- Consider a fluid particle of unit volume in a stably-stratified medium of density $\rho(z)$, located at altitude z at equilibrium position.

- Move the particle adiabatically upwards to altitude $z+\eta$. The particle is no longer in equilibrium, being subject to the *buoyancy force :*
  $\rho(z+\eta)\, g - \rho(z)\, g \approx g\, d\rho/dz\, \eta\ (<0)$

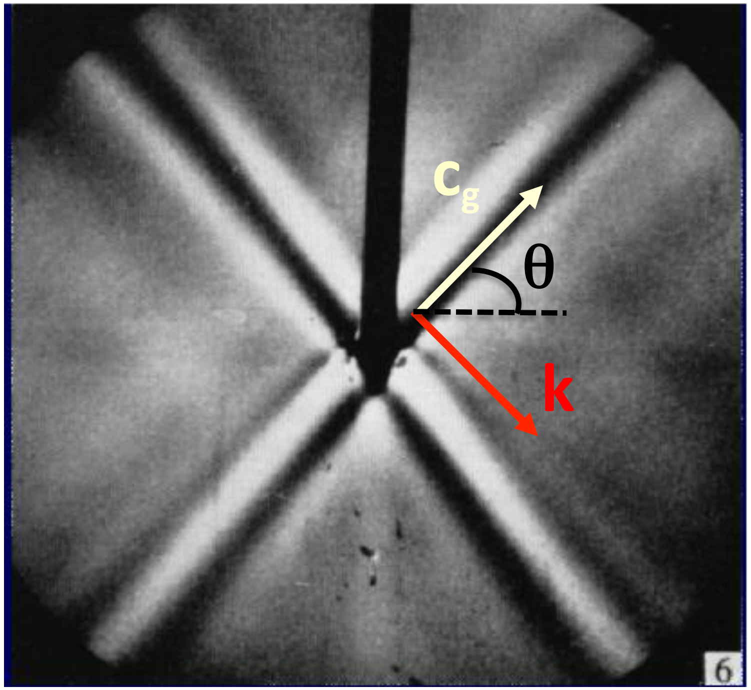
- Apply Newton's law to the particle :
  $\rho(z)\, d^2\eta/dt^2 = g\, d\rho/dz\, \eta$ or

$$\boxed{\begin{array}{c} \mathbf{d^2\eta/dt^2 + N^2\,\eta = 0} \\ \mathbf{with\ N^2 = -\,g/\rho(z)\, d\rho/dz} \end{array}}$$

N(z) is the Brunt-Vaisala (or buoyancy) frequency and the buoyancy force can be written as: $-\rho(z)\, N^2\, \eta$

Dispersion relation of internal gravity waves

Mowbray and Rarity (J. Fluid Mech., 1967)



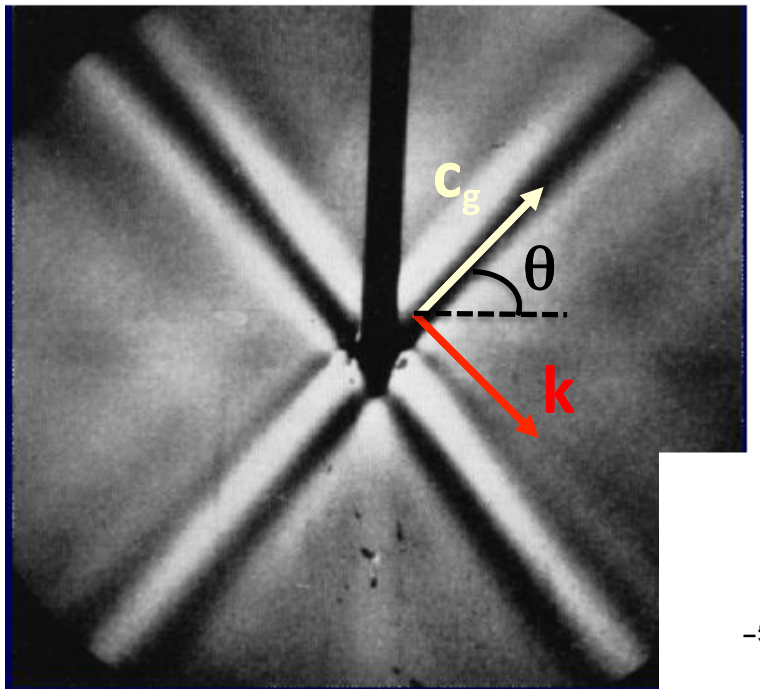
→ $\mathbf{c}_g$ (group velocity) $\perp$ **k**

→ **no length scale selection** from the dispersion relation (only a time scale N^{-1})

$$\omega^2 = N^2 \sin^2 \theta$$

Dispersion relation of internal gravity waves

Mowbray and Rarity (J. Fluid Mech, 1967)

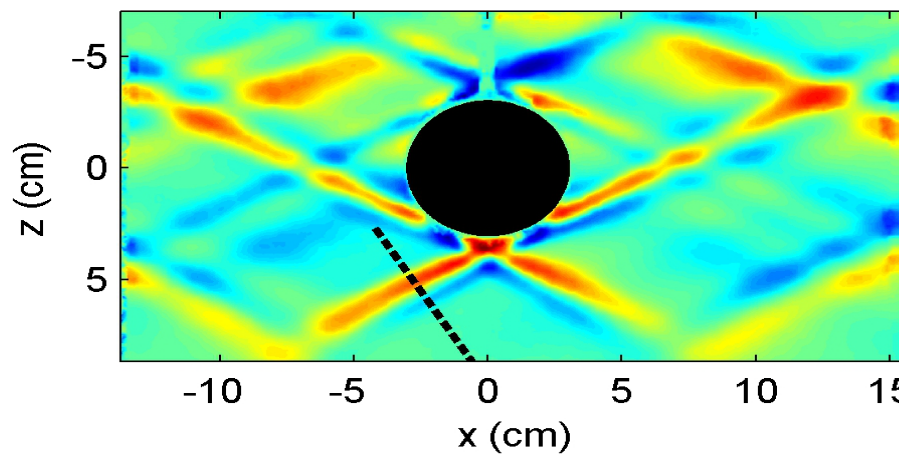


$$\omega^2 = N^2 \sin^2\theta$$

→ $\mathbf{c_g}$ (group velocity) $\perp$ **k**

→ **no length scale selection** from the dispersion relation (only a time scale N^{-1})

Gostiaux, PhD, 2007



Dispersion relation of internal gravity waves

Dispersion of internal gravity waves in a stably-stratified rotating medium

In a stably-stratified rotating medium, $\overline{E}_k \neq \overline{E}_p$, and the simplest way to derive the dispersion relation is to rely on the talk of Jérôme Noir:

In a **purely rotating medium**, the restoring force is the Coriolis force and the dispersion relation is: $\omega^2 = (2\,\Omega)^2 \cos^2\theta$, where Ω is the angular velocity of the rotating container.

On the Earth, what matters is the **projection of the angular velocity of the Earth Ω_E on the local vertical**, denoted f (Coriolis parameter):

$f = 2\,\Omega_E \sin\Phi$ (Φ: latitude) (0 ≤ f ≤ 1.5 10^{-4} rad/s, with f=10^{-4} rad/s at 45° latitude)

In a stably-stratified rotating medium, the dispersion relation is:

$$\boxed{\omega^2 = N^2 \sin^2\theta + f^2 \cos^2\theta}$$

→ f ≤ ω ≤ N

Ocean: N/f ≈ 10 (taking f=10^{-4} rad/s and N=10^{-3} rad/s) (10^{-4} (bottom) ≤ N [rad/s] ≤ 10^{-2} (thermocline))

Atmosphere: N/f ≈ 100 (taking f=10^{-4} rad/s and N=10^{-2} rad/s)

Dispersion relation of internal gravity waves

Dispersion of internal gravity waves in a stably-stratified rotating medium

In a stably-stratified rotating medium, $\overline{E}_k \neq \overline{E}_p$, and the simplest way to derive the dispersion relation is to rely on the talk of Jérôme Noir:

In a **purely rotating medium**, the restoring force is the Coriolis force and the dispersion relation is: $\omega^2 = (2\,\Omega)^2 \cos^2\theta$, where Ω is the angular velocity of the rotating container.

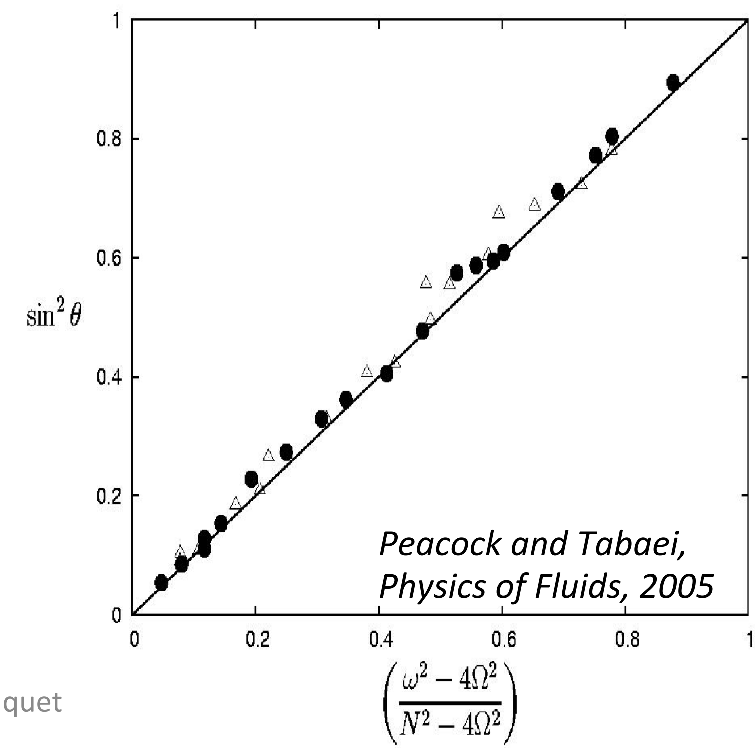
On the Earth, what matters is the **projection of the angular velocity of the Earth Ω_E on the local vertical**, denoted f (Coriolis parameter):

$f = 2\,\Omega \sin\Phi$ (Φ: latitude)

