Excitation of Internal Gravity Waves by Convection

Daniel Lecoanet
Princeton Center for Theoretical Science

L.-A. Couston, B. Favier, M. Le Bars

Internal Gravity Wave Excitation by Turbulent Stellar Convection

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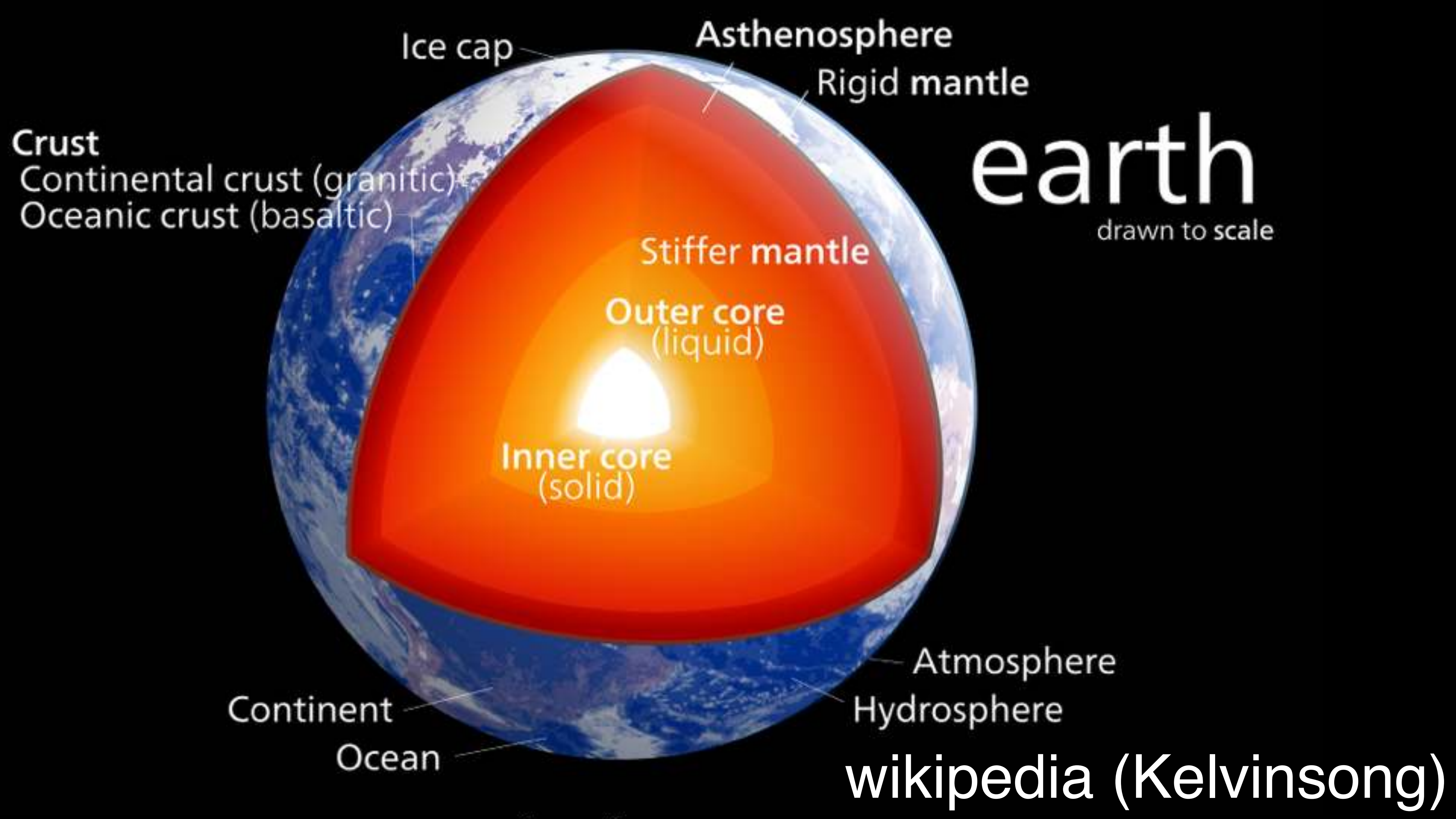
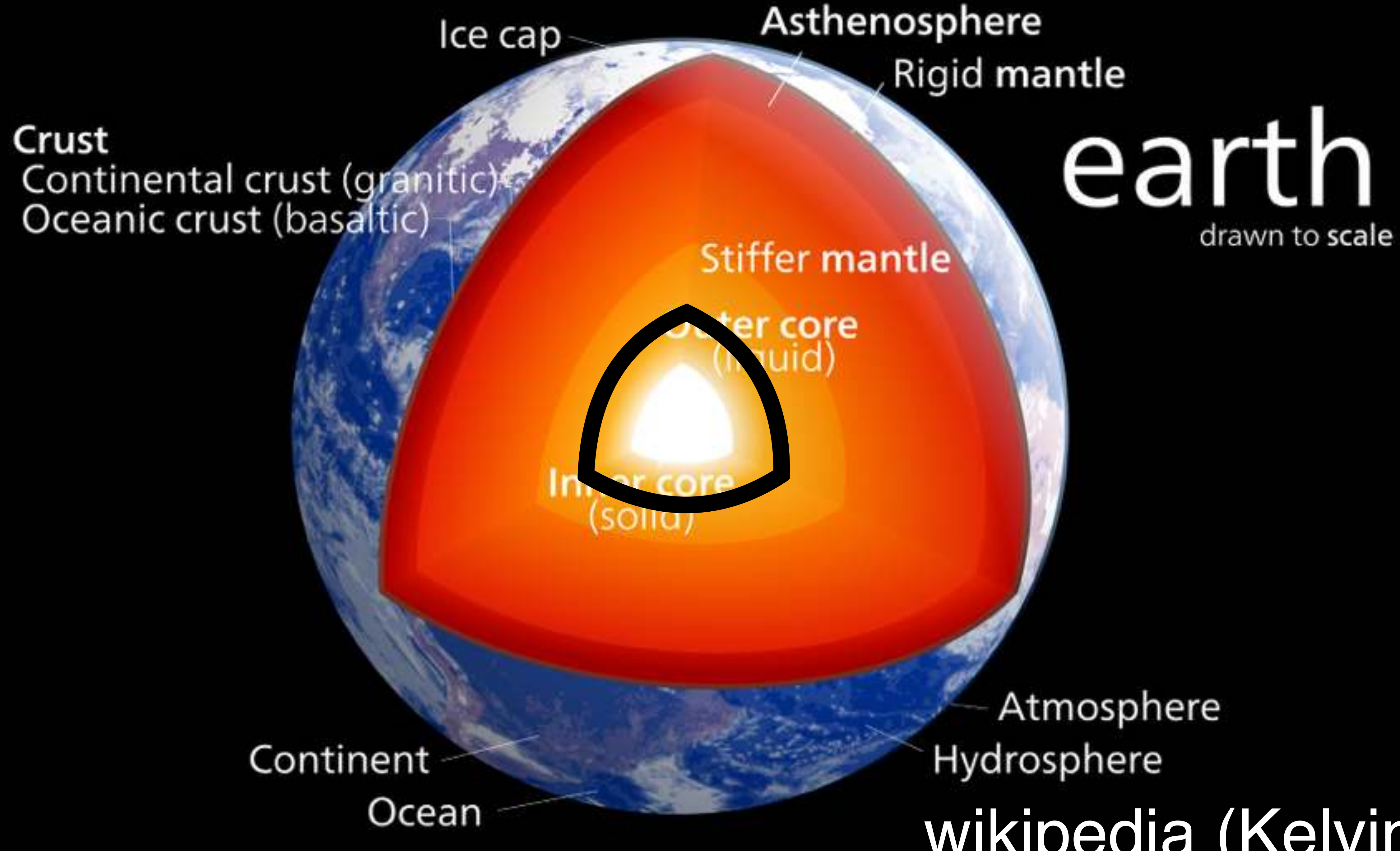


wikipedia (Kelvinsong)



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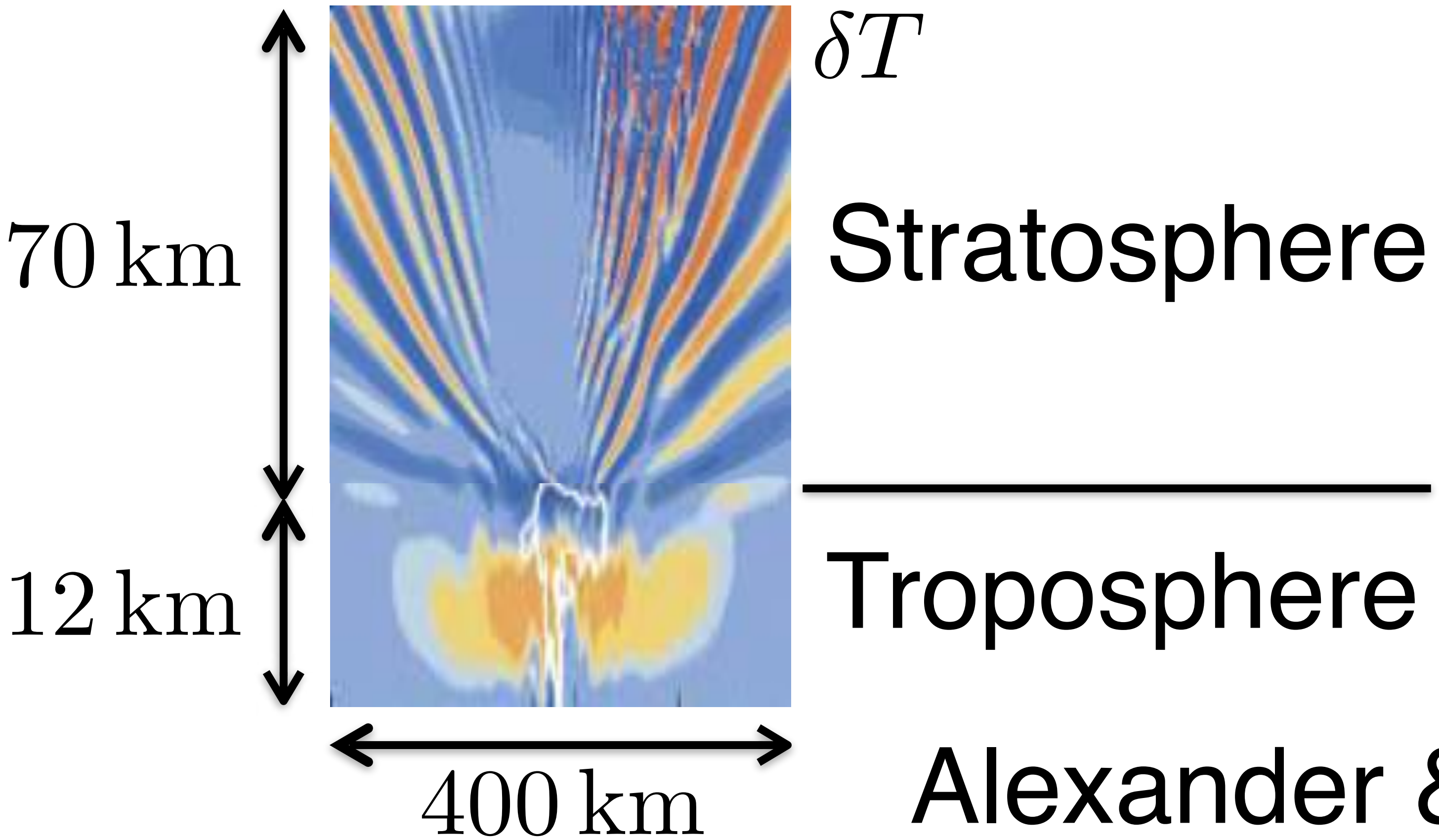
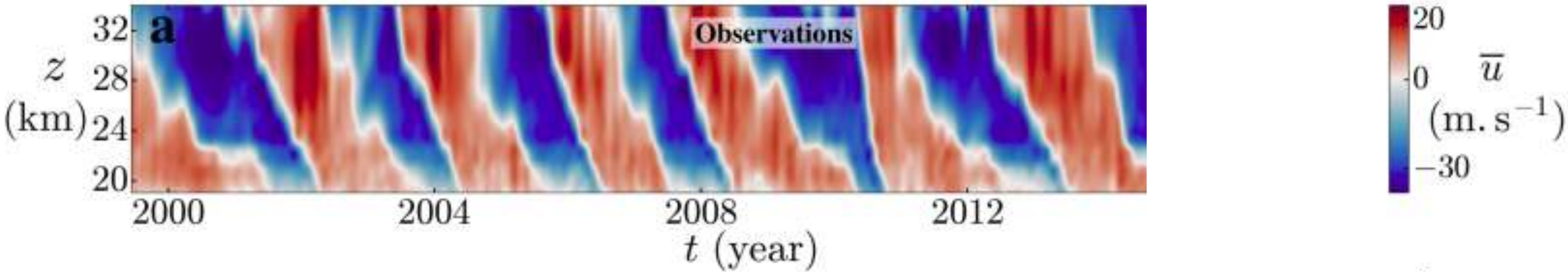
Earth’s Atmosphere

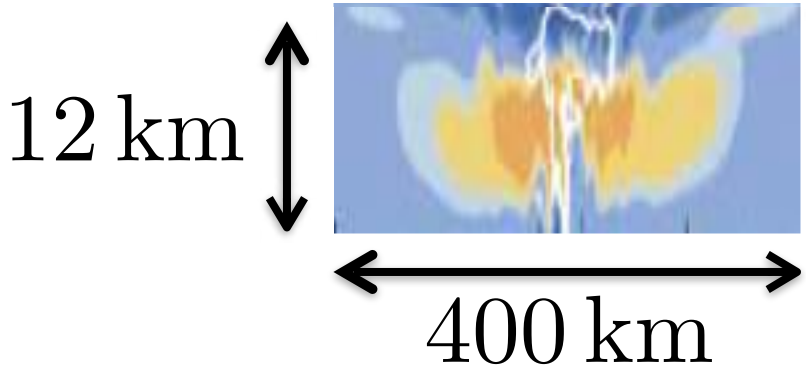


Alexander & Barnet (2006)

Earth’s Atmosphere

Renaud et al (2019)





Troposphere

Alexander & Barnet (2006)

Earth’s Atmosphere

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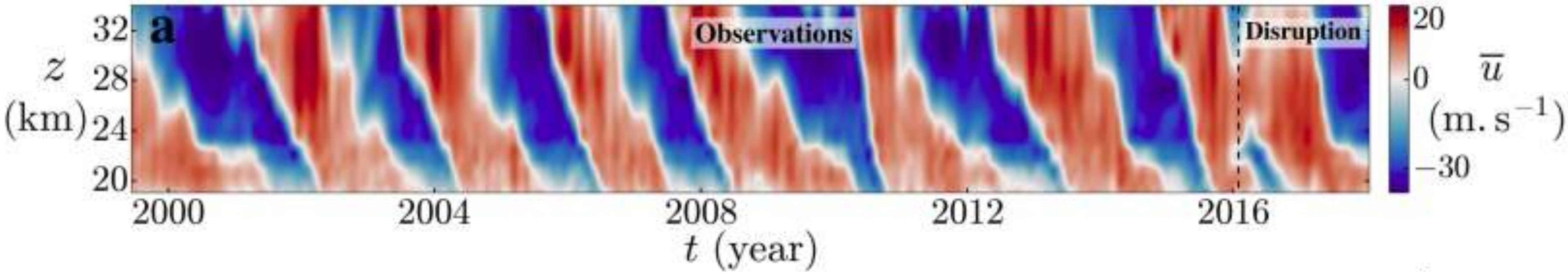