In a stably-stratified rotating medium, the dispersion relation is:

$$\boxed{\omega^2 = N^2 \sin^2\theta + f^2 \cos^2\theta}$$

→ $f \leq \omega \leq N$



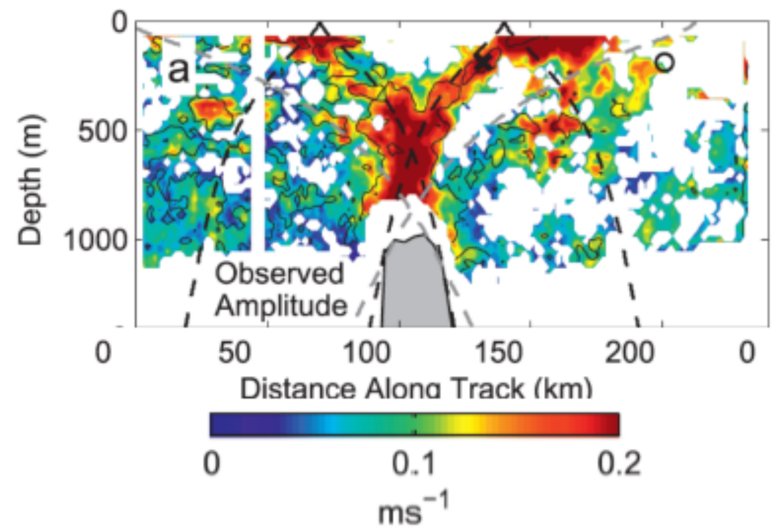
Peacock and Tabaei, Physics of Fluids, 2005

Internal gravity waves in the atmosphere and the ocean

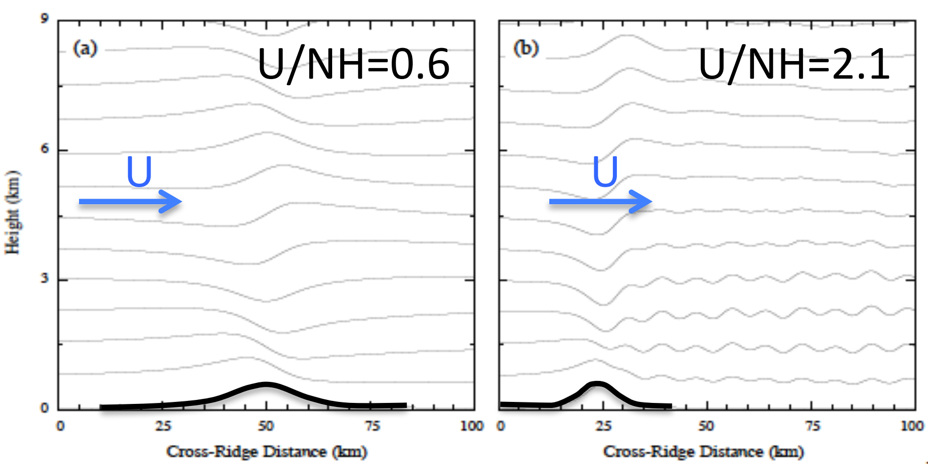
UNIVERSITE Grenoble Alpes

The structure of internal gravity waves depends upon their source

- **Oscillating source**: the waves propagate as beams → this is the case for the internal wave field produced by the tide flowing over topography (the « **internal tide** »).



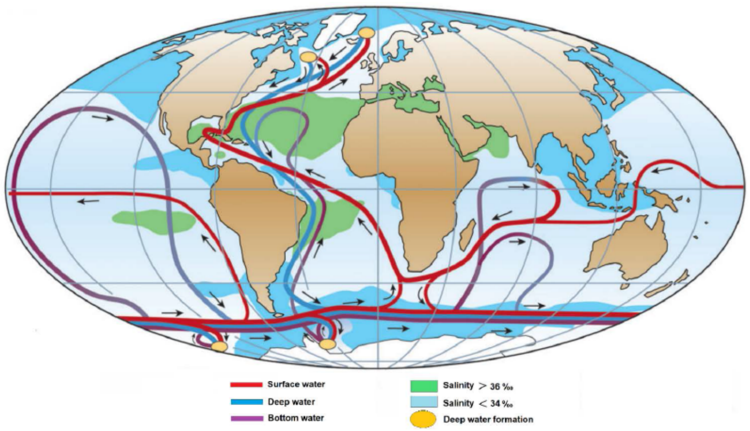
Pickering & Alford (J. Phys. Ocean., 2012)



D. Durran (2003)

* **Steady source** such as a uniform wind blowing over topography : the wave field does not propagate as beams and is steady wrt the topography (« **lee waves** »)

Internal gravity waves in the ocean: motivations

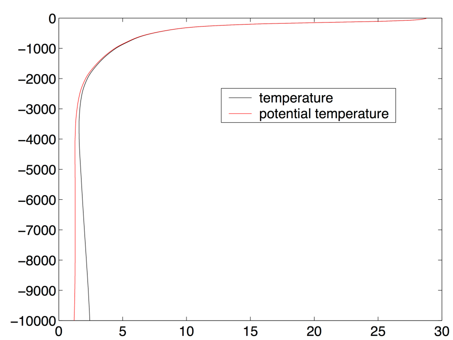


Schematic representation of the **meridional overturning circulation** *(Rahmstorf, Nature, 2002)*

- Cold-surface waters sink down to the bottom of the ocean during the winter hemisphere (North. Hemi.: Nordic sea (Norway/Greenland) and Labrador sea -- South. Hemi.: Ross sea and Weddell sea)

- These cold waters eventuallly raise to the surface (by mass conservation)

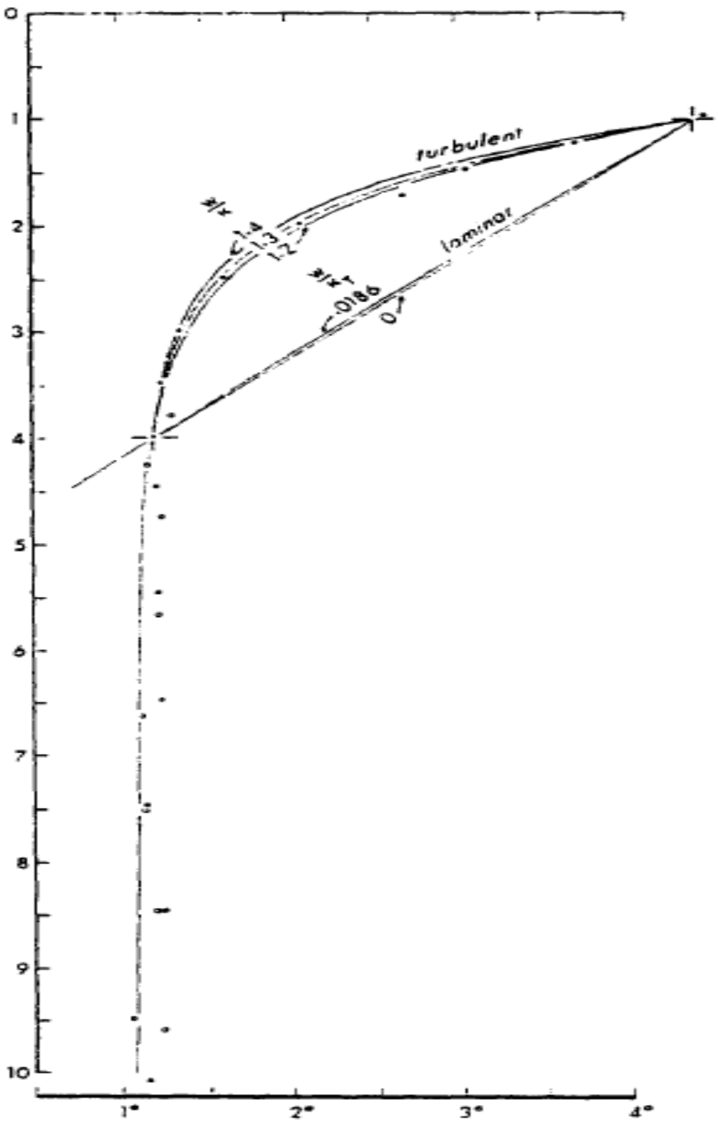
- The global temperature profile in the ocean implies that raising occurs through **mixing processes** *(Munk, J. Phys. Ocean., 1966)*



- The fact that the ocean is in a turbulent state was shown by Munk (1966)

Evidence for the turbulent state of the ocean

UNIVERSITE Grenoble Alpes



Munk (1966)

Munk 1966

Potential temperature as function of depth (km) at two stations in the Central Pacific (Mindanao Deep) recorded in 1930 (●) and 1951 (o).

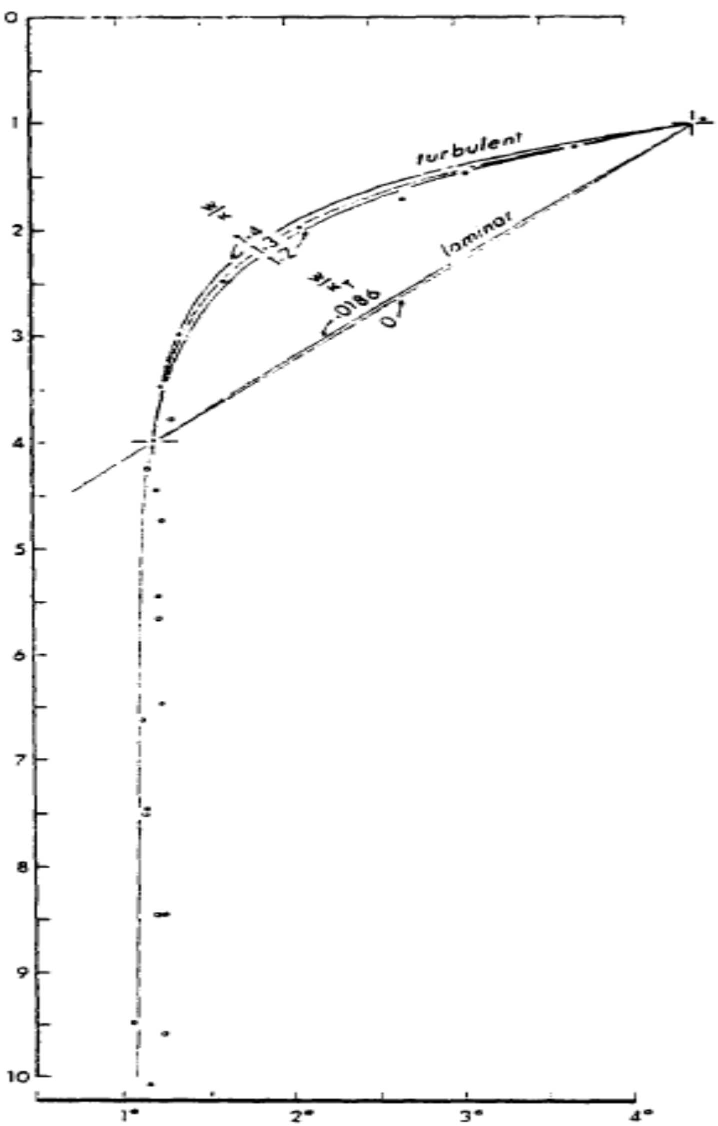
Assume that the temperature profile is a solution of the balance equation: $\underbrace{w\frac{dT}{dz}}_{(1)} - \underbrace{\kappa\frac{d^2T}{dz^2}}_{(2)} = 0$

with (1): upward transport of heat due to slowly rising waters

and (2): downward transport of heat by vertical diffusion

How to determine κ ?