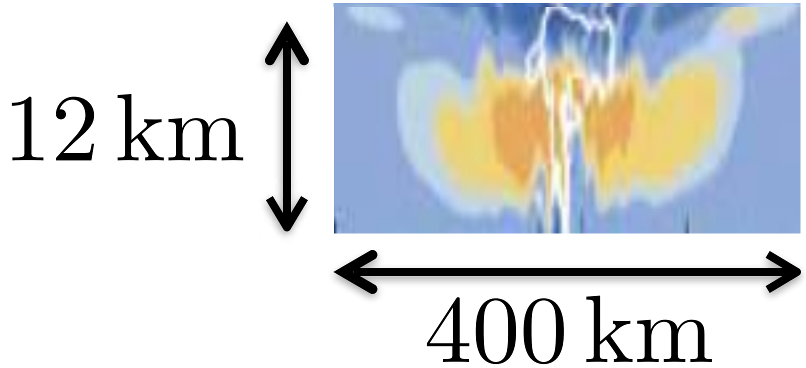

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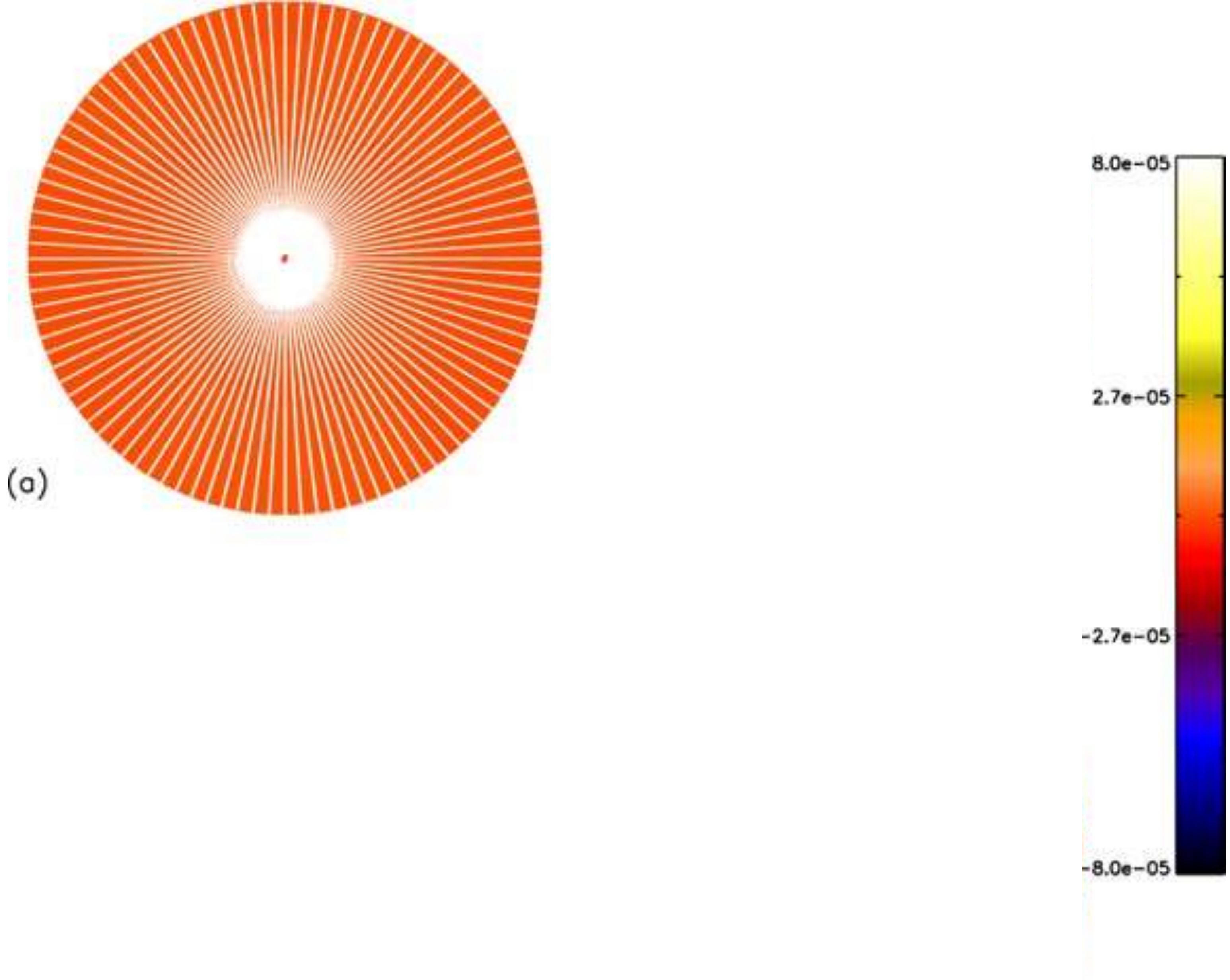
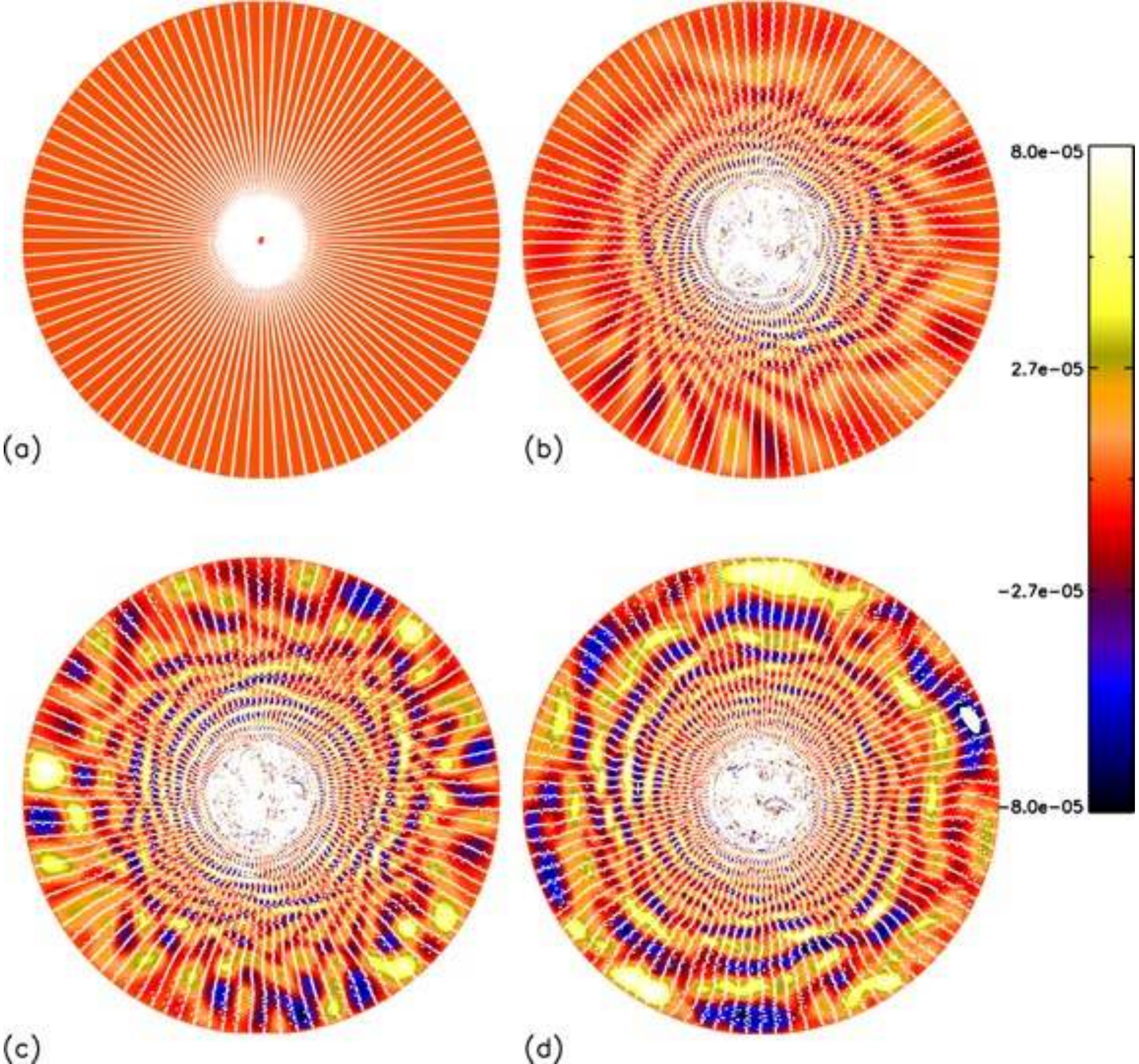


Wave Mixing

Rogers et al (2017)

Wave Mixing



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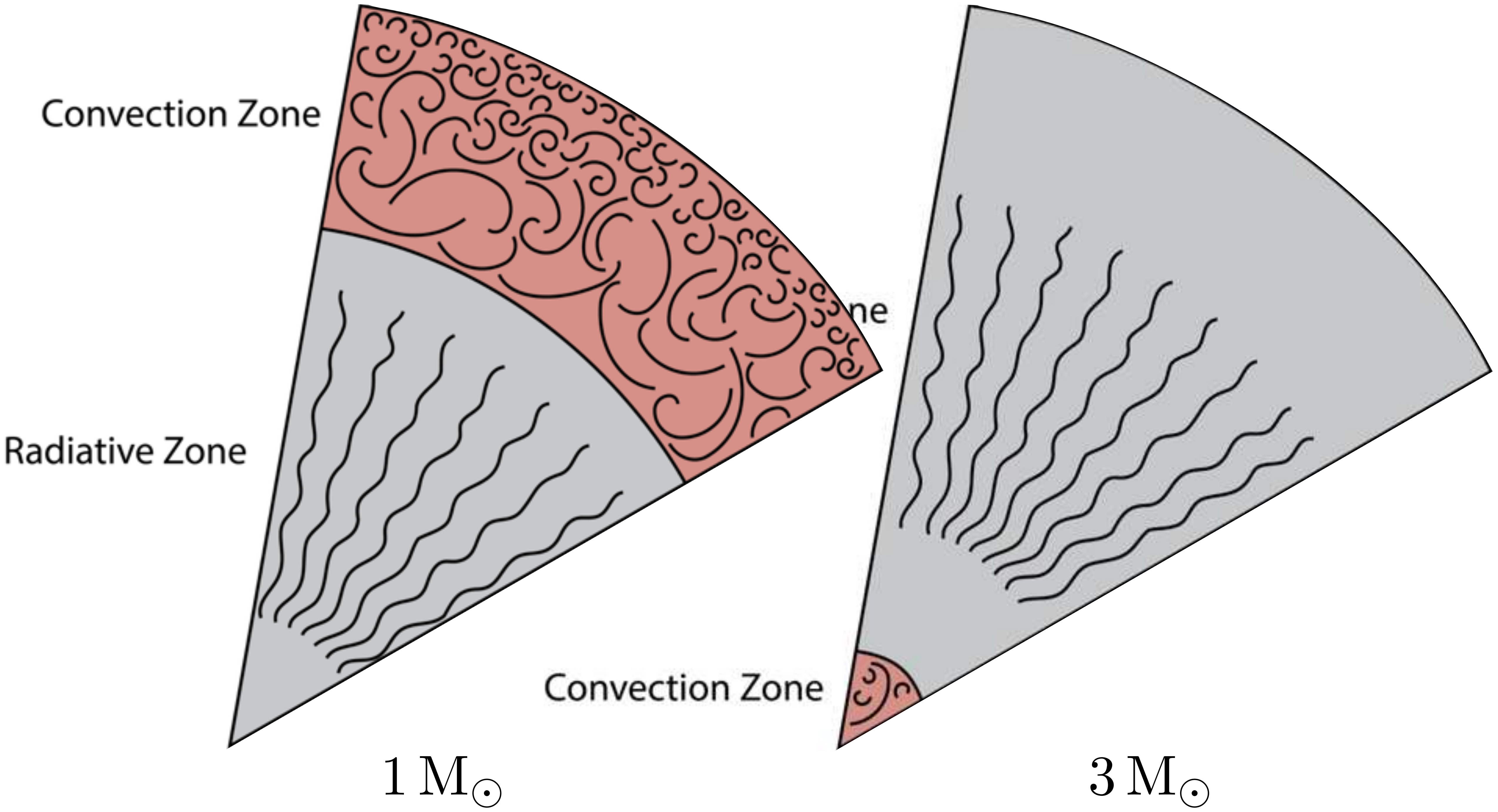

Convection Zone
Radiative Zone
Convection Zone
$1\,M_\odot$
$3\,M_\odot$

Space-Based Photometry

CoRoT

Dec 2006

27 cm diameter

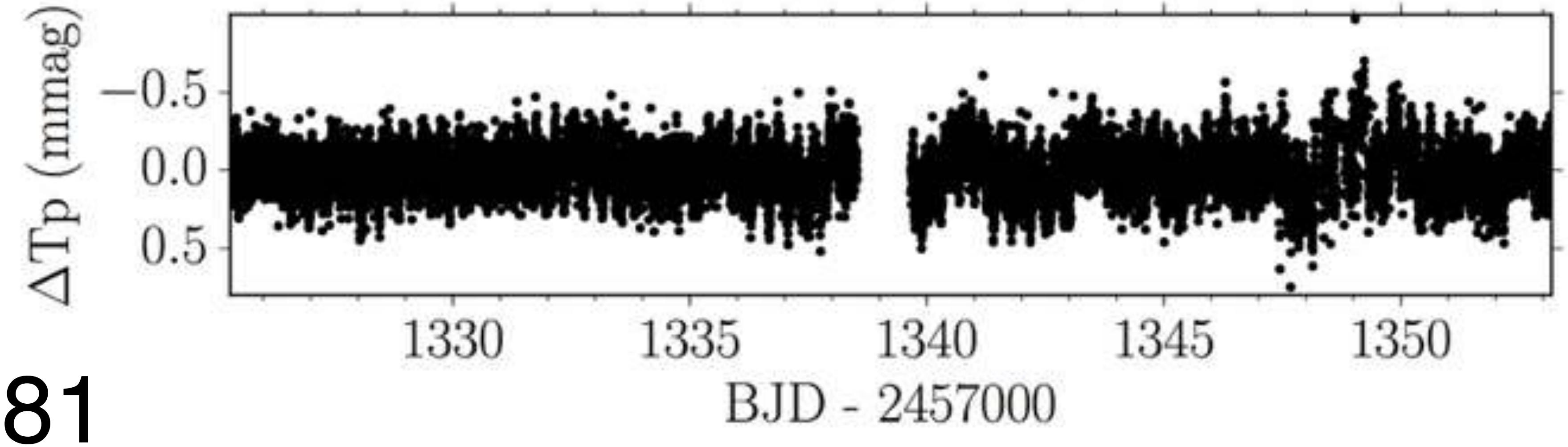
Kepler

Mar 2009

95 cm diameter

TESS

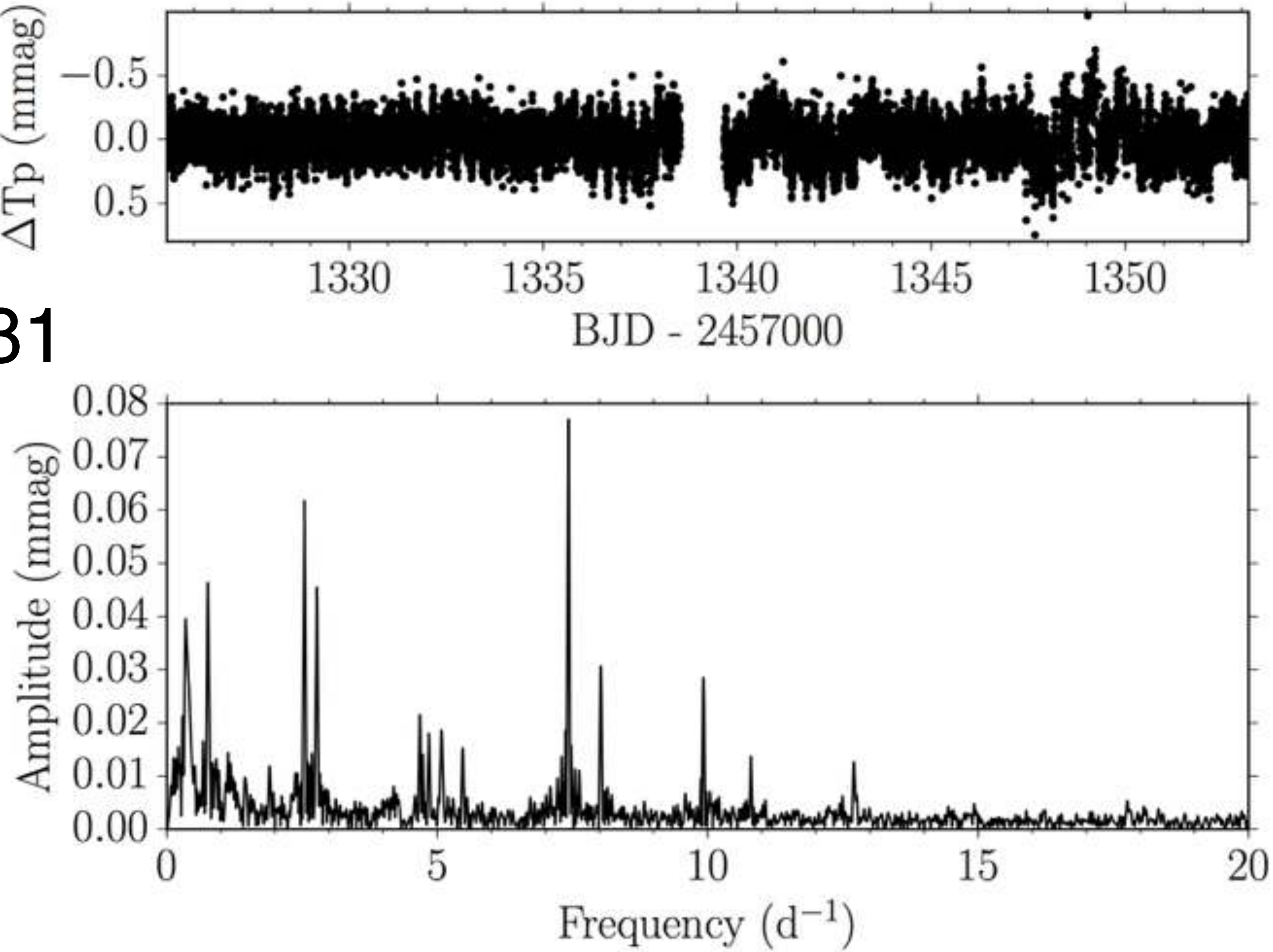
HD 212581



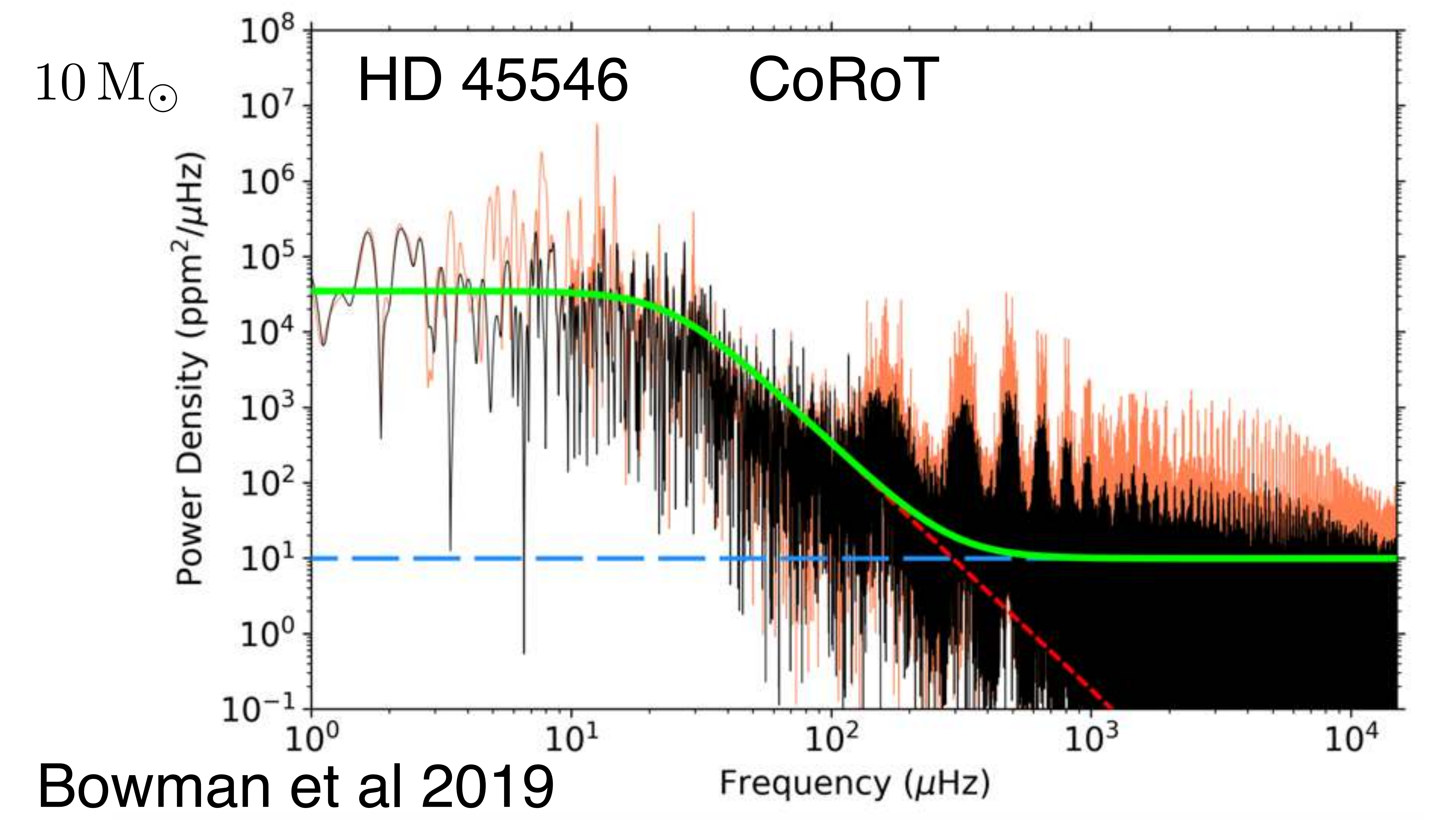
Pedersen et al 2019

TESS

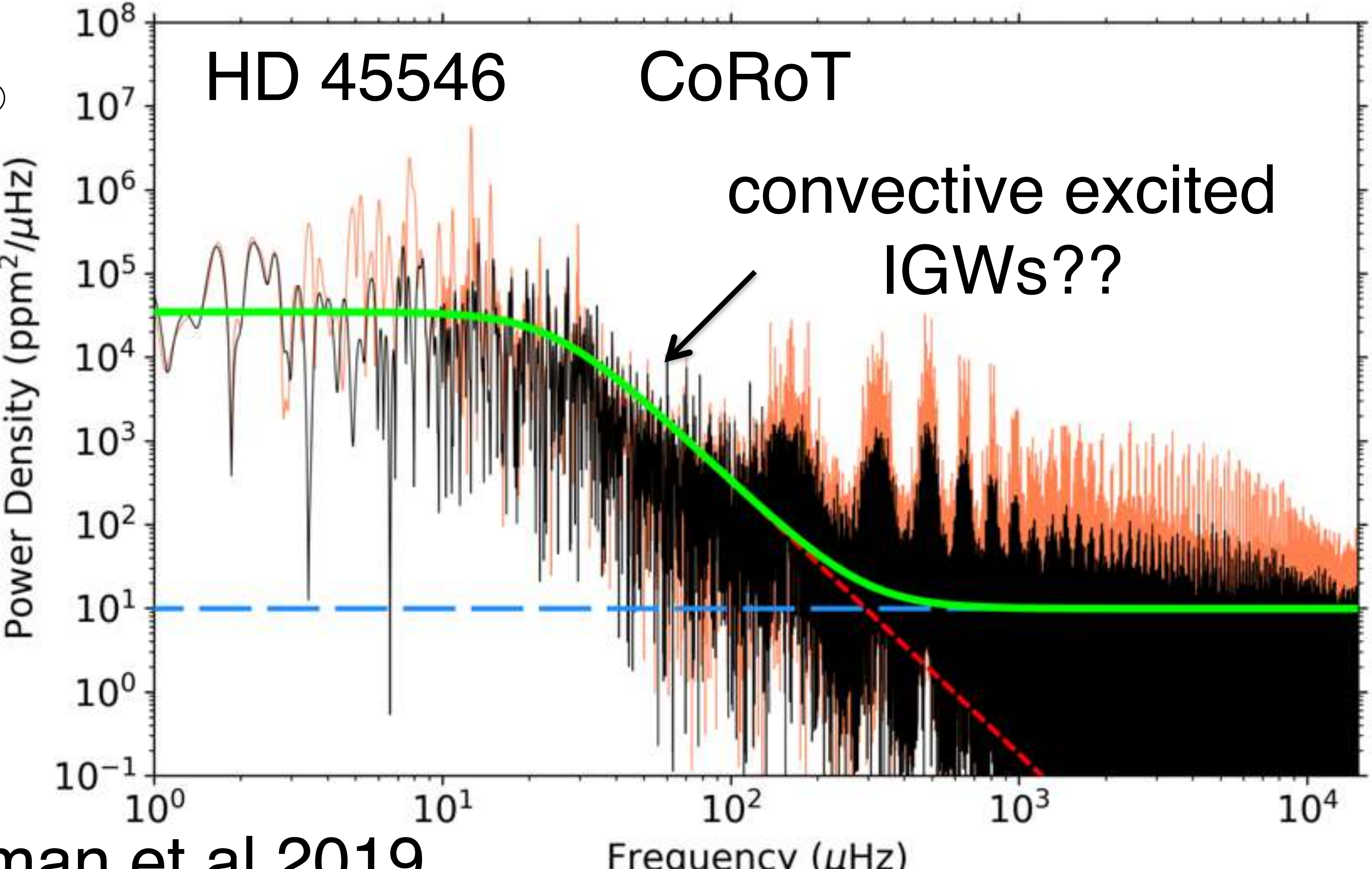
HD 212581



Pedersen et al 2019


10 $M_\odot$
HD 45546
CoRoT
Power Density (ppm^2/μHz)
Frequency (μHz)
Bowman et al 2019

$10\,\mathrm{M}_\odot$



Bowman et al 2019

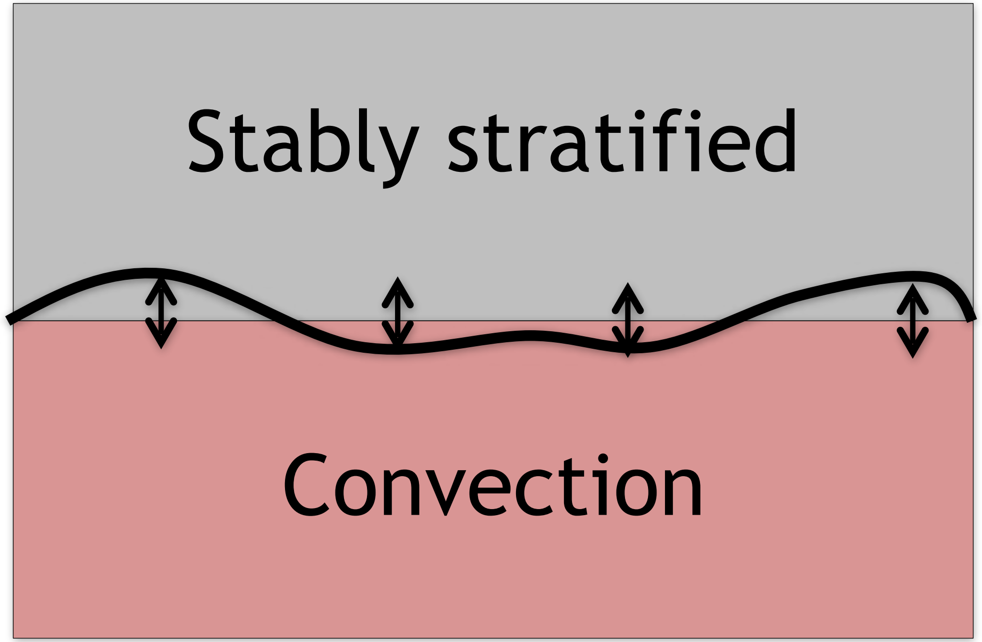
I: How?

II: Which?

III: What?