Evidence for the turbulent state of the ocean



Munk (1966)

Assume that the temperature profile is a solution of the balance equation:

$$w\,\frac{dT}{dz} - \kappa\frac{d^2T}{dz^2} = 0$$

How to determine κ ?

1. Fit of the T-profile with the solution of the equation yields $\kappa/w \approx 0.8$ km.

2. Fit of vertical profile over 4000 m of ^{14}C (with decay term) yields $\kappa/w^2 \approx 200$ years
→ $\kappa=10^{-4}$ m²/s and w=1.2 cm/day

3. « Check »: this value of w is consistent with the upward flux of Radium 226 in deep water.

→ the value of **κ** is 1000 times larger than the molecular diffusivity of temperature in water

→ the deep ocean is in a state of turbulent flux.

Note: the laminar solution (with $\kappa = 10^{-7}$ m²/s) yields a linear profile, which is not consistent with the observed profile.

Evidence for the turbulent state of the ocean

Munk (1966) estimate of $\boldsymbol{\kappa_t} = 10^{-4}$ m^2/s was obtained with **a simple model**, assuming **κ is uniform** and fitting data **in the Pacific ocean** below 1 km from the surface.

Is $\kappa_t = 10^{-4}$ m^2/s representative of the turbulent diffusivity in the deep ocean?

→ How to « measure » κ_t in the deep ocean?

A classical model for κ_t, assuming a steady-state has been reached, is *(see the lecture by Sonya Legg)*

$$\kappa_t = \gamma \frac{\epsilon}{N^2}$$

where $\gamma \approx 0.2$ is the mixing efficiency (change in background potential energy due to mixing relative to the amount of energy brought to the system)

ε is the dissipation rate of kinetic energy

N is the buoyancy frequency

→ How to measure ε ?

Obtained using estimate of velocity shear

- from measurement of velocity components at centimeter-scale (microstructure instrument)
- from vertical velocity profiles via « fine-scale parameterization » (scale: ≥ 10 m).

Evidence for the turbulent state of the ocean

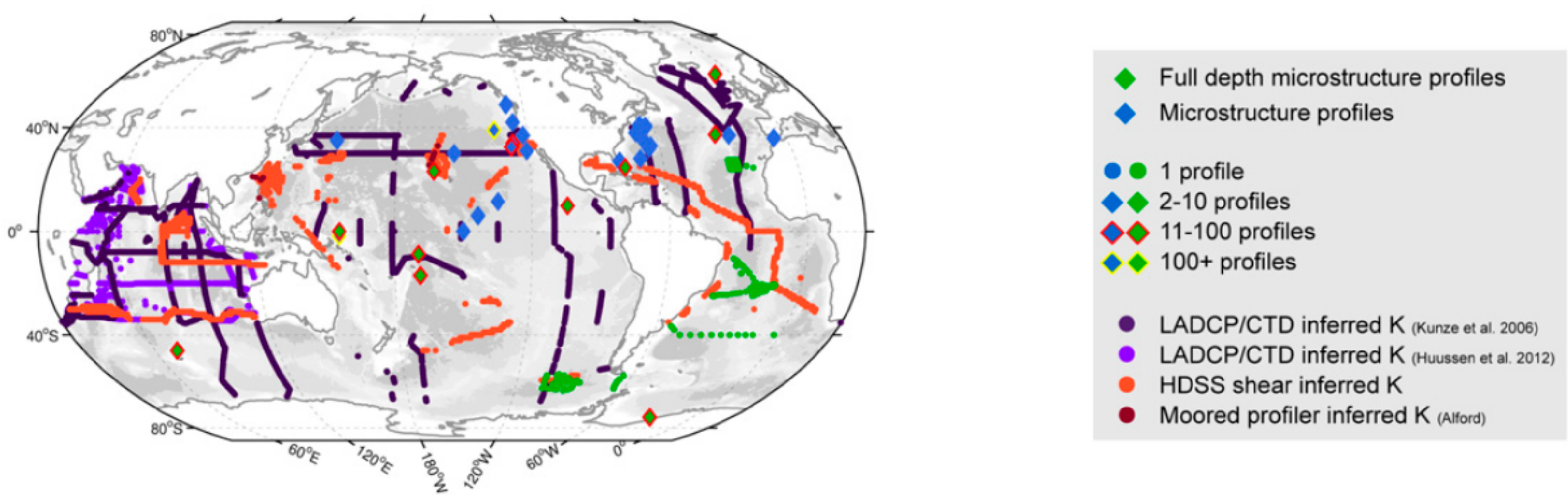
UNIVERSITE Grenoble Alpes

Global Patterns of Diapycnal Mixing from Measurements of the Turbulent Dissipation Rate

AMY F. WATERHOUSE,[*] JENNIFER A. MACKINNON,[*] JONATHAN D. NASH,[+] MATTHEW H. ALFORD,[#] ERIC KUNZE,[#] HARPER L. SIMMONS,[@] KURT L. POLZIN,[&] LOUIS C. ST. LAURENT,[&] OLIVER M. SUN,[&] ROBERT PINKEL,[*] LYNNE D. TALLEY,[*] CAITLIN B. WHALEN,[*] TYCHO N. HUUSSEN,[*] GLENN S. CARTER,[**] ILKER FER,[++] STEPHANIE WATERMAN,[##,@@] ALBERTO C. NAVEIRA GARABATO,[&&] THOMAS B. SANFORD,[#] AND CRAIG M. LEE[#]

[*] *Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California*
[+] *College of Oceanic and Atmospheric Sciences, Oregon State University, Corvallis, Oregon*
[#] *Applied Physics Laboratory and School of Oceanography, University of Washington, Seattle, Washington*
[@] *University of Alaska Fairbanks, Fairbanks, Alaska*
[&] *Department of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts*
[**] *Department of Oceanography, University of Hawai'i at Mānoa, Honolulu, Hawaii*
[++] *Geophysical Institute, University of Bergen, Bergen, Norway*
[##] *Department of Earth, Ocean and Atmospheric Sciences, University of British Columbia, Vancouver, British Columbia*
[&&] *National Oceanography Centre, University of Southampton, Southampton, United Kingdom*

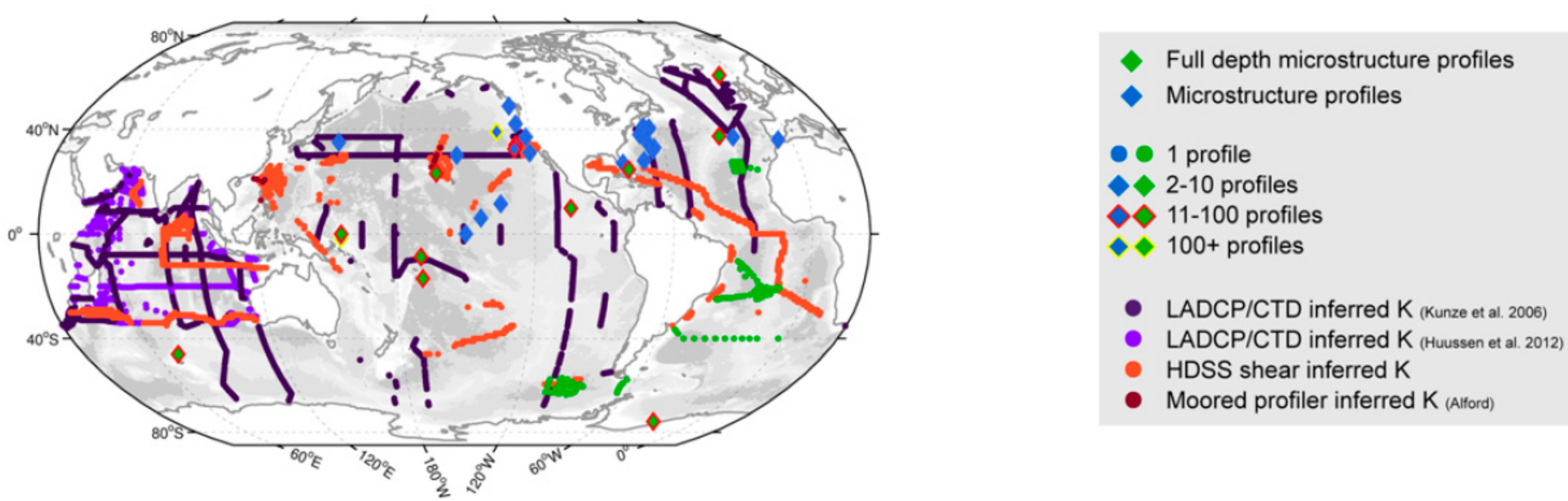
J. Phys. Ocean., 2014



Evidence for the turbulent state of the ocean

Waterhouse et al (2014): used over 5200 vertical profiles of **microscale shear** (scale : 1 cm) and **vertical profiles of velocity** (resolution: ≥ 10 m) to estimate the turbulent dissipation rate ε and the diffusivity K.

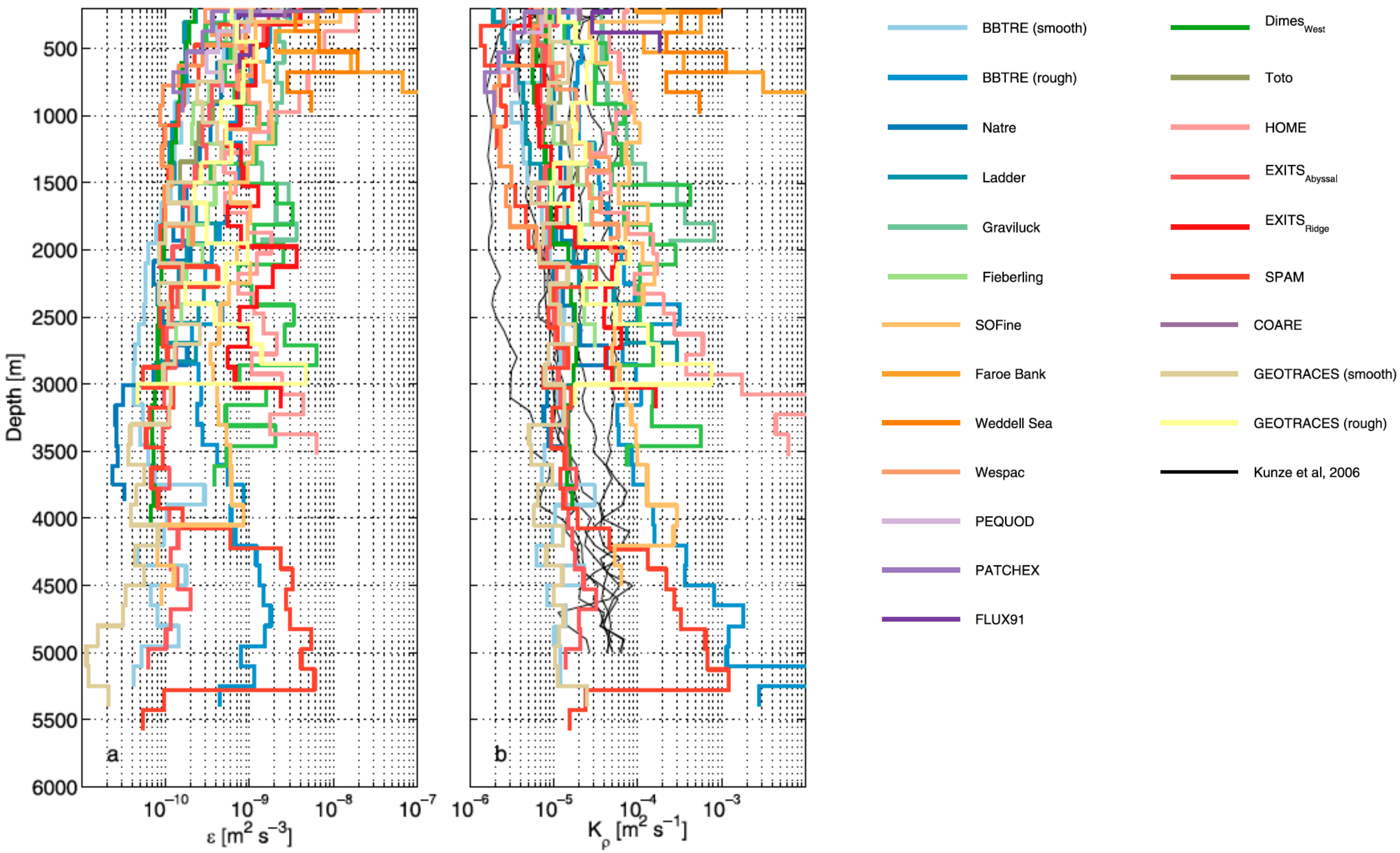
-**microscale shear**: provides « direct » measurements of ε
-**vertical profiles of velocity**: indirect estimates of ε inferred from finescale parameterization