Models for wave generation

(e.g., Fritts & Alexander 2003)
Interface forcing



Models for wave generation

(e.g., Fritts & Alexander 2003)
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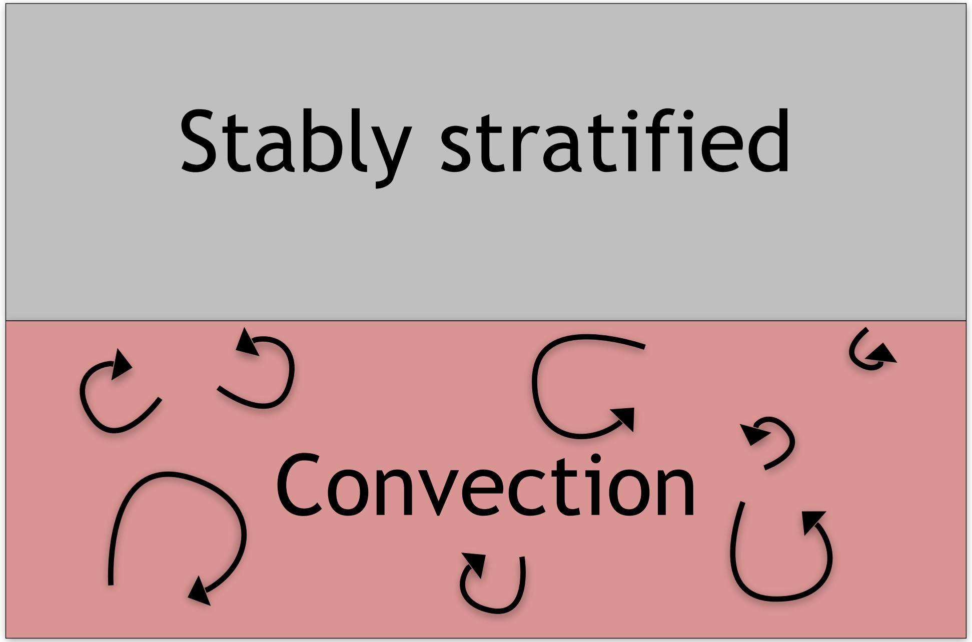
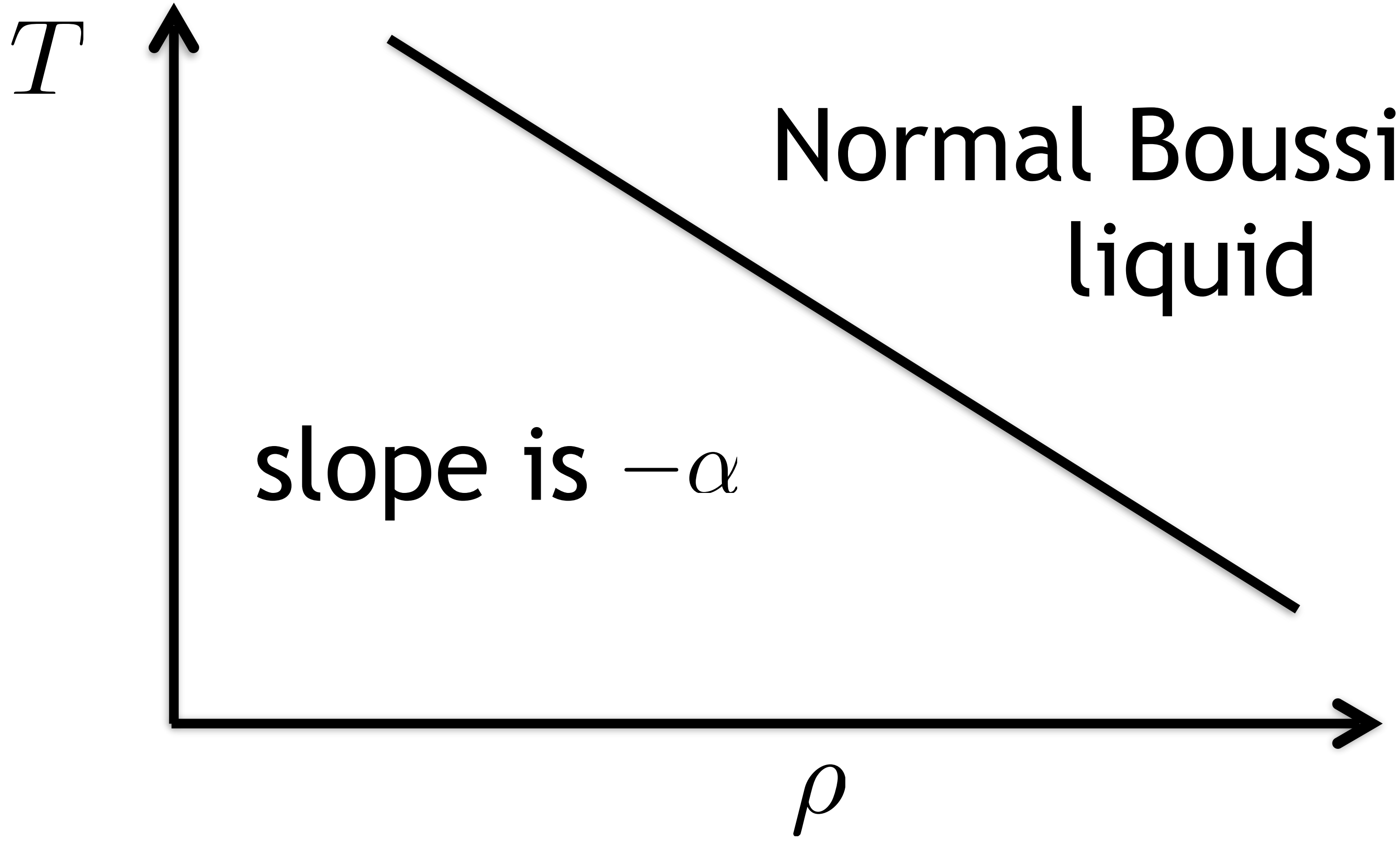
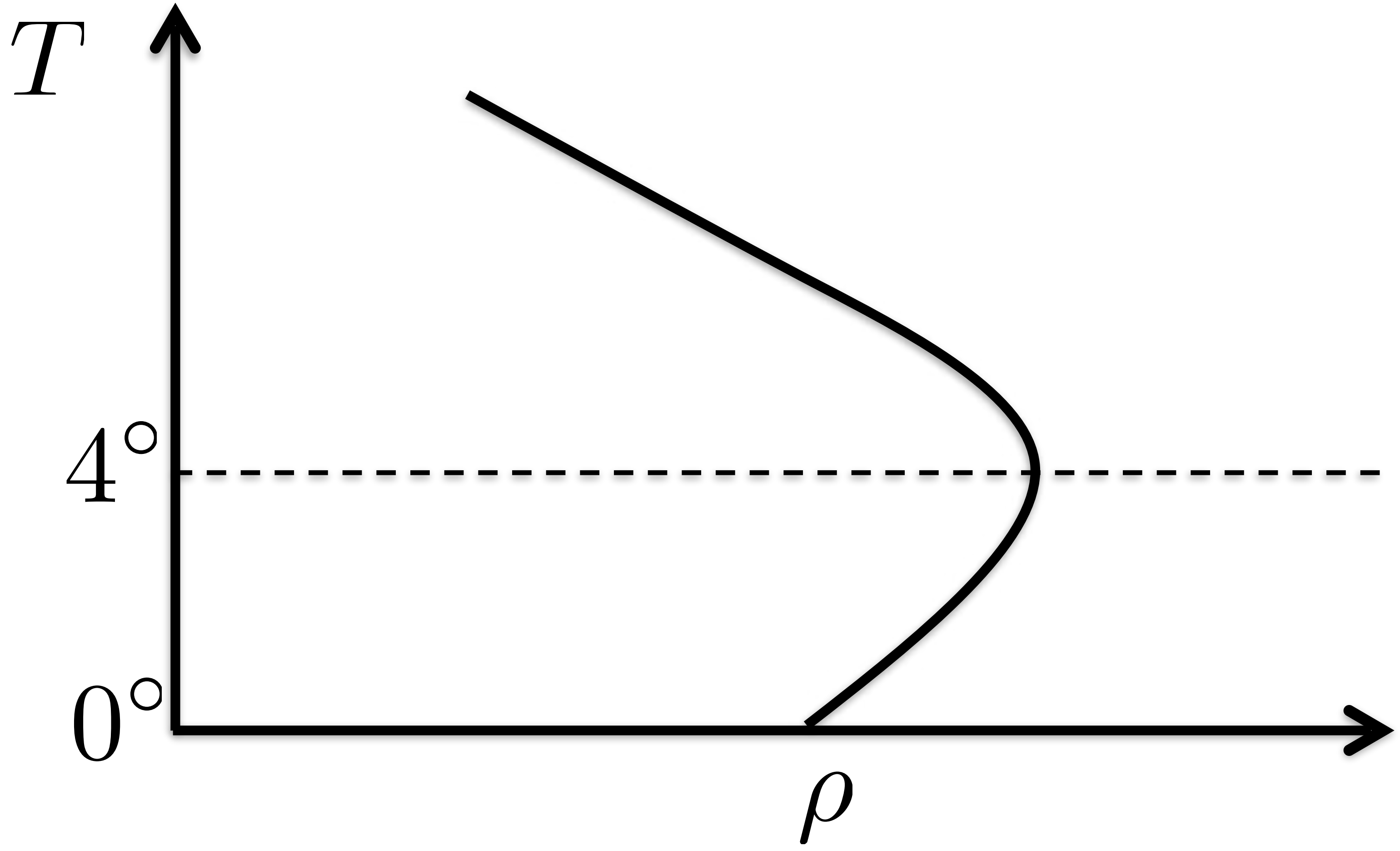


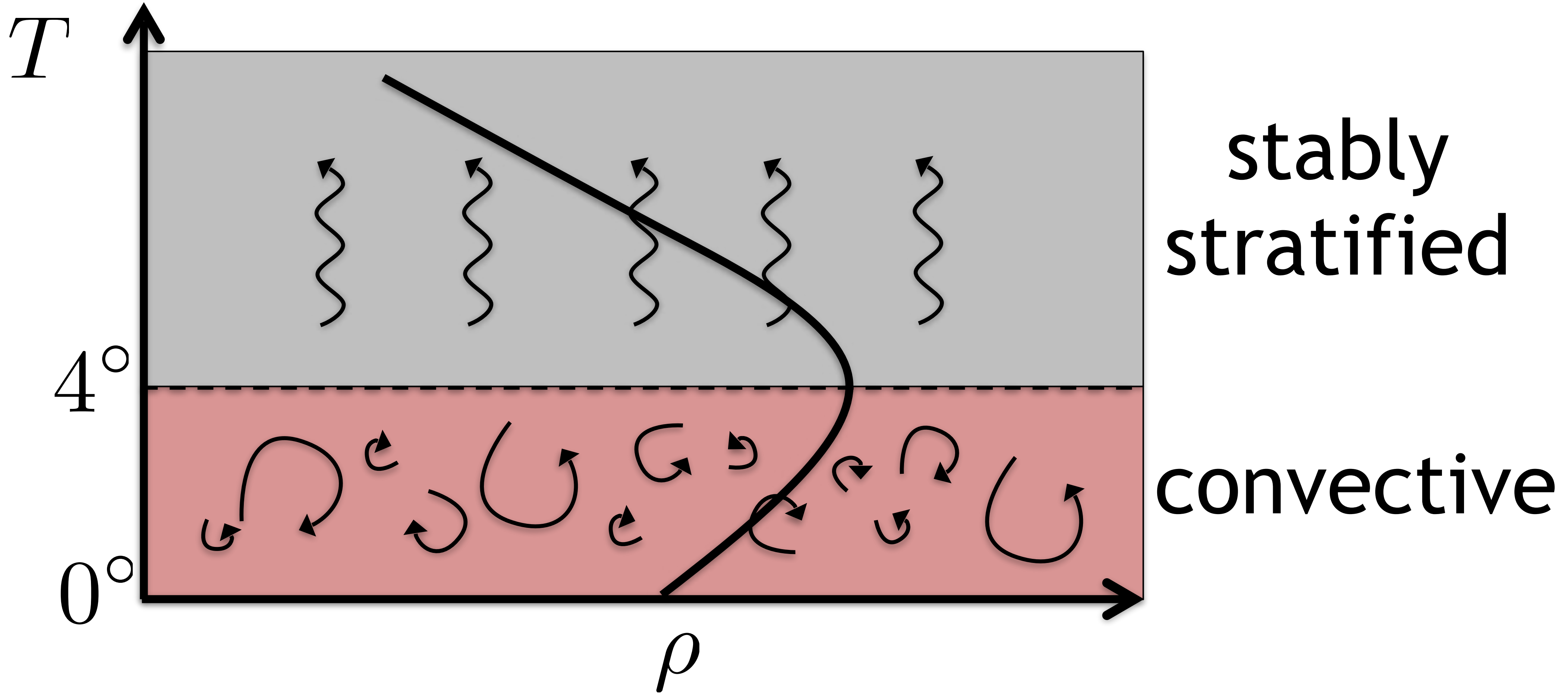
Equation of State



Equation of State of water

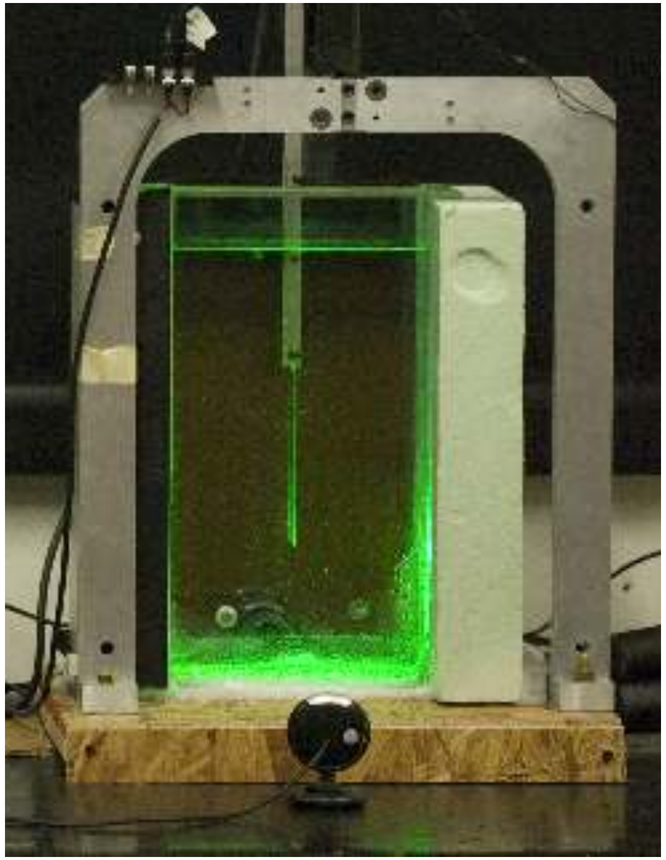


Equation of State of water



Water Experiment

Le Bars et al. 2015

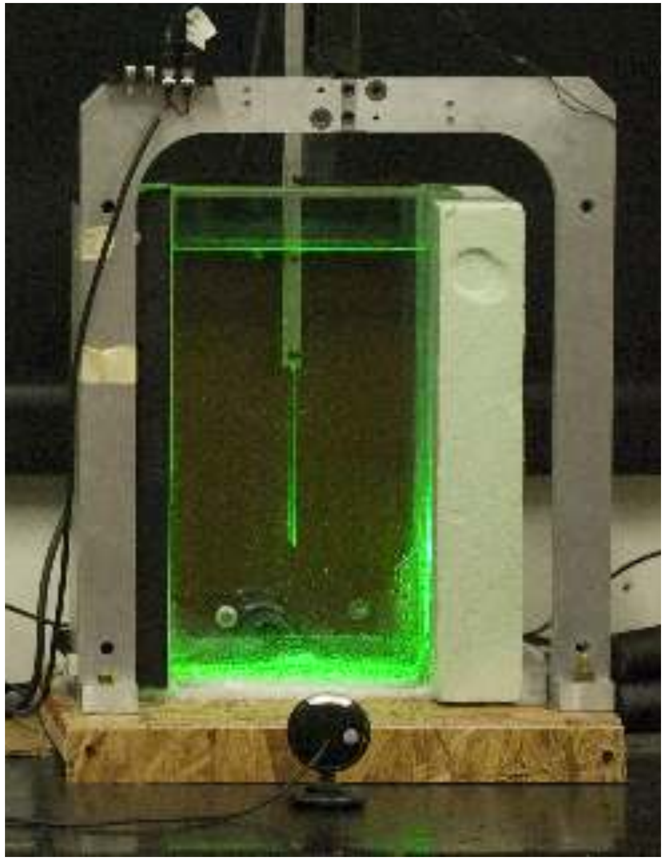


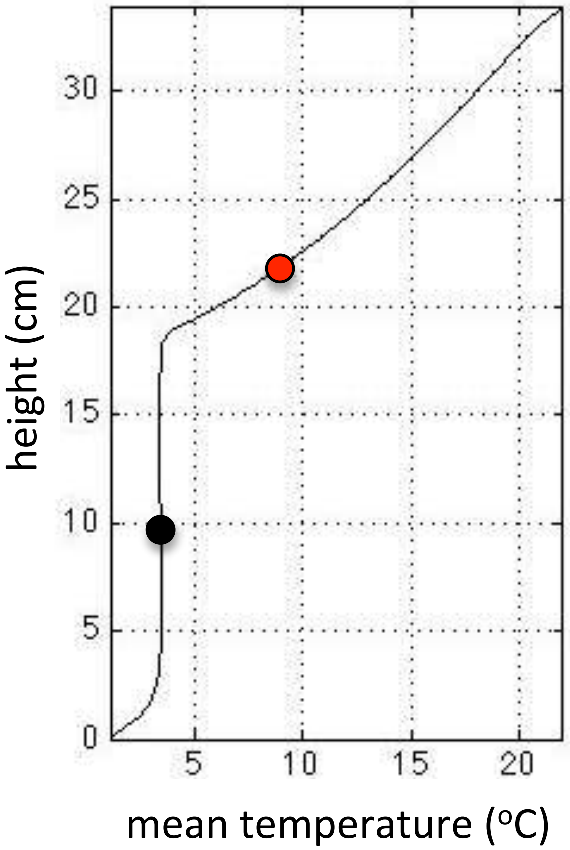
Dimensions: 20 x 4 x 35 cm^3

Ra ~ $2x10^7$ - $2x10^8$

Water Experiment

Le Bars et al. 2015





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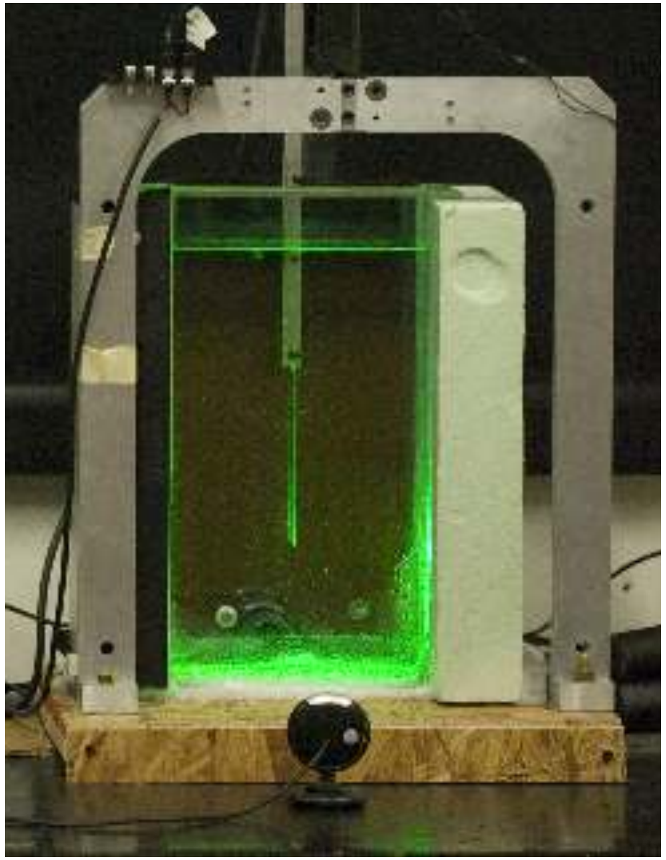

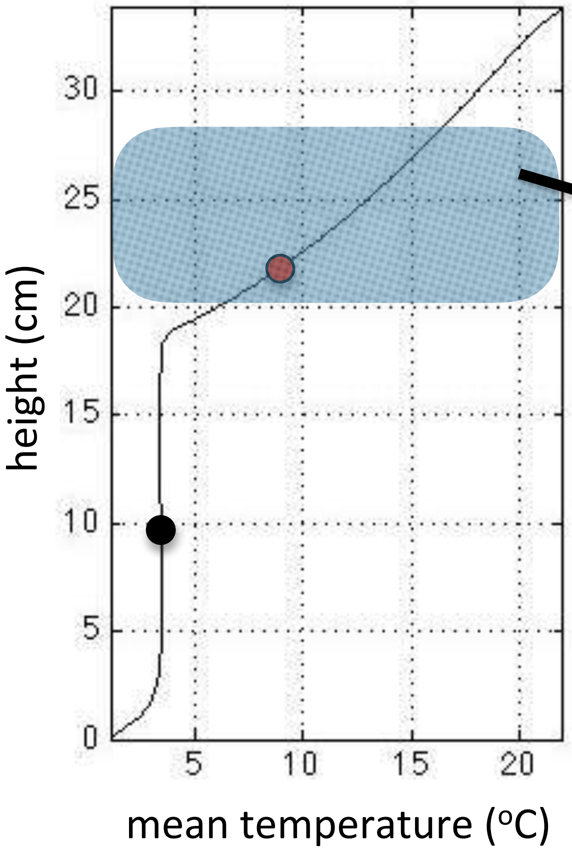

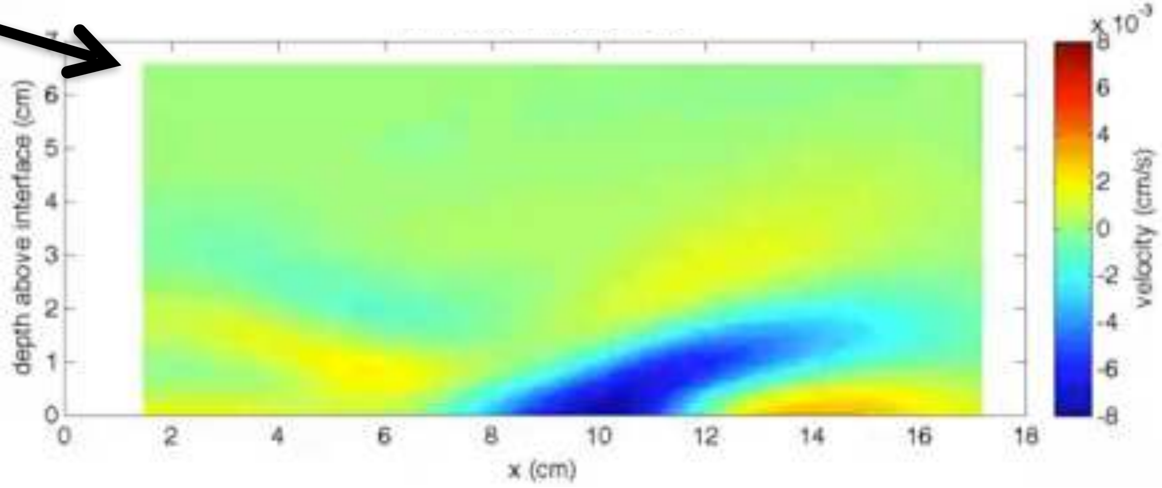

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Ra ~ 2×10^7 - 2×10^8


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SOLVING DIFFERENTIAL EQUATIONS
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The team so far

Daniel Lecoanet (Princeton) Keaton Burns (Flatiron)

Jeff Oishi (Bates) Ben Brown (Colorado)

Geoff Vasil (Sydney)

Australian Government

Australian Research Council

NSF

Water Simulation

$$\partial_t \boldsymbol{u} + \boldsymbol{\nabla} p - \nabla^2 \boldsymbol{u} = -\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} + \mathrm{Ra} \boldsymbol{e}_z (T - T_0)^2$$

$$\partial_t T - \mathrm{Pr}^{-1} \nabla^2 T = -\boldsymbol{u} \cdot \boldsymbol{\nabla} T$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

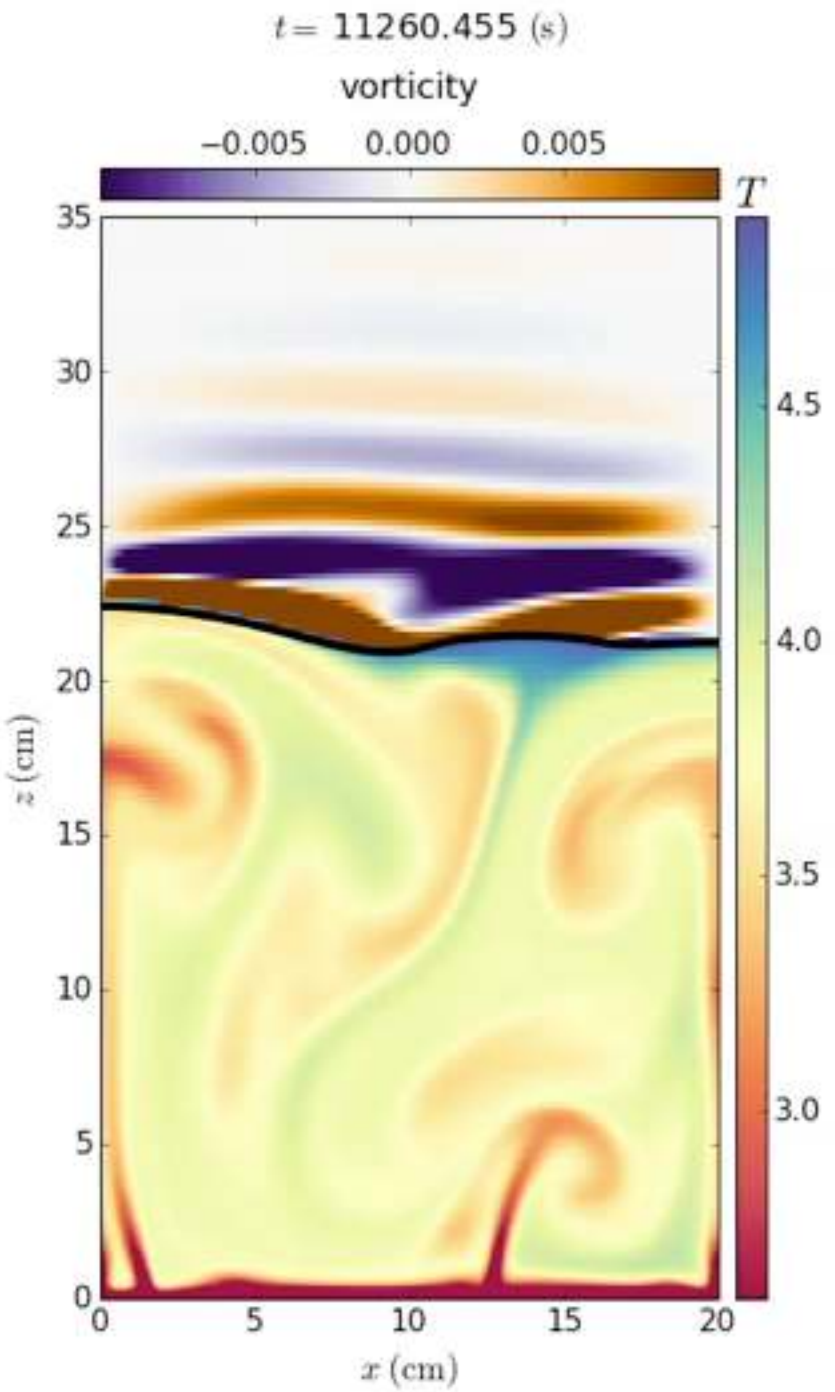
Water Simulation

$$\partial_t \boldsymbol{u} + \boldsymbol{\nabla} p - \nabla^2 \boldsymbol{u} = -\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} + \mathrm{Ra} \boldsymbol{e}_z (T - T_0)^2$$

$$\partial_t T - \mathrm{Pr}^{-1} \nabla^2 T = -\boldsymbol{u} \cdot \boldsymbol{\nabla} T$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

```
# Equations
prob.add_equation("dt(u) - dx(dx(u)) - dz(uz) + dx(p) = - u*dx(u) - w*uz")
prob.add_equation("dt(w) - dx(dx(w)) - dz(wz) + dz(p) = - u*dx(w) - w*wz + Ra*(T-T0)*(T-T0)")
prob.add_equation("dt(T) - Pr**(-1)*(dx(dx(T)) + dz(Tz)) = - u*dx(T) - w*Tz")
prob.add_equation("dx(u) + wz = 0”)
prob.add_equation("Tz - dz(T) = 0")
prob.add_equation("uz - dz(u) = 0")
prob.add_equation("wz - dz(w) = 0")
```



Lecoanet et al 2015

Interface Forcing

Solve in stable layer:

$$\nabla^2 \partial_t^2 \xi_z - N^2(z) \nabla_\perp^2 \xi_z = 0$$

Force with BC:

$$\xi_z(x, z_{\rm int}) = z_{\rm int}(x) - \bar{z}_{\rm int}$$

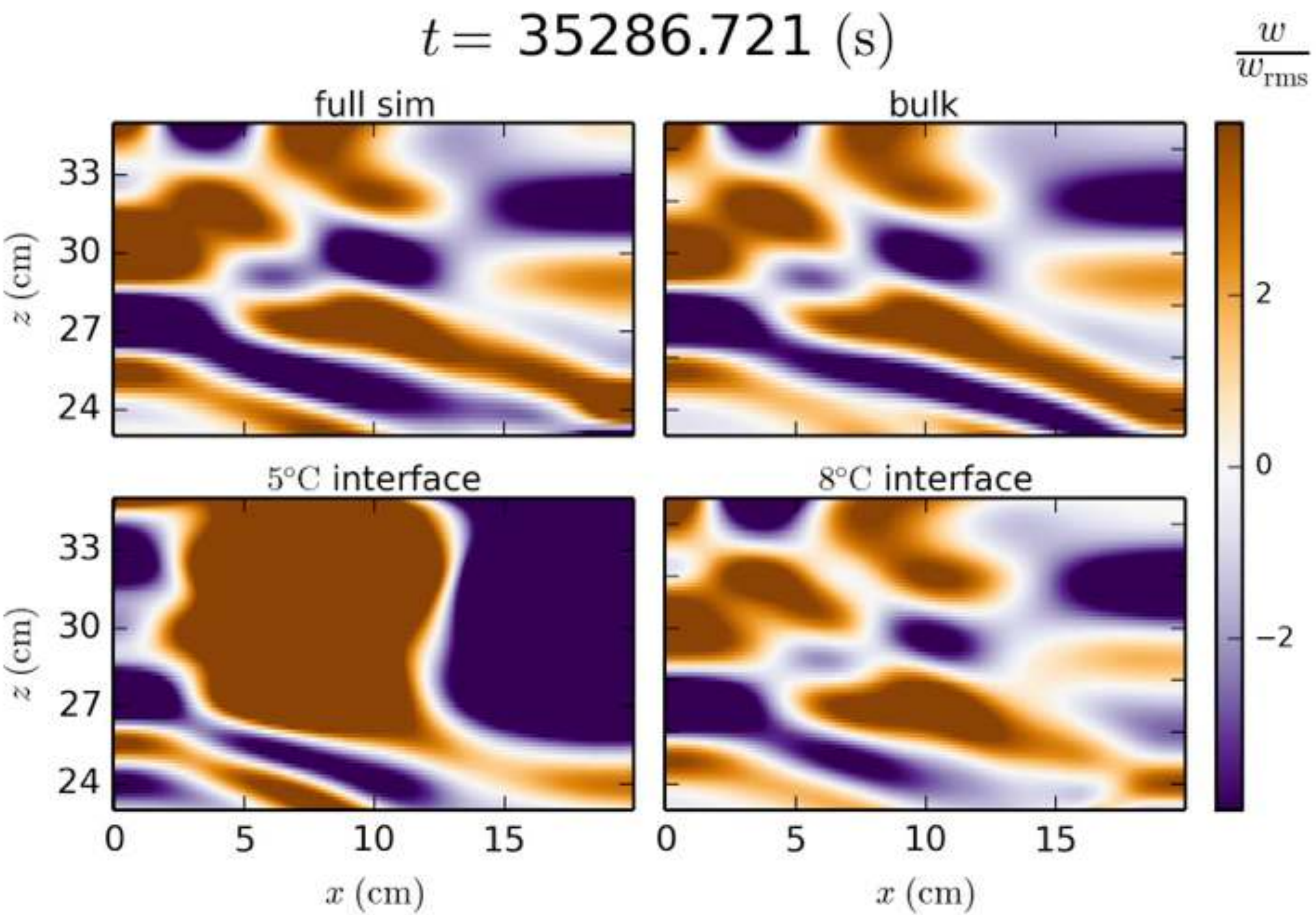
Bulk Forcing

Solve in the full domain:

$$\nabla^2 \partial_t^2 \xi_z - N^2(z) \nabla_\perp^2 \xi_z = S$$

S related to Reynolds stresses

Also include viscosity


$t = 35286.721$ (s)
$\frac{w}{w_{\text{rms}}}$
full sim
bulk
5°C interface
8°C interface
z (cm)
x (cm)
33
30
27
24
0
5
10
15
2
0
−2

I: How?

II: Which?

III: What?

Bulk Forcing

Seems to work — can use to estimate wave spectrum

$$\xi_z \sim \int G * S$$

Comes from wave eigenfunctions in stable & conv regions

$$(-\nabla^2 \boldsymbol{e}_z + \partial_z \nabla) \cdot (\boldsymbol{u}_c \cdot \nabla \boldsymbol{u}_c)$$

Lecoanet & Quataert 2013

IGW Basics

Want to calculate **wave flux**

$$F = \langle wp \rangle$$

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$$F = \langle wp \rangle = \rho_0 c_{g,z} \frac{1}{2} |\mathbf{u}|^2 \qquad c_{g,z} = \frac{\partial \omega}{\partial k_z} = -\frac{\omega k_z}{k^2}$$

IGW Basics

Want to calculate **wave flux**

$$F = \langle wp \rangle = \rho_0 c_{g,z} \frac{1}{2} |\mathbf{u}|^2 \qquad c_{g,z} = \frac{\partial \omega}{\partial k_z} = -\frac{\omega k_z}{k^2}$$

Absent dissipation stays constant!!