Evidence for the turbulent state of the ocean

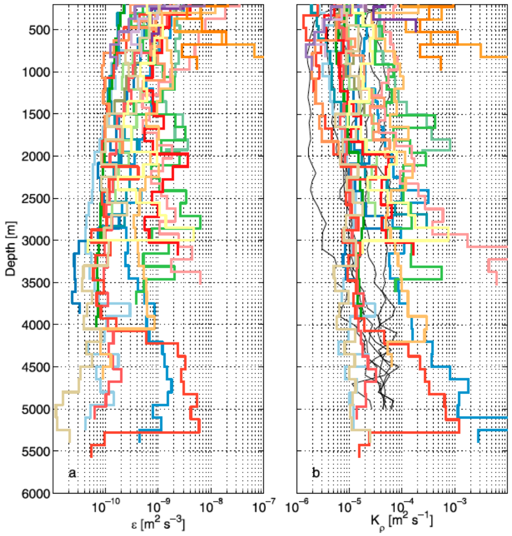
UNIVERSITE Grenoble Alpes

Vertical profiles of ε and κ_t (Waterhouse et al 2014)



Evidence for the turbulent state of the ocean

UNIVERSITE Grenoble Alpes



Waterhouse et al (2014)

	Depth range	K (m^2 s^{-1})	Projects included in average
Full depth	From MLD to bottom	$3.3\ (0.2–8.6) \times 10^{-4}$	WESPAC, GEOTRACES (smooth and rough), Fieberling, NATRE, BBTRE (smooth and rough), HOME, GRAVILUCK (above 2000 m), LADDER, TOTO, SOFine, DIMES (DP and West), EXITS (ridge and abyssal), and MIXET
Upper ocean	From MLD to 1000-m depth	$0.3\ (0.2–0.4) \times 10^{-4}$	PEQUOD, PATCHEX, FLUX91, COARE, WESPAC, GEOTRACES (smooth and rough), Fieberling, NATRE, BBTRE (smooth and rough), HOME, GRAVILUCK (above 2000 m), LADDER, TOTO, SOFine, DIMES (DP and West), EXITS (ridge and abyssal), and MIXET
Deeper ocean	From 1000-m depth to bottom	$4.3\ (0.4–11.5) \times 10^{-4}$	WESPAC, GEOTRACES (smooth and rough), Fieberling, NATRE, BBTRE (smooth and rough), HOME, GRAVILUCK (above 2000 m), LADDER, TOTO, SOFine, DIMES (DP and West), EXITS (ridge and abyssal), and MIXET

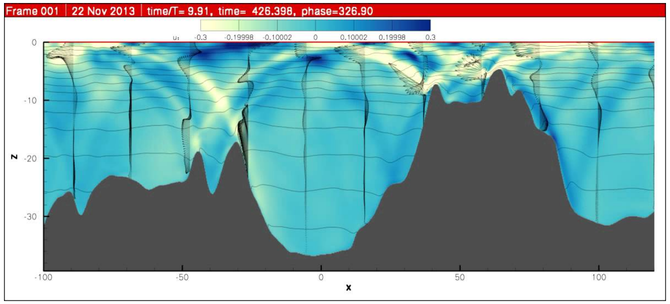
Conclusion: the global-averaged diapycnal diffusivity

- above 1000-m depth is O(10^{-5}) m^2 s^{-1} and
- **below 1000 m is O(10^{-4}) m^2 s^{-1}**

… in agreement with Munk (1966) estimate.

What are the mixing processes in the deep ocean?

Munk proposed several mechanisms to account for mixing in the deep ocean, among them being **the shear associated with the internal tide** ***(see Sonya Legg's lecture)***

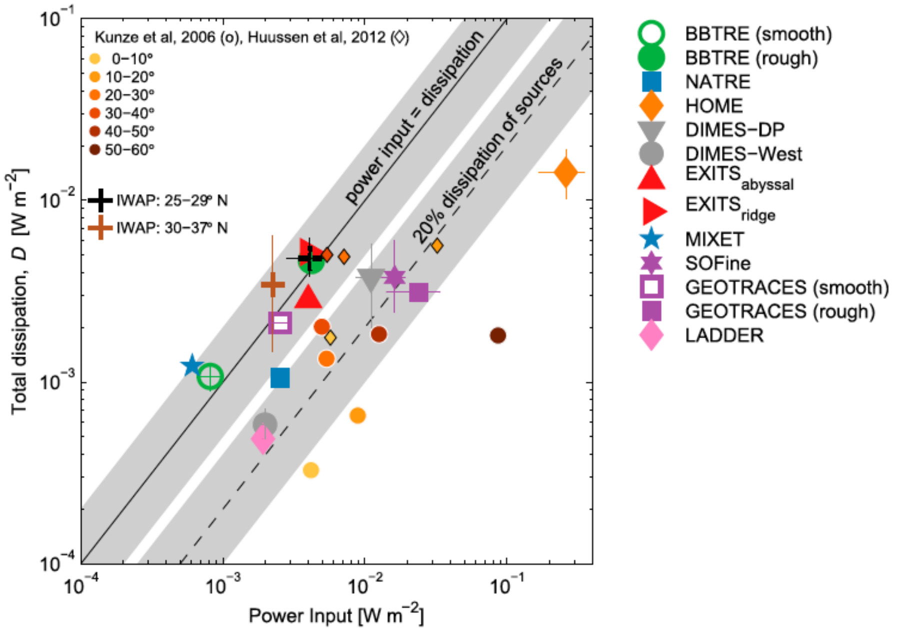


Waterhouse et al (2014)**: the depth-integrated ε shows spatial variability related to internal wave generation**

→ Waterhouse et al (2014) investigated the link between **local energy dissipation** and the **flux of energy into the local internal wave field**.

Conclusion:

- at most locations, **total power lost through turbulent dissipation < input into internal wave field**.
- **Mixing in the deep ocean can be attributed to internal gravity waves**, with 70% of the total power input being dissipated in the interior (and the remainder at the boundaries).



What are the sources of internal waves in the ocean?

UNIVERSITE Grenoble Alpes

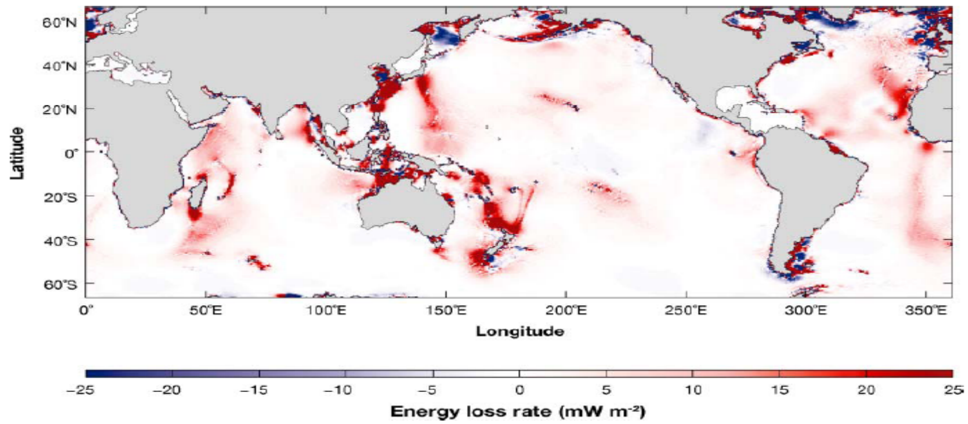
Munk and Wunsch (J. Phys. Ocean., 1998): the tide and the wind are the only possible sources of mechanical energy to drive the interior mixing

… via the radiation of an internal wave field

1. Tide

The barotropic tide flowing over topography radiates the «internal tide»

25-30% of the barotropic tidal energy is lost to internal waves, mainly at mid-ocean ridges and continental slopes (*Garrett & Kunze, Ann. Rev. Fluid Mech. 2007*).



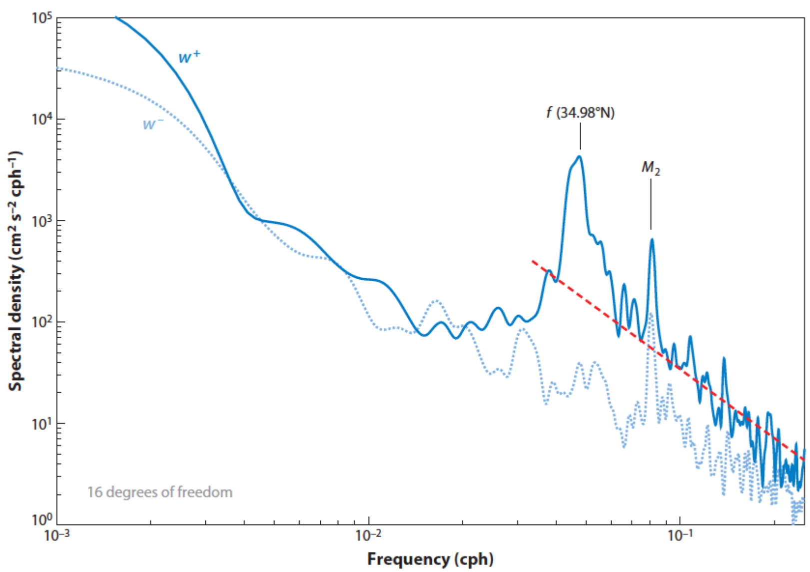
Estimate of tidal energy dissipation from TOPEX-Poseidon altimeter data (*Egbert & Ray, J. Geophys. Res., 2001*)

What are the sources of internal waves in the ocean?