IGW Basics

Dissipation rate:

$$\tau_d \sim (\kappa k^2)^{-1}$$

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$$\ell_d \sim c_{g,z}\tau_d \sim \frac{1}{\kappa k^2}\omega\frac{k_z}{k^2}$$

IGW Basics

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$$\ell_d \sim c_{g,z}\tau_d \sim \frac{1}{\kappa k^2}\omega\frac{k_z}{k^2} \sim \frac{1}{\kappa}\frac{k_z}{k_\perp^4}\frac{\omega^5}{N^4}$$

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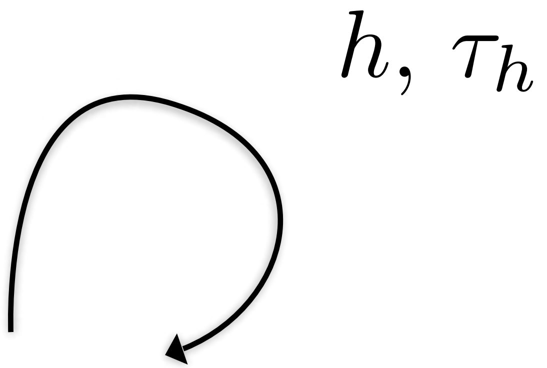
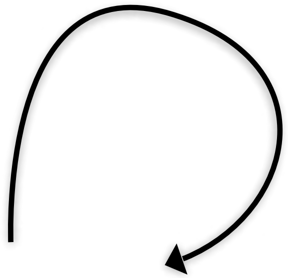
$$\ell_d^{-1} \sim \kappa\frac{k_\perp^3 N^3}{\omega^4}$$

Single Eddy



Single Eddy

h, τ_h

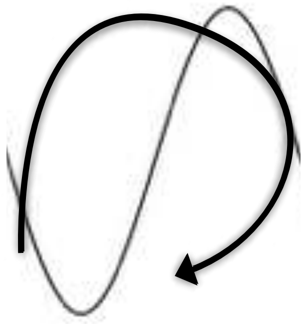


Which waves are excited?

$k_\perp, \omega$

Single Eddy

h, τ_h



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$k_\perp h > 1$

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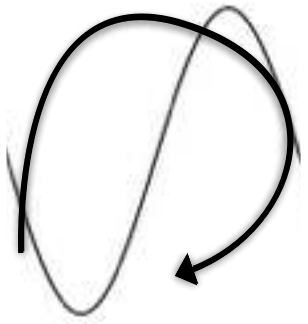

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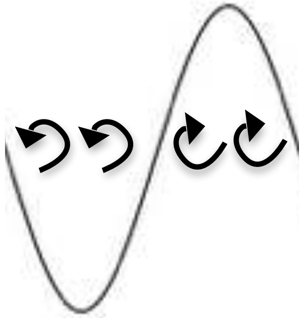

Which waves are excited?

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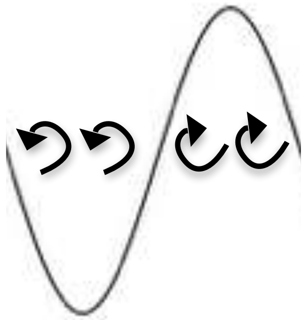
~~$k_\perp h > 1$~~ $k_\perp h < 1$

Single Eddy

h, τ_h



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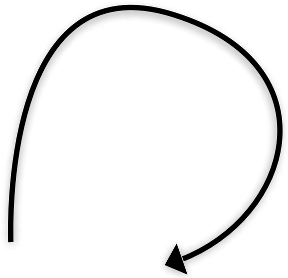
$h, \, \tau_h$



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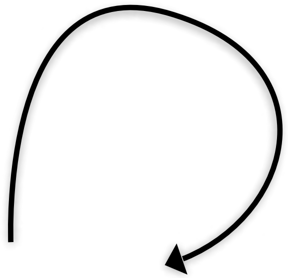


Which waves are excited?

$k_\perp, \omega$

$$F_w \sim \rho_0 \left(\frac{h}{\tau_h}\right)^3 \frac{\omega}{N_0} (k_\perp h)^4$$

~~$k_\perp h > 1$~~ $k_\perp h < 1$

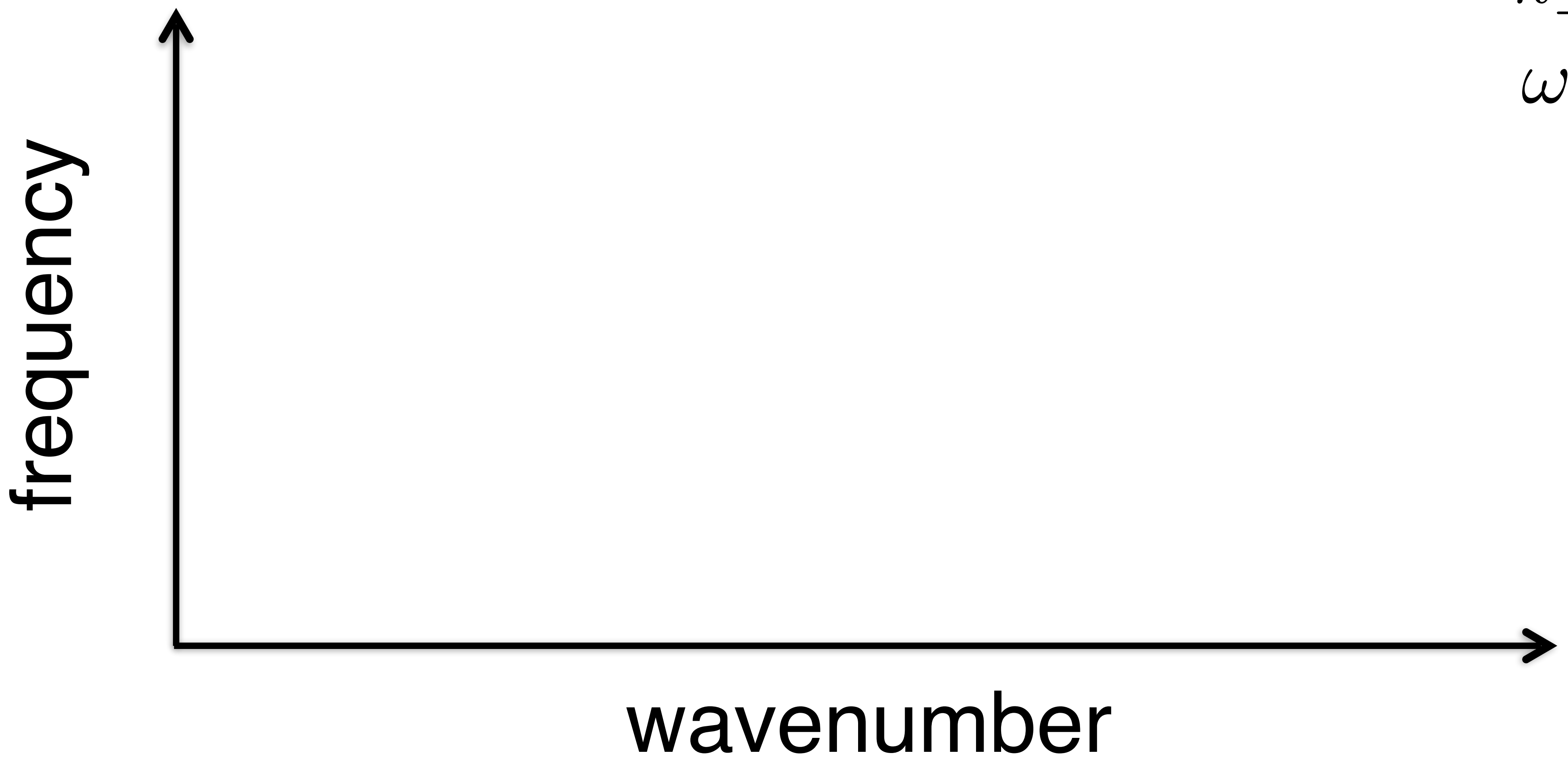
~~$\omega \tau_h > 1$~~ $\omega \tau_h < 1$

Spectrum

$$F_w \sim \rho_0 \left(\frac{h}{\tau_h} \right)^3 \frac{\omega}{N_0} (k_\perp h)^4$$

$$k_\perp h < 1$$

$$\omega \tau_h < 1$$



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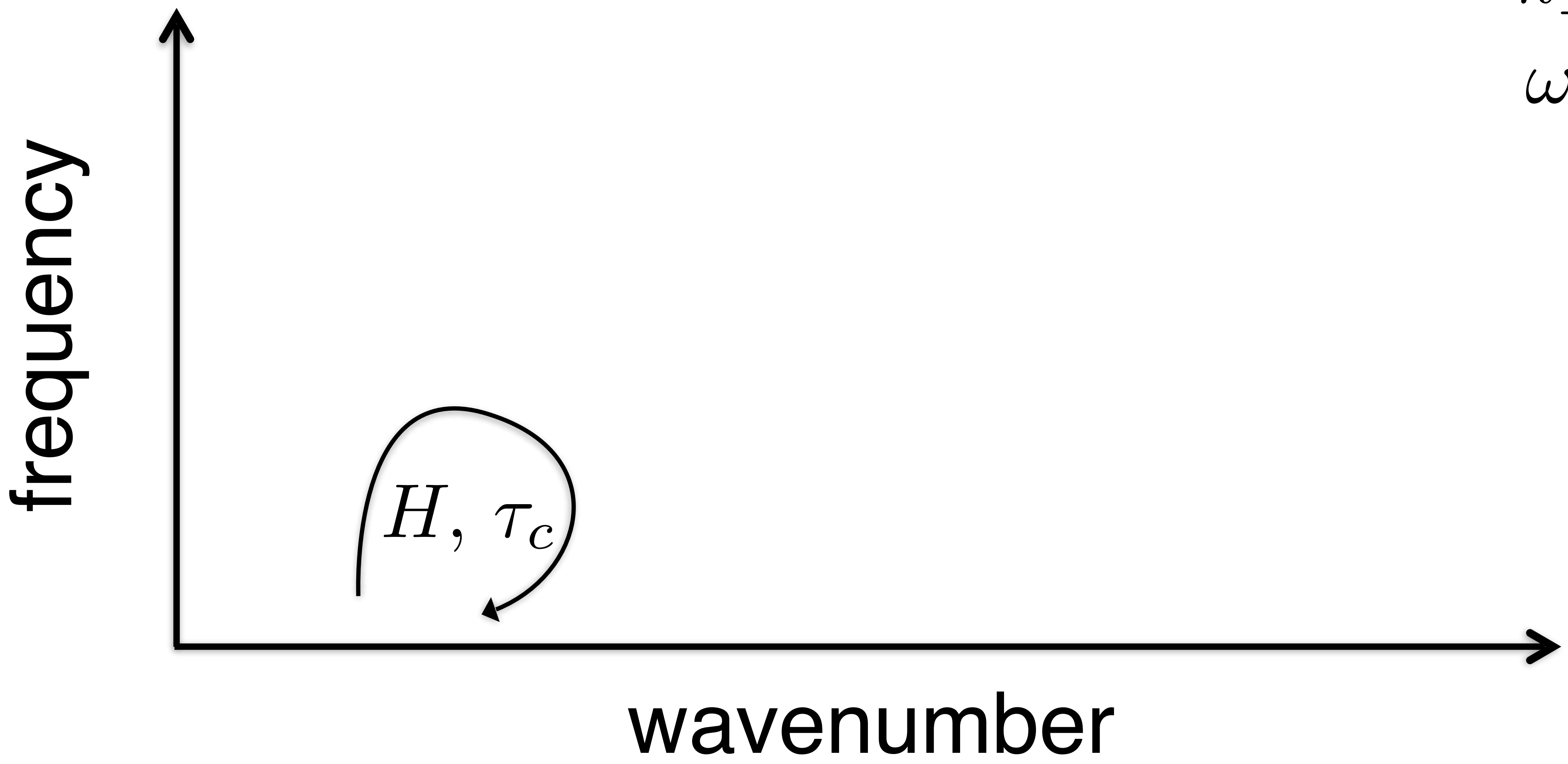

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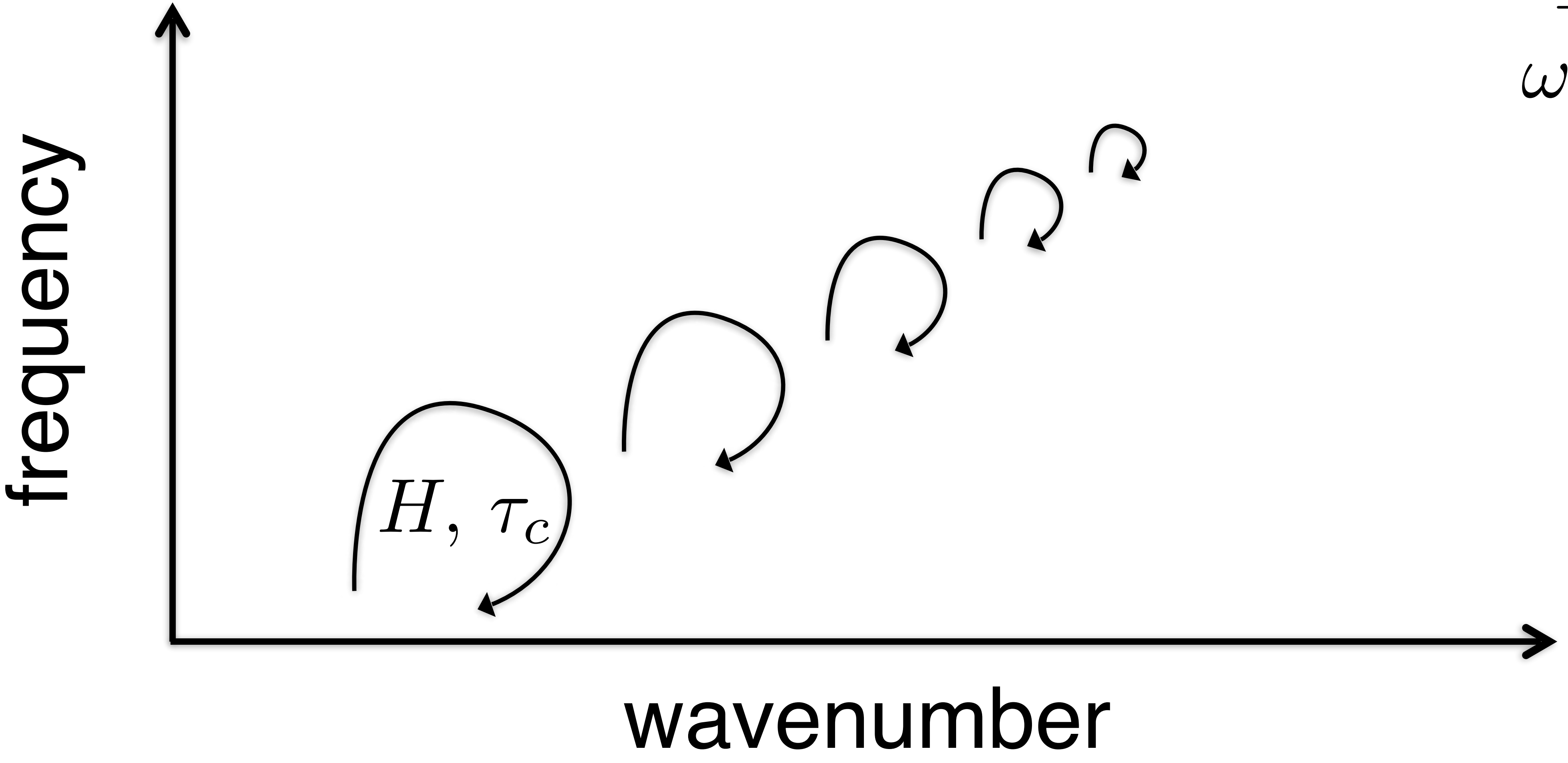