Munk and Wunsch (1998): the tide and the wind are the only possible sources of mechanical energy to drive the interior mixing … via the radiation of an internal wave field

2. Wind

Because of the Coriolis force in the equations of motions, any forcing whose Fourier spectrum involves the Coriolis frequency will resonantly excite inertial oscillations in the surface layer. This is the case for the wind.



Alford et al, Ann. Rev. Fluid Mech., 2016

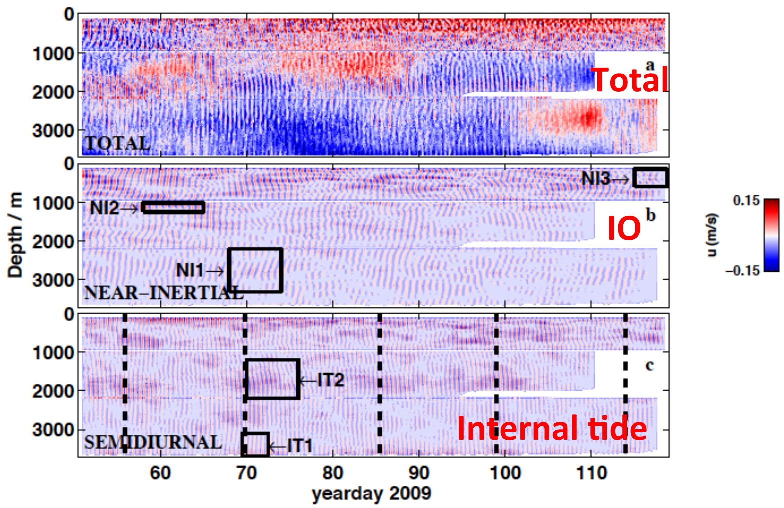
Velocity spectrum at 261-m depth from current-meter data.
Blue line: clockwise motion associated with downward energy propagation (namely, inertial oscillations).
Dashed blue line: counterclockwise motions.

What are the sources of internal waves in the ocean?

UNIVERSITE Grenoble Alpes

2. Wind (cont'd)

These inertial oscillations (IOs) can propagate in the deep ocean



Bandpassing used to separate near-inertial and semi-diurnal signals (Alford M., private comm., 2010)

Mechanism of deep propagation:

(*Danioux et al, J. Phys. Ocean (JPO), 2007; Danioux and Klein (JPO), 2008; Danioux et al (JPO), 2011*)

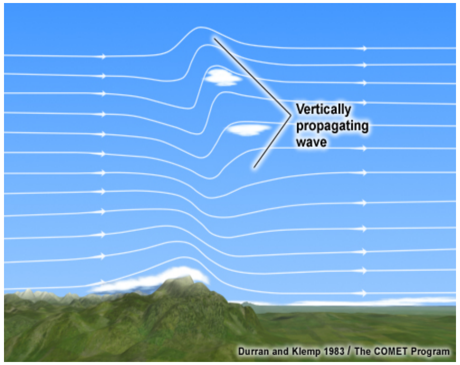
- Inertial oscillations (freq. f) in a field of mesoscale vortices (vort. ζ): local relative wave frequency is f+ζ → IOs escape in a cyclonic vortex (ζ>0), are trapped in an anticyclonic vortex (Kunze 1985)
- IOs are refracted in an anticyclonic vortex → larger (downward) vertical group velocity → propagating into the deep ocean
- Resonant interaction of IOs with the anticyclonic vortex → generation of waves with maximum wave-induced energy at 2500 m and frequency 2f

What are the sources of internal waves in the ocean?

UNIVERSITE Grenoble Alpes

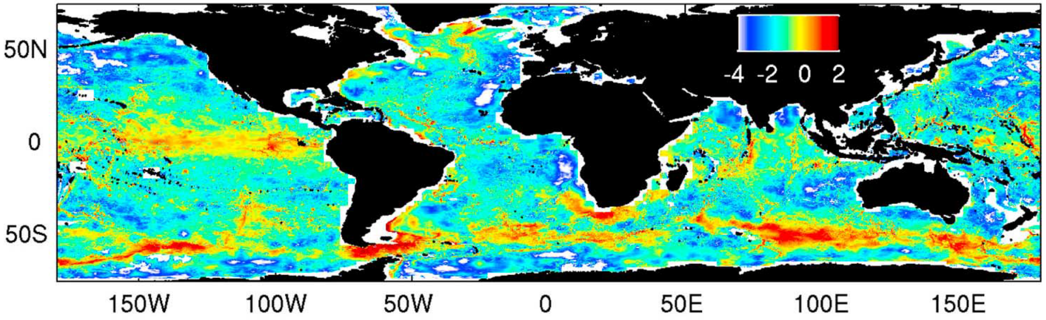
3. Interaction of geostrophic motions with rough bottom topography in the Southern Ocean

→ Generation of **lee waves**, as does the wind blowing over a mountain range



D. Durran 2002

Estimate of energy flux into lee waves (Nikurashin & Ferrari, Geophys. Res. Lett. 2011): 50 % of the flux of energy in the lee wave field is in the Southern Ocean)



Relative importance of the internal wave fields

Supply of internal wave energy into the ocean (Watherhouse et al, 2014):

Power transferred from the barotropic tide to **the internal tide**: 0.7-1.3 TW

Power transferred from the wind to **inertial oscillations**: 0.3-1.5 TW

Power transferred from geostrophic motions to **lee waves**: 0.2 TW*

→ **Internal waves generated by the tide and the wind** are supposed to be the main contributors to mixing in the ocean

* There is actually a paper by Wright et al (2014) which estimates a higher global conversion rate into the lee wave field, of **0.75 ± 0.19 TW**. It is based on *in situ* data for the bottom currents and also shows a prominent role of the Southern Ocean.

Breaking processes of large amplitude internal waves

In the ocean, rotation effets should be taken into account to describe wave breaking.

We start with the simplest case: a monochromatic plane wave propagating in a medium at rest.

Without rotation, N constant:

Breaking occurs via convective instability, in the plane normal to the propagation plane.

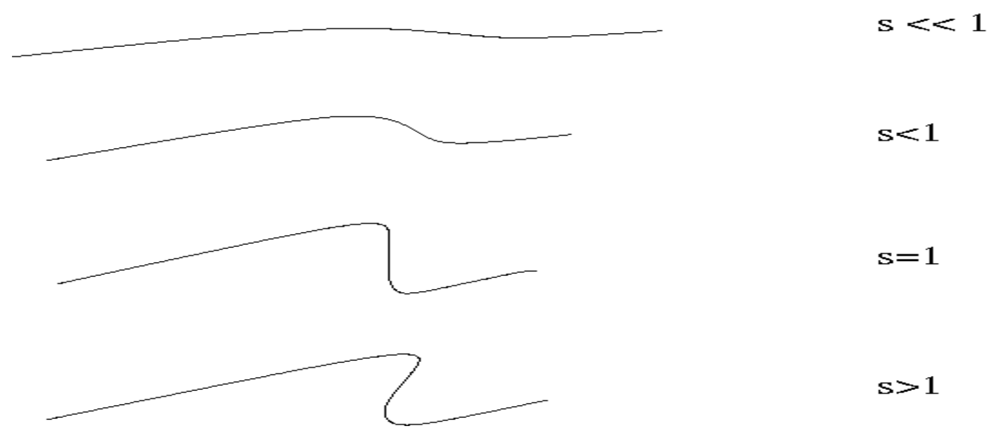
This instability occurs when the wave steepness (s) >1.

The normalized amplitude or steepness of the wave is defined as:

$$s = u_{max} / c_x$$

where c_x is the phase velocity in the *x*-direction.

Propagation plane of the wave



Breaking processes of large amplitude internal waves

In the ocean, rotation effets should be taken into account to describe wave breaking.

We start with the simplest case: a monochromatic plane wave propagating in a medium at rest

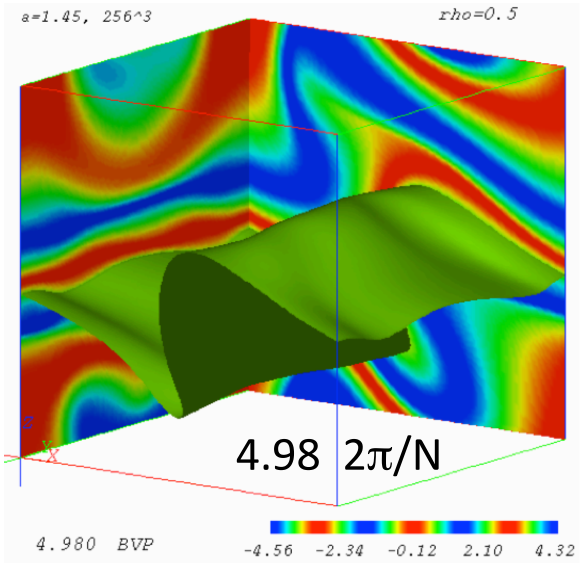
Without rotation, N constant:

Breaking occurs via convective instability, in the plane normal to the propagation plane.

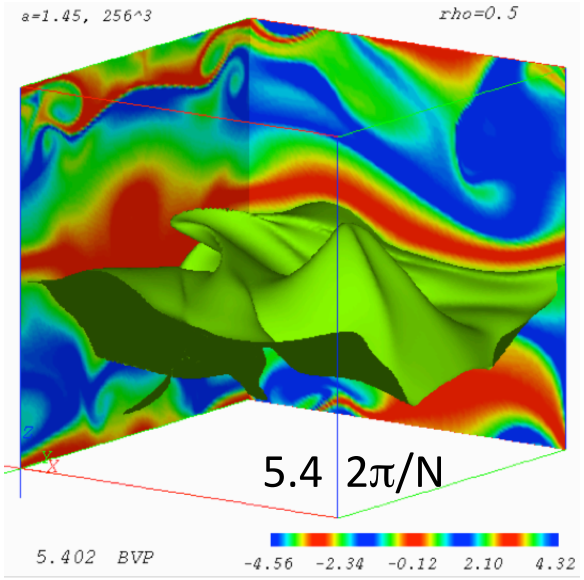
This instability occurs when the wave steepness (s) >1.

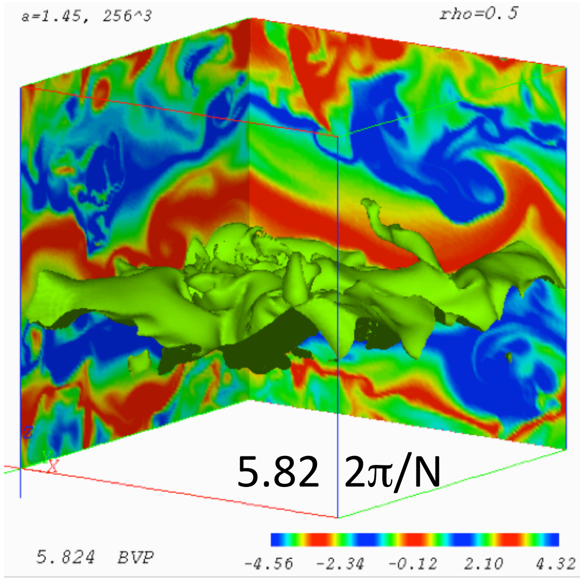
Illustration: PhD work of C. Koudella (1999)

Breaking of a plane monochromatic internal gravity wave with s=1.45
No rotation (Koudella, PhD, 1999)



Surface of constant density and
(two) planes of density contours





Breaking processes of large amplitude internal waves

In the ocean, rotation effets should be taken into account to describe wave breaking.

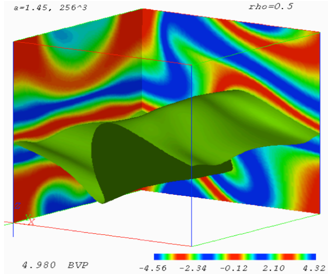
We start with the simplest case: a monochromatic plane wave propagating in a medium at rest

Without rotation, N constant:

Breaking occurs via convective instability, in the plane normal to the propagation plane.

This instability occurs when s (wave steepness) >1.

Illustration: PhD work of C. Koudella (1999)



With rotation, f/N constant:

from the dispersion relation: $f < \omega < N$

- If $f \ll \omega < N$: non-rotating case recovered (see above) → convective instability
- If $f < \omega \ll N$: shear instability dominates

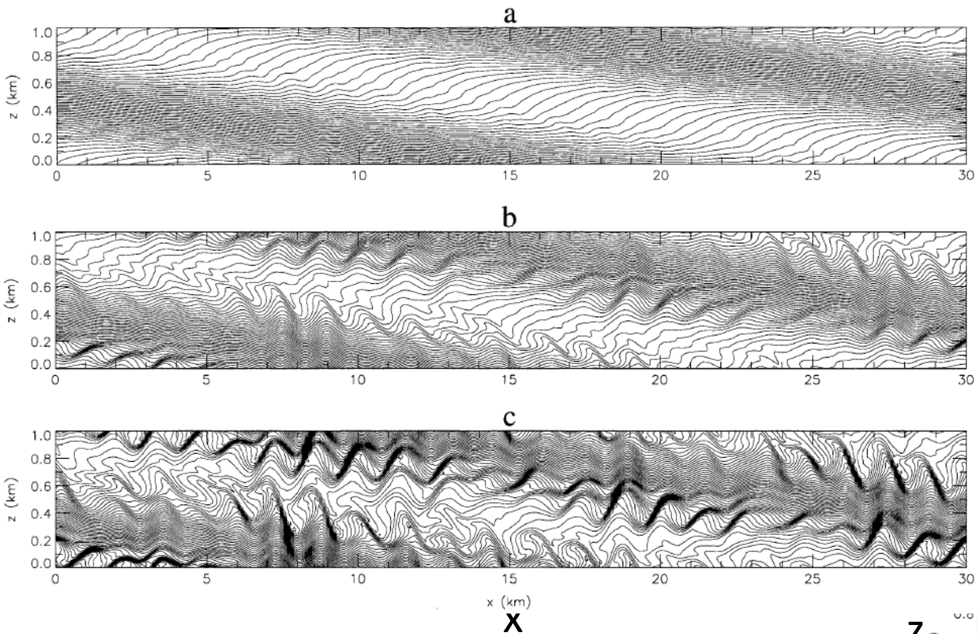
Reminder: necessary condition for instability (proved for steady parallel shear flow): Ri<1/4 somewhere in the flow (Miles and Howard, J. Fluid Mech., 1964), with the Richardson number Ri defined by: $Ri = N^2/(dU/dz)^2$

Illustration: Lelong and Dunkerton (J. Atmos. Sci., 1998)

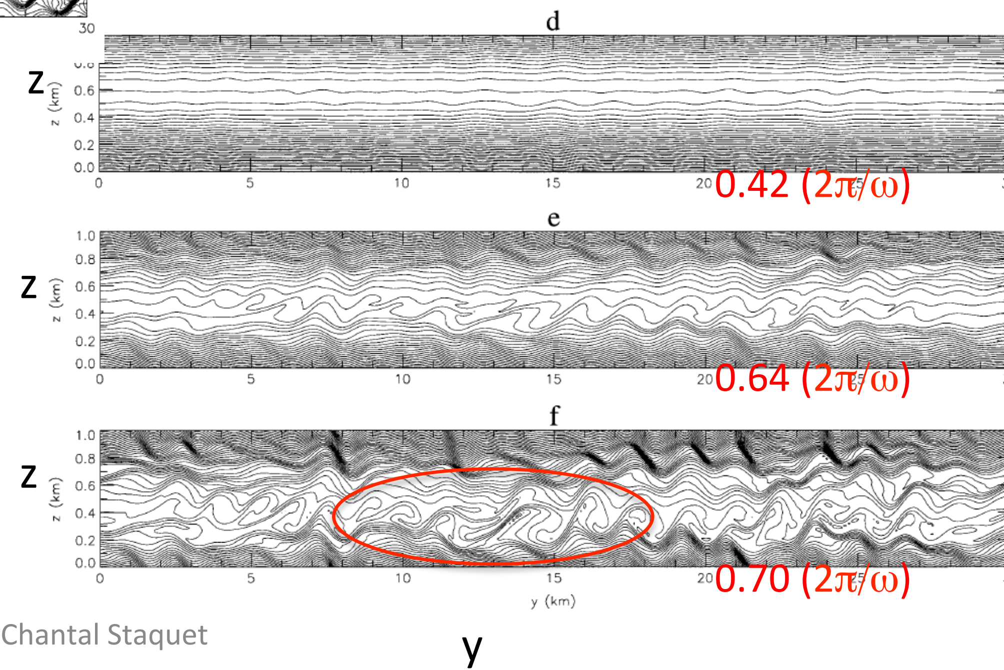
Breaking of a plane monochromatic internal gravity wave with s=0.7
Near-inertial wave: ω/f=1.05 (Lelong and Dunkerton, 1998)

Initial condition: monochromatic wave propagating in the (x,z) plane
(all fields of the form $A\, e^{i(k_x\, x + k_z\, z - \omega\, t)}$)
+ 3D white noise

(x,z) vertical plane



(y,z) vertical plane



Breaking processes of large amplitude internal waves

UNIVERSITE Grenoble Alpes

In the ocean, rotation effets should be taken into account to describe wave breaking.

We start with the simplest case: a monochromatic plane wave propagating in a medium at rest

Without rotation, N constant:

Breaking occurs via **convective instability**, in the plane normal to the propagation plane.
This instability occurs when the wave steepness (s) >1.

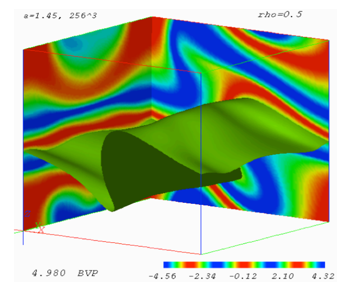


Illustration: PhD work of C. Koudella (1999)

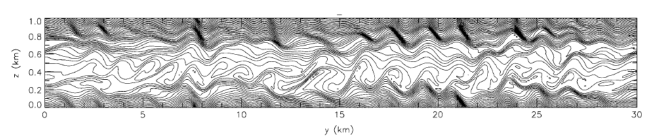
With rotation, f/N constant:

from the dispersion relation: $f<\omega<N$

- If $f << \omega < N$: non-rotating case recovered (see above) → convective instability
- If $f < \omega << N$: shear instability dominates

Reminder: necessary condition for instability (proved for steady parallel shear flow): Ri<1/4 somewhere in the flow (Miles 1964), with the Richardson number Ri defined by: $Ri = N^2/(dU/dz)^2$

Illustration: Lelong and Dunkerton (1998)



Very simplified view. See Lelong & Dunkerton (1998) for stability analysis and numerical simulation of a large amplitude plane wave in a stratified rotating fluid.

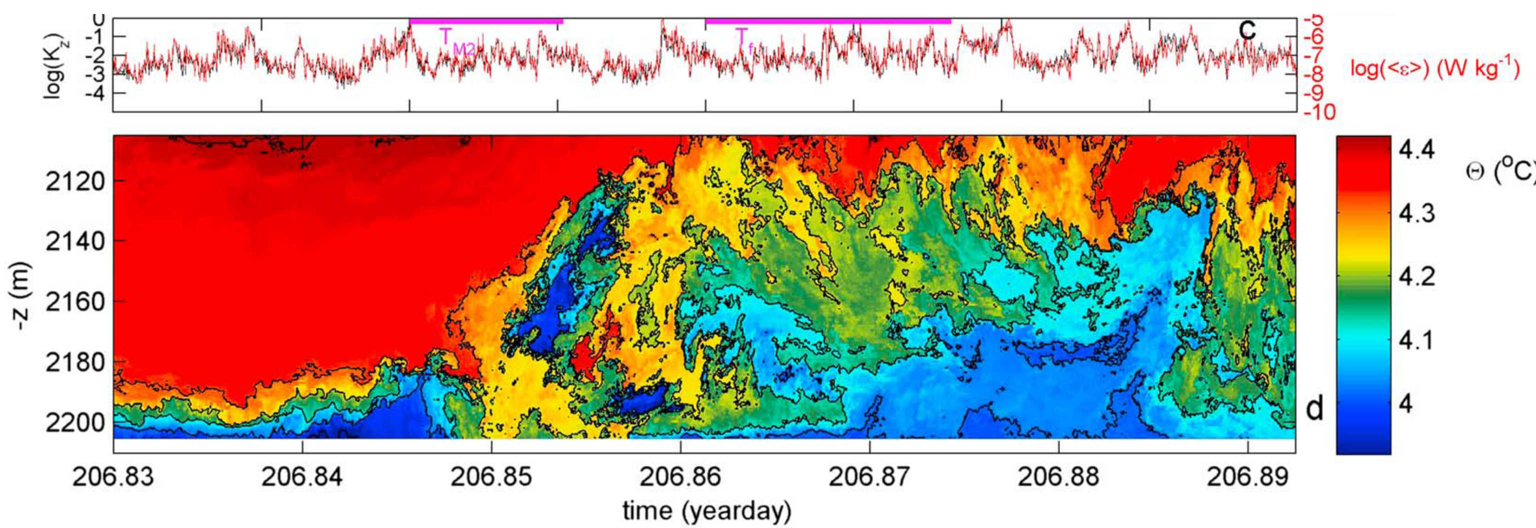
Breaking processes of internal waves in the ocean

In the **atmosphere**, the **wave amplitude increases** as the wave propagates upward because the density decreases.