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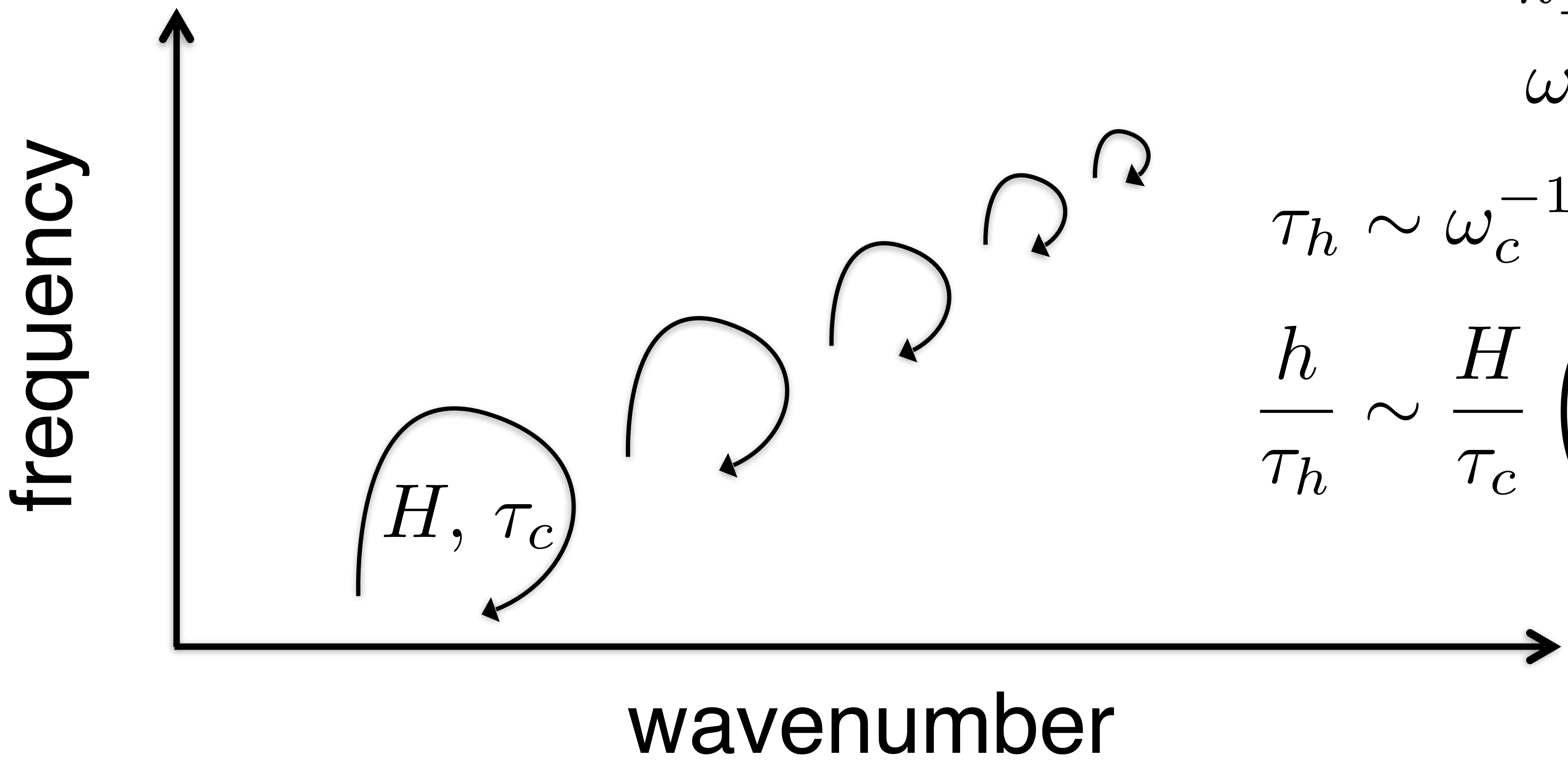

$$\tau_h \sim \omega_c^{-1}(h/H)^{2/3}$$

$$\frac{h}{\tau_h} \sim \frac{H}{\tau_c}\left(\frac{h}{H}\right)^{1/3}$$

Spectrum

$$F_w \sim \rho_0 \left(\frac{h}{\tau_h}\right)^3 \frac{\omega}{N_0} (k_\perp h)^4$$

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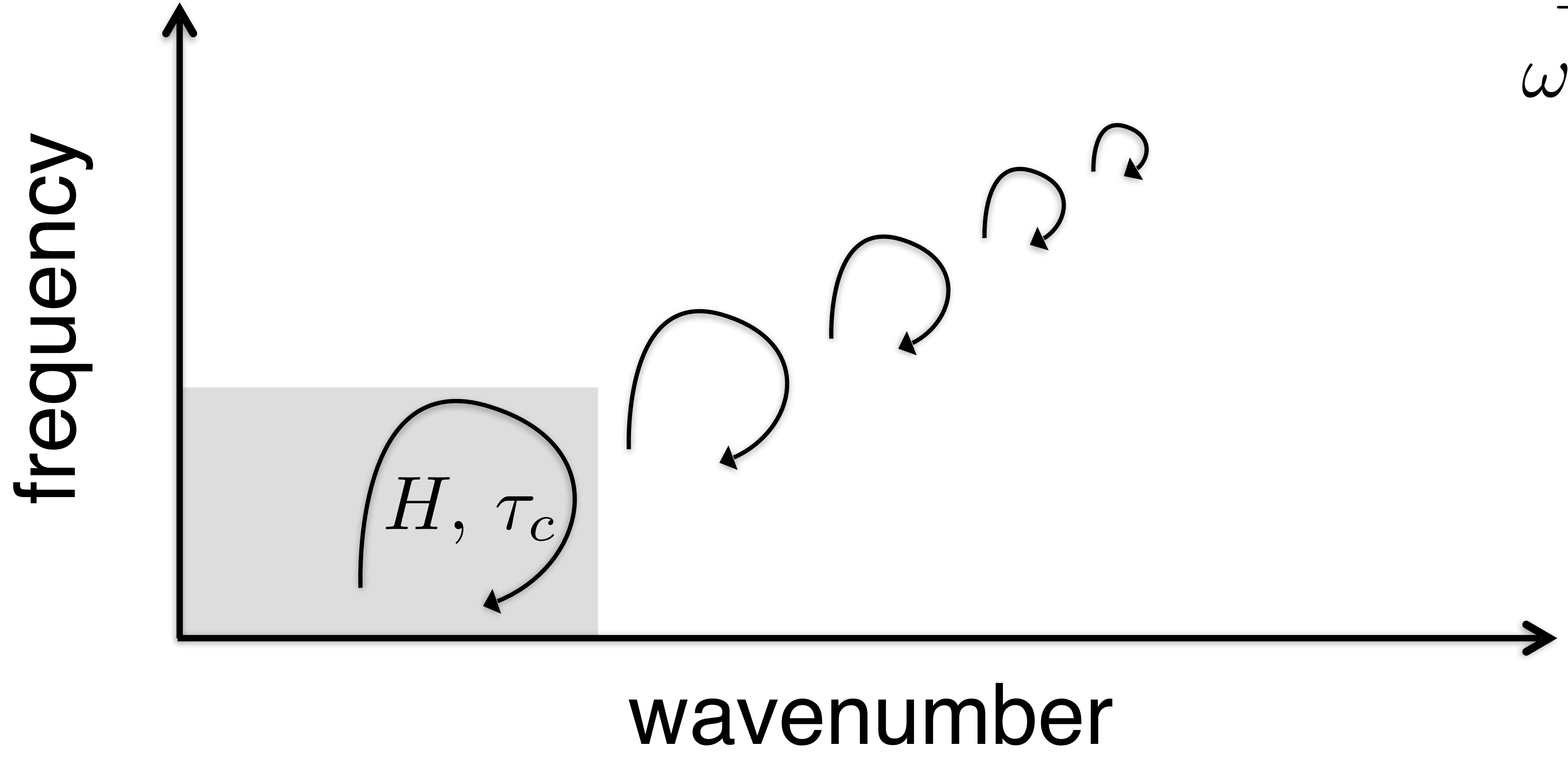

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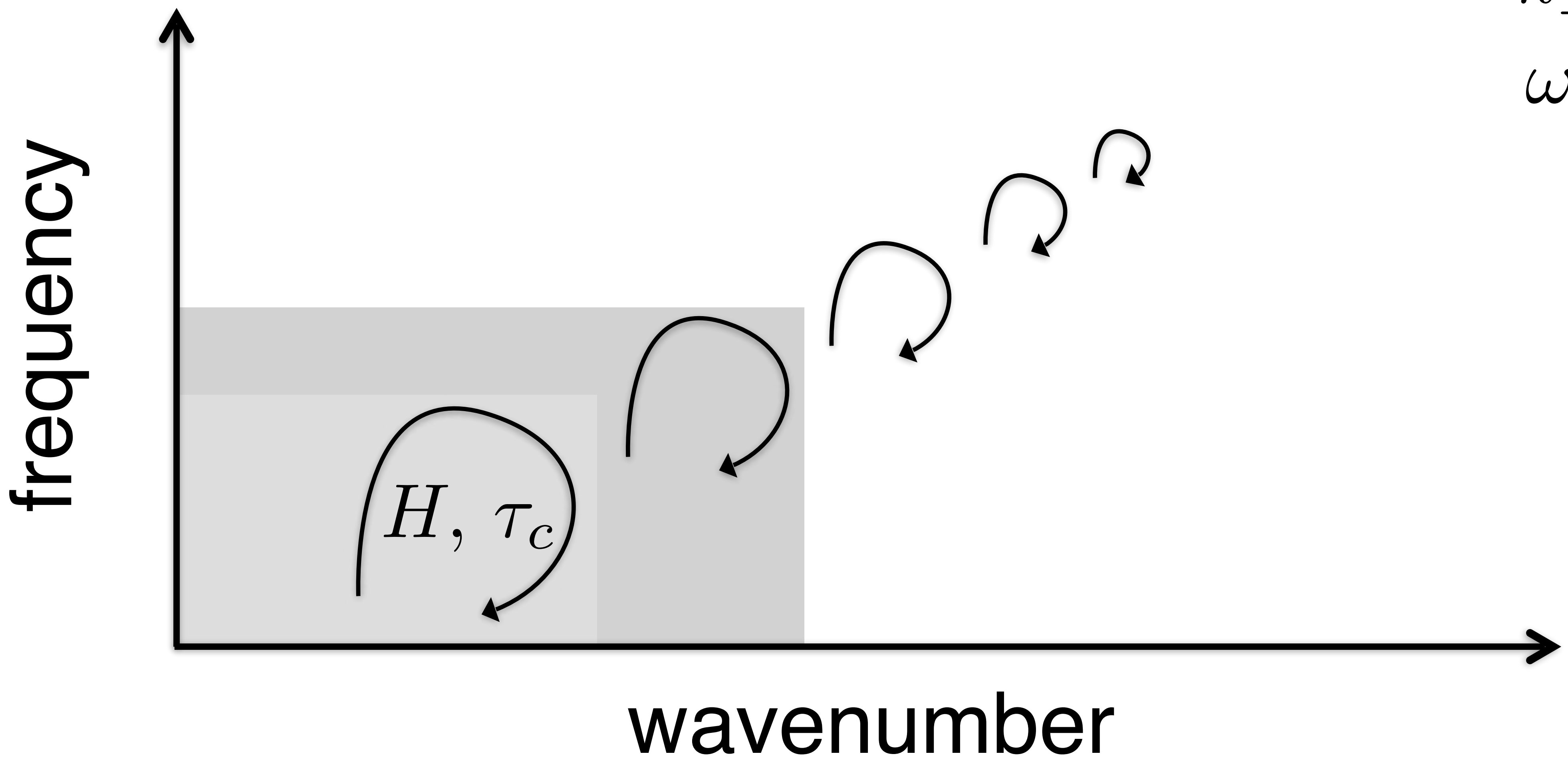

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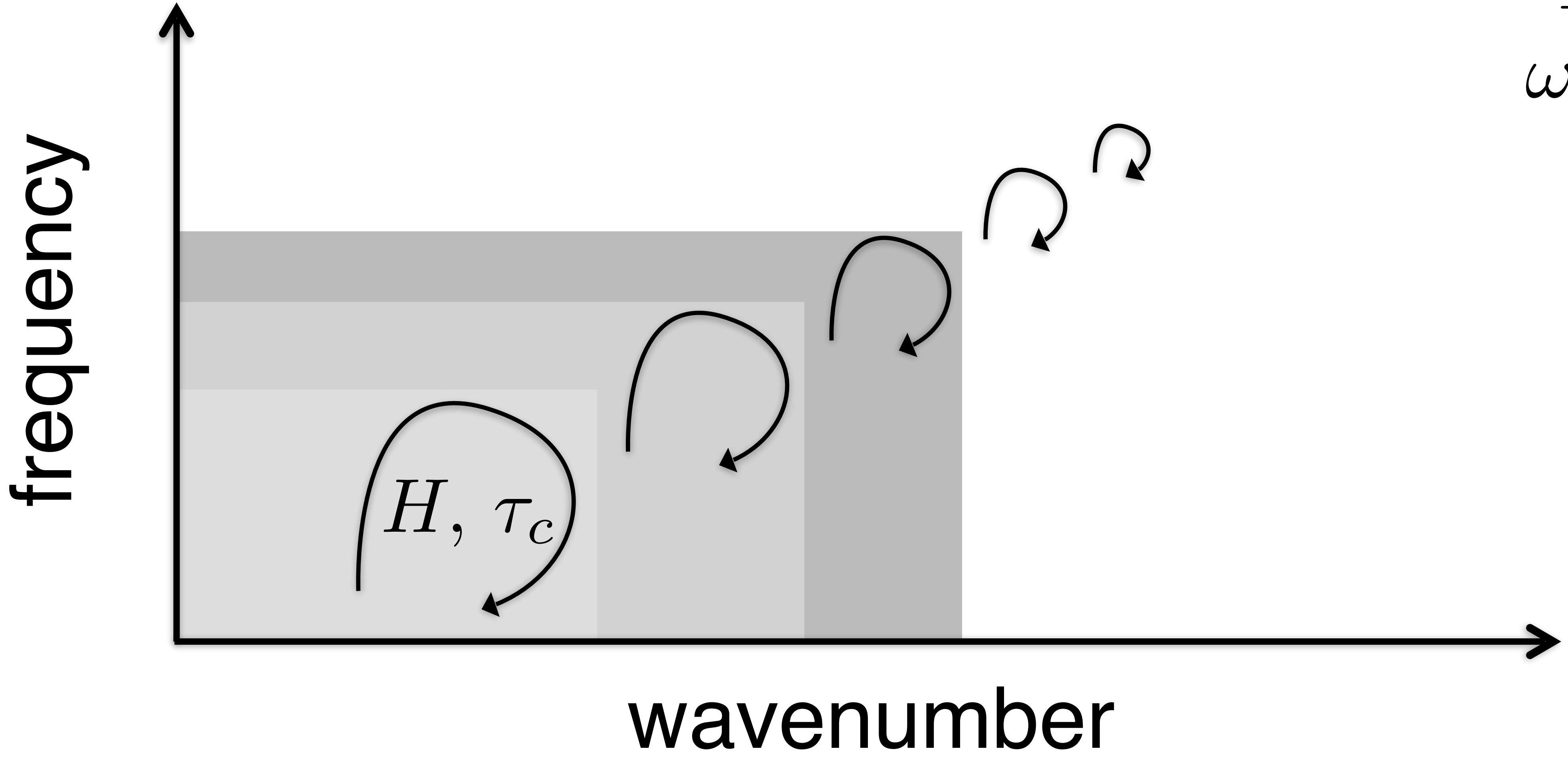