No such process in the ocean → how can the wave amplitude increase locally?

Currents are mainly located in the upper part of the ocean → **no interaction with currents** can be invoked as an ubiquitous process to make wave amplitude increase (as opposed to interaction with winds in the atmosphere).

→ One should look for a wave amplification process **in a medium at rest** (possibly involving the topography).



Climatoribus, van Haren and Gostiaux, Geophys. Res. Lett., 2015

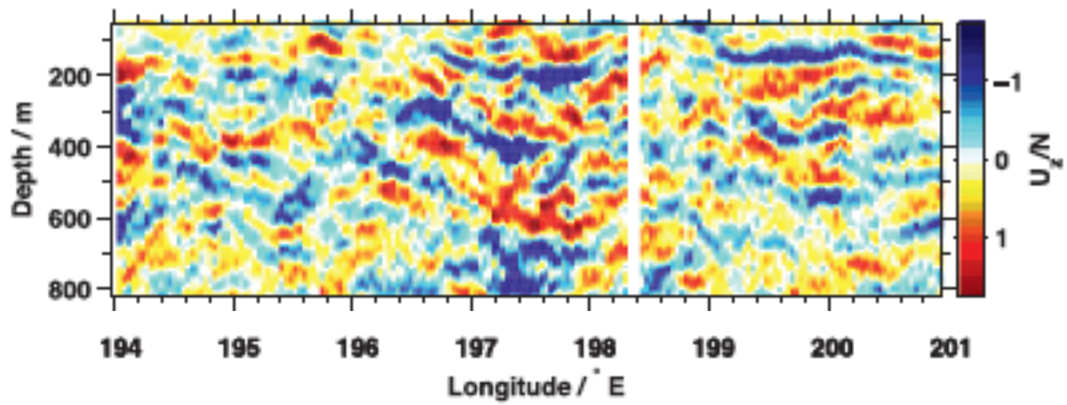
Breaking processes of internal waves in the ocean

→ One should look for a wave amplification process **in a medium at rest** (possibly involving the topography).

- **For near-inertial wave**, Kelvin-Helmholtz instability could possibly occur if $Ri = N^2/U_z^2 < 1/4$ locally (namely $U_z/N > 2$). Is it the case ?

Field campaign in Pacific Ocean (MacKinnon et al, J. Phys. Ocean., 2013):

Vertical shear U_z scaled by N, with U_z and N <u>averaged</u> over the depth [50 m, 950 m] from the surface - Latitude: 28.8°



→ Shear instability likely to occur locally, at least at this latitude.

- **And for f << ω <N ?**
  Parametric sub-harmonic instability (PSI) can be such a process.
  → **what is PSI?**

UNIVERSITE Grenoble Alpes

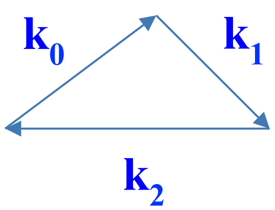
Preambule: resonant interactions among a wave triad

We consider a wave triad $[(\mathbf{k}_0,\omega_0), (\mathbf{k}_1,\omega_1), (\mathbf{k}_2,\omega_2)]$, namely ω_i and $\mathbf{k}_i$ satisfy the dispersion relation. The wave amplitudes are supposed to be infinitesimal.

The triad is said to be **resonant** if

$\mathbf{k}_0 + \mathbf{k}_1 + \mathbf{k}_2 = 0$ (nonlinear interactions involve triads of wave vectors)

$\omega_0 + \omega_1 + \omega_2 = 0$ (**temporal resonance relation**)



When both relations are satisfied, the energy flux among the waves of the triad is increased, on a slow time scale proportional to the inverse of the wave amplitude

Ref.: Phillips O.M. The dynamics of the upper ocean, CUP, 1977 (2nd edition).

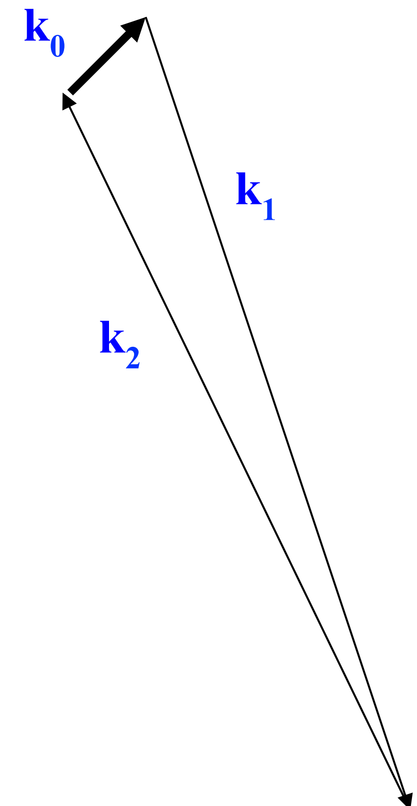
Breaking processes of internal waves in the ocean: PSI

UNIVERSITE Grenoble Alpes

We consider a **resonant** internal wave triad $[(\mathbf{k}_0,\omega_0), (\mathbf{k}_1,\omega_1), (\mathbf{k}_2,\omega_2)]$:

$\mathbf{k}_0 + \mathbf{k}_1 + \mathbf{k}_2 = 0$ (spatial resonance relation)

$\omega_0 + \omega_1 + \omega_2 = 0$ (temporal resonance relation)



When two waves of the triad, say $\mathbf{k}_1$ and $\mathbf{k}_2$, are of much smaller length scale than the third wave, $\mathbf{k}_0$: $|\mathbf{k}_1|, |\mathbf{k}_2| >> |\mathbf{k}_0|$,
the ($\mathbf{k}_0,\omega_0$) wave is unstable if it has the highest frequency in the triad (Hasselmann 1970).

In this case, $|\omega_1| \approx |\omega_2| \Rightarrow$ from the temporal resonance relation:
$|\omega_1| \approx |\omega_2| \approx |\omega_0|/2$, namely **the instability is of the parametric subharmonic type** (PSI).

It can be shown that **a monochromatic internal gravity wave of infinitesimal amplitude is unstable to PSI, whatever the stratification level of the fluid** (Drazin 1977).

This instability is associated with a **direct energy transfer** to small scale waves (no cascade process). When the steepness of these small-scale waves > 1, breaking occurs through convective instability.

How does parameteric subharmonic instability manifest itself ?

Ref.:
- Sommeria and Staquet, 2002, Ann. Rev. Fluid Mech.
-Koudella and Staquet, 2006, J. Fluid Mech.

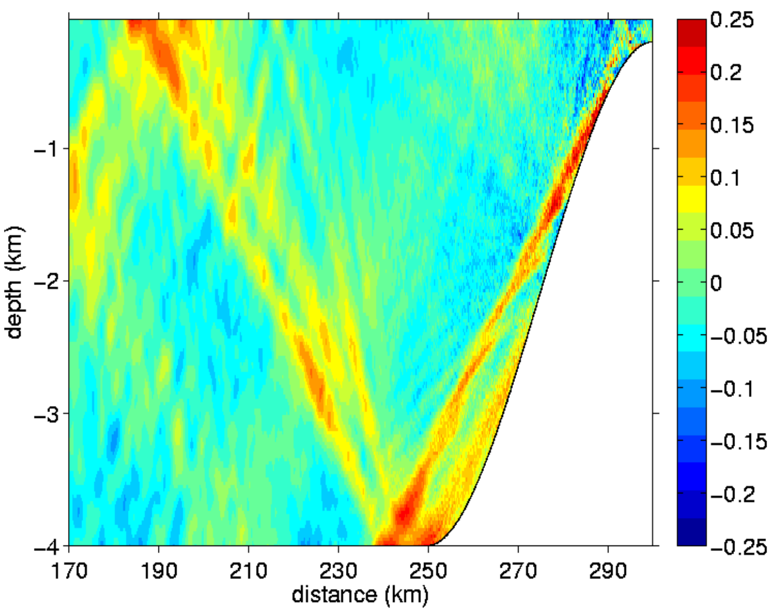
Breaking processes of internal waves in the ocean: PSI

UNIVERSITE Grenoble Alpes

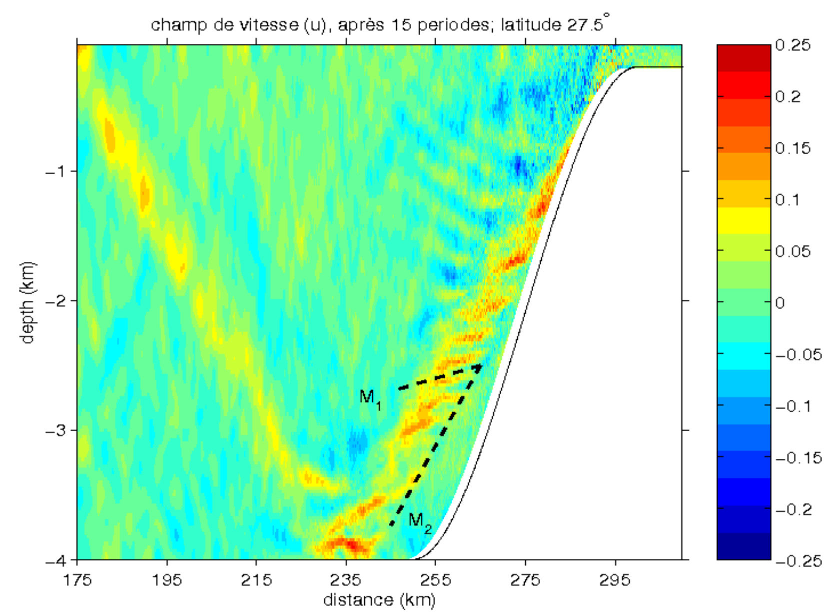
Numerical simulation in a vertical plane of internal tide generation:
a « tidal forcing » is continously applied on the left boundary, at frequency « M_2 »
(with consistent oscillating mass flux on the right boundary).
No initial velocity, N constant.

→ Generation of internal gravity waves, with a beam structure, tangent to the continental shelf.

Total horizontal velocity component, after 7.5 days



f = 0
(latitude ϕ=0° → « equator »)



f = 6.7 10^{-5}
(latitude ϕ=27.3°)

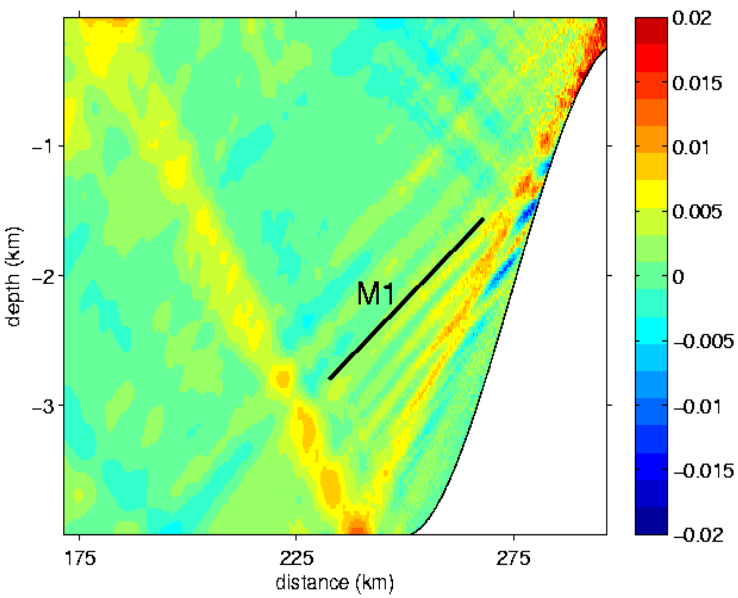
Breaking processes of internal waves in the ocean: PSI

UNIVERSITE Grenoble Alpes

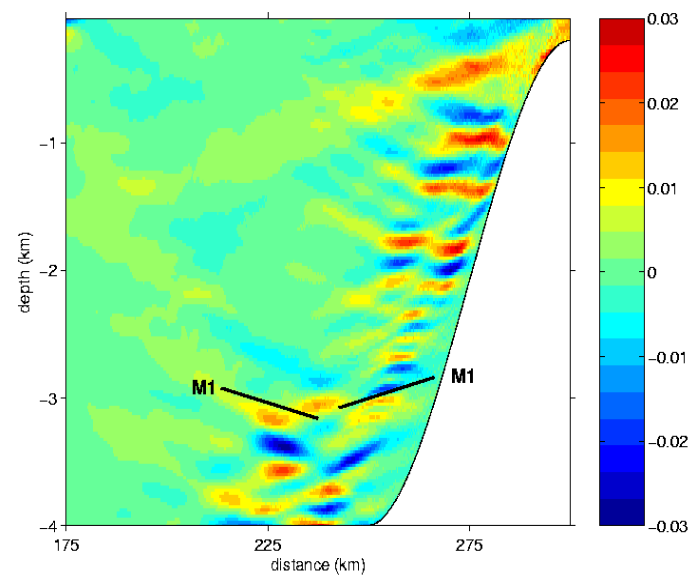
The perturbation excited by PSI has frequency equal to $M_2/2$ (denoted M_1).
For PSI to occur, one should have $f \leq M_2/2 \leq N \Rightarrow$ PSI occurs at latitudes f such that $\boxed{f \geq f_c = M_2/2}$.

The latitude at which $\mathbf{f_c = M_2/2}$ is the « critical latitude » (=28.8°).

Horizontal velocity component filtered
at half the forcing frequency $M_1=M_2/2$



f = 0
(latitude ϕ=0 « equator »)

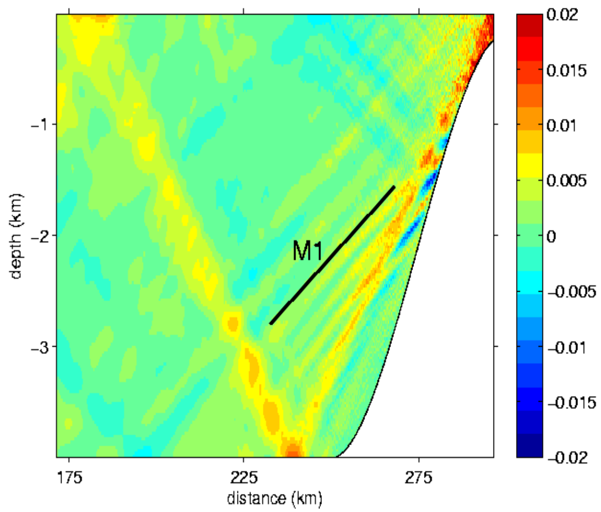


f = 6.7 10^{-5} rad/s
(latitude ϕ=27.3°)

Breaking processes of internal waves in the ocean: PSI

The perturbation excited by PSI has frequency equal to $M_2/2$ (denoted M_1).
For PSI to occur, one should have $f \leq M_2/2 \leq N \Rightarrow$ PSI occurs at latitudes f such that $\boxed{f \geq f_c = M_2/2}$.

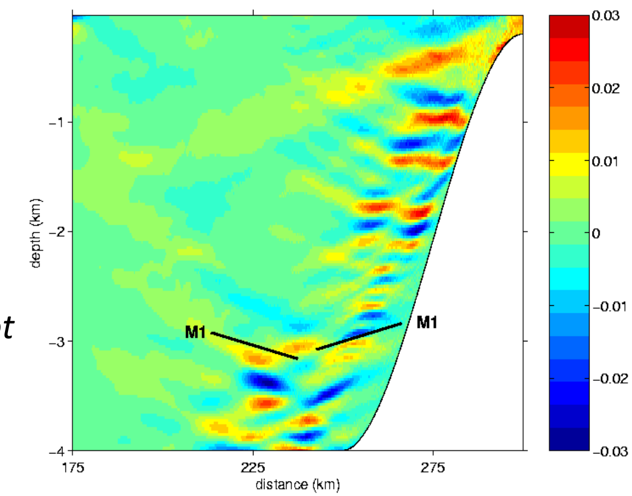
The latitude at which $\mathbf{f_c = M_2/2}$ is the « critical latitude » (=28.8°).



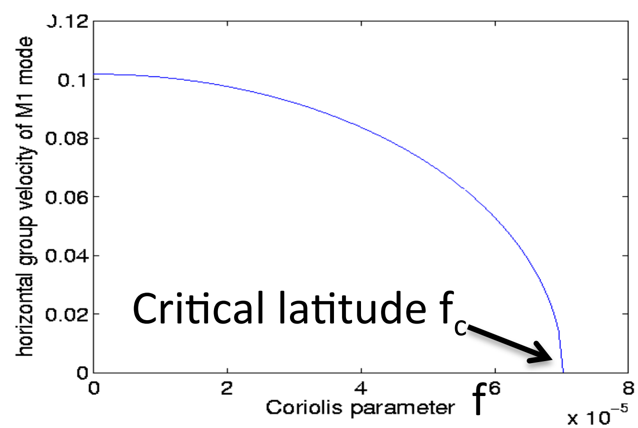
f = 0
(latitude ϕ=0 « equator »)

Horizontal velocity component filtered at half the forcing frequency $M_1=M_2/2$

Gerkema, Staquet, Bouruet-Aubertot Geophys. Res. Lett. 2006



f = 6.7 10^{-5} rad/s
(latitude f=27.3°)



Horizontal component of group velocity of the M_1 wave versus Coriolis parameter
(for k_z=1/300; N=0.002 rad/s)

Breaking processes of internal waves in the ocean: PSI

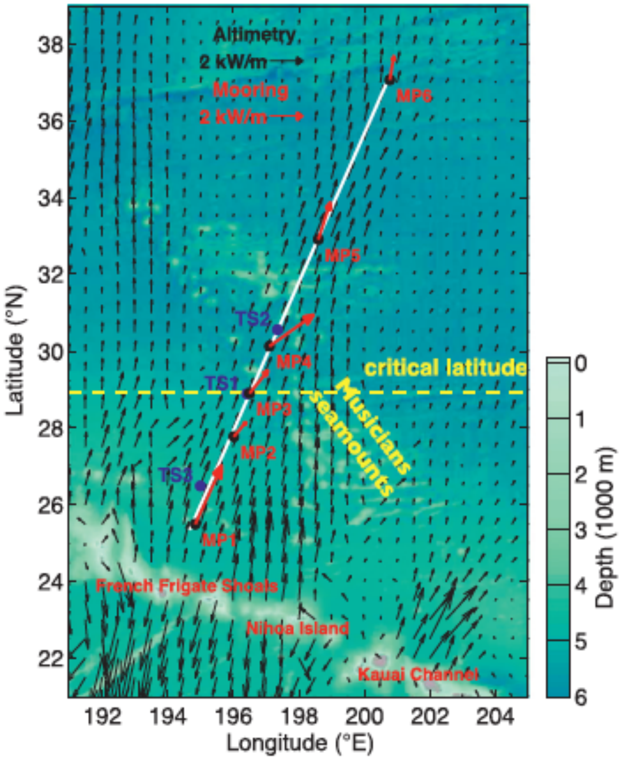
Does PSI occur in the ocean?

How to track it?

What does happen at the critical latitude? Enhanced mixing?

Breaking processes of internal waves in the ocean: PSI

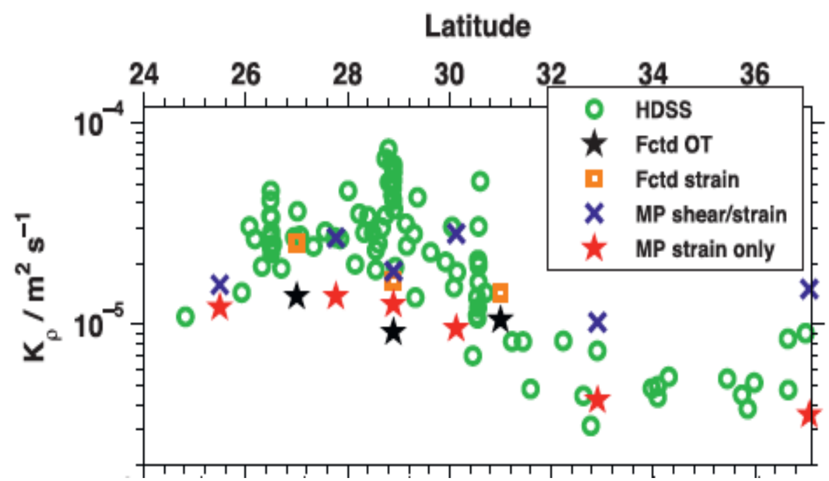
UNIVERSITE Grenoble Alpes



MacKinnon et al, J. Phys. Ocean., 2013

IWAP (Internal Waves Across the Pacific) field campaign. Measurements from 21° to 39°N (about 2000 km) → across the critical latitude.

Question: does the internal tide lose « significant » energy near the critical latitude?



Depth-averaged diffusivity as a function of latitude for several instruments and methods

Conclusions

- Essentiel role of internal gravity waves in the deep ocean: the global temperature and salinity profiles result from mixing processes mainly contributed by inertial waves.

- Without mixing, the ocean would turn, within a few thousand years, into a stagnant pool of cold salty water with near-surface mixing (Munk & Wunsch 1998) → **internal waves maintain the meridional overturning circulation.**

- Mixing by internal gravity waves need to be parametrized in an appropriate manner in large-scale circulation models (already shown by Bryan, J. Phys. Ocean. 1987) …
and in climate models as well.