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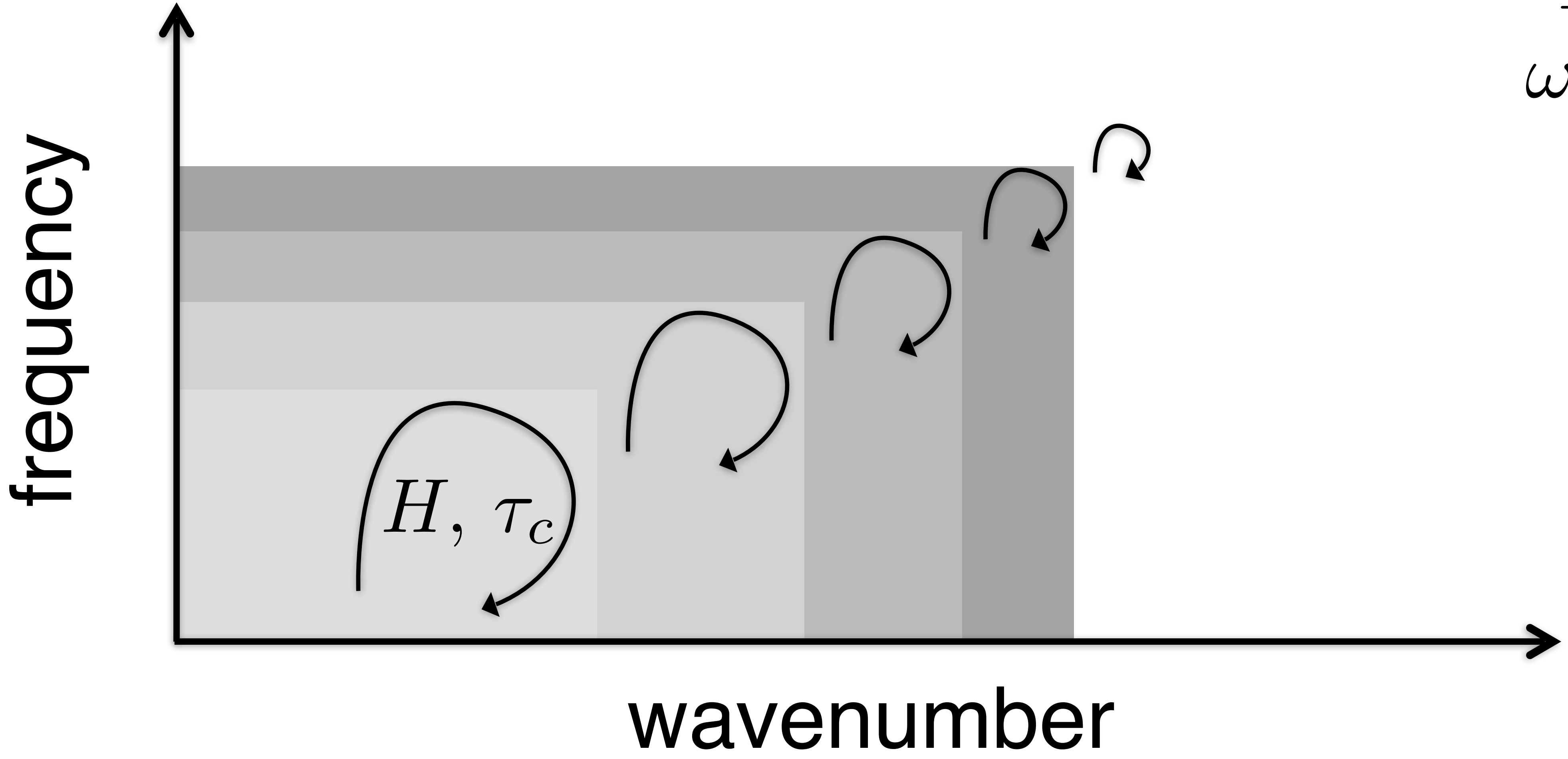

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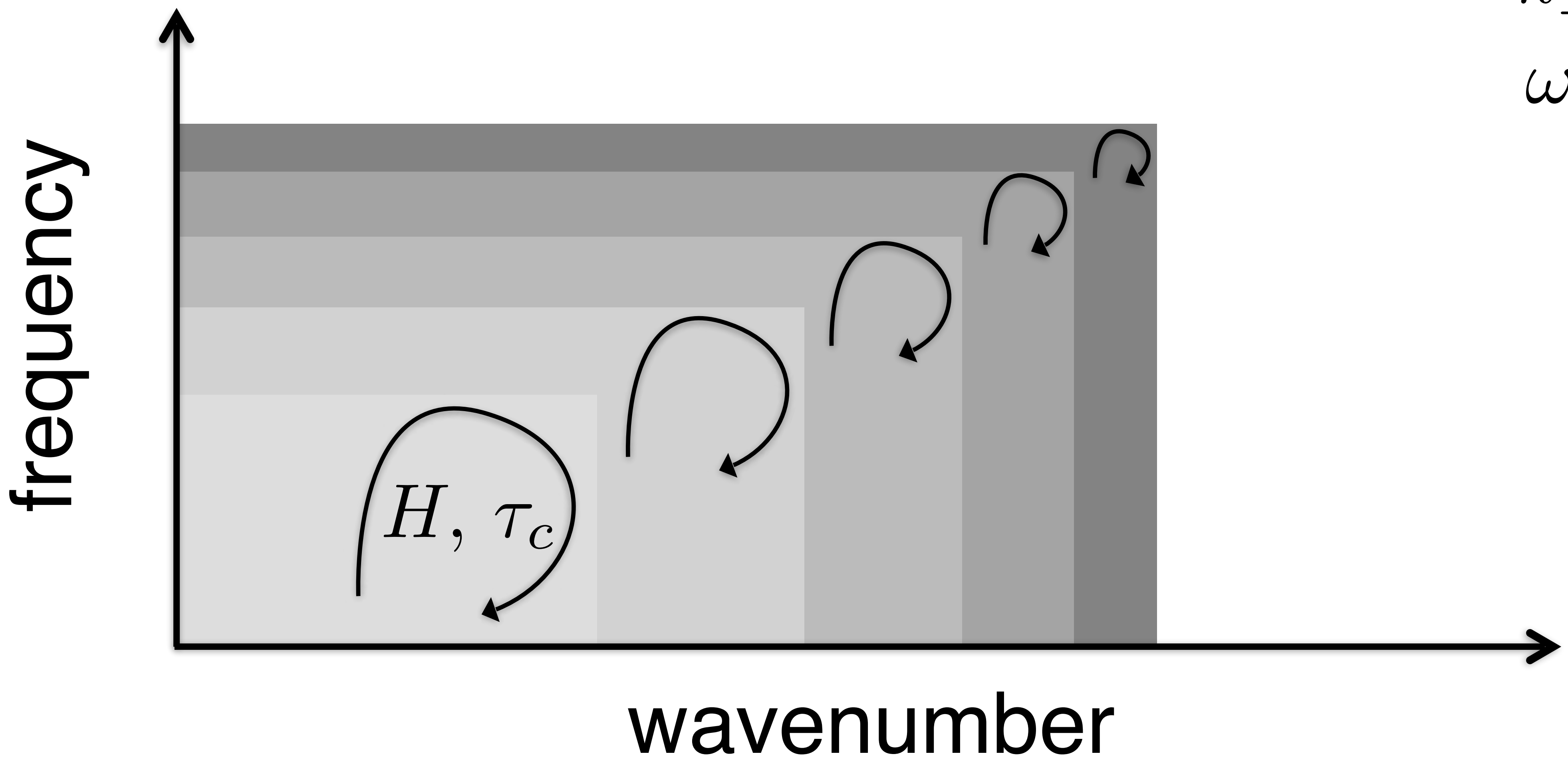


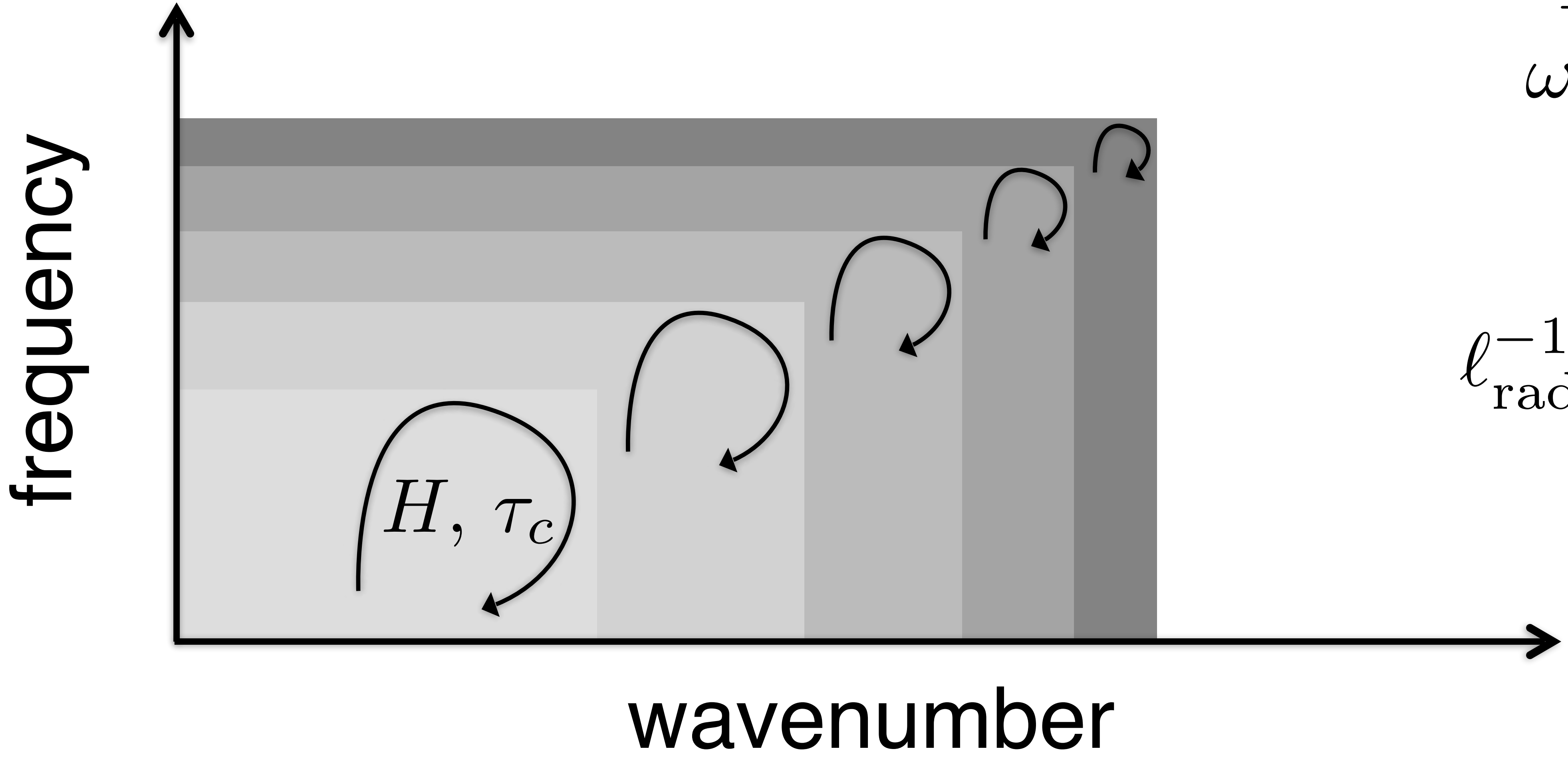
Spectrum

$$F_w \sim \rho_0 \left(\frac{h}{\tau_h}\right)^3 \frac{\omega}{N_0}(k_\perp h)^4$$

$$k_\perp h < 1$$

$$\omega \tau_h < 1$$

$$\ell_{\rm rad}^{-1} \sim \frac{k_\perp^3}{\omega^4}$$



Spectrum

$$F_w \sim \rho_0 \left(\frac{h}{\tau_h}\right)^3 \frac{\omega}{N_0} (k_\perp h)^4$$

$$k_\perp h < 1$$

$$\omega \tau_h < 1$$

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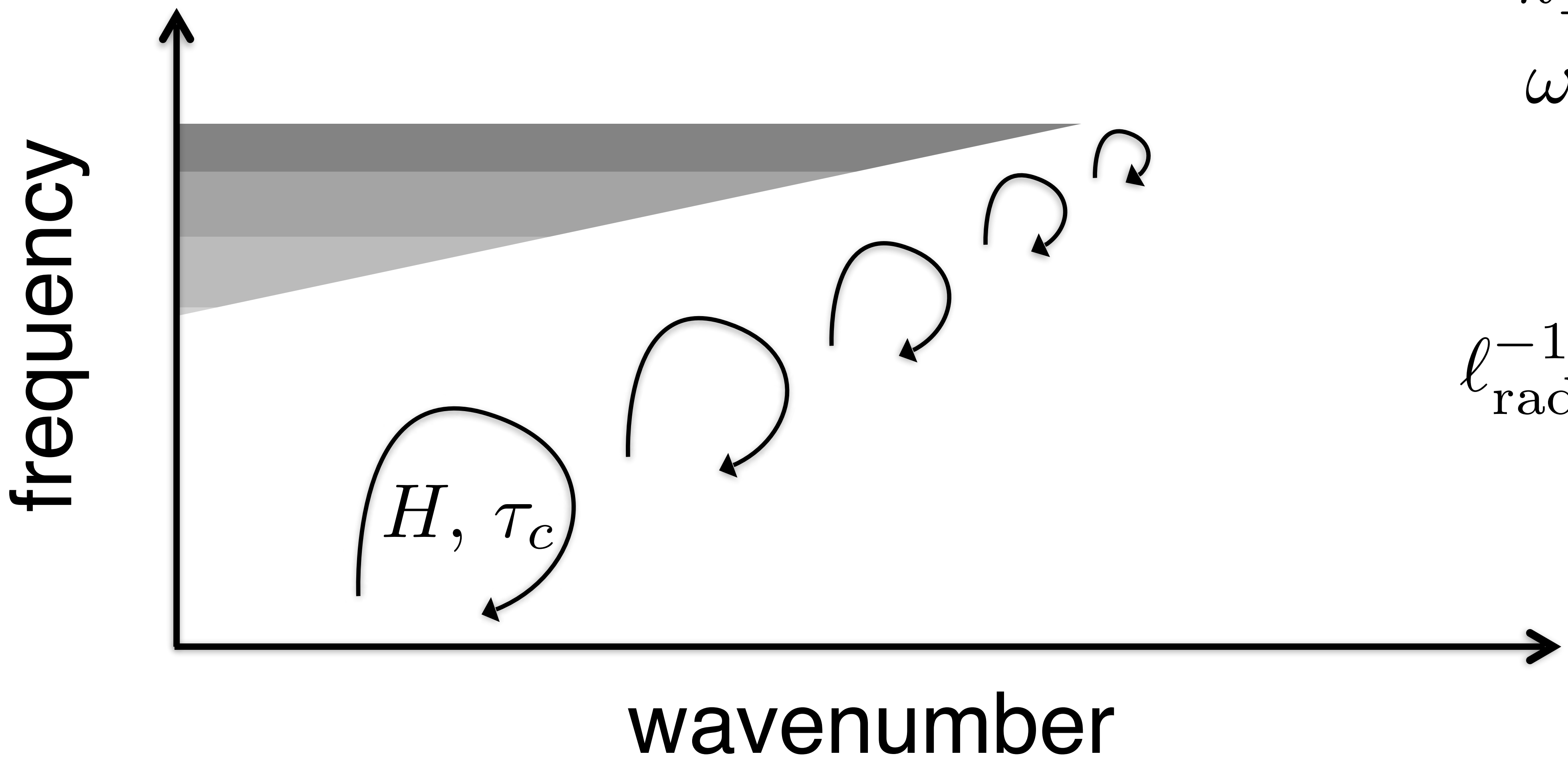

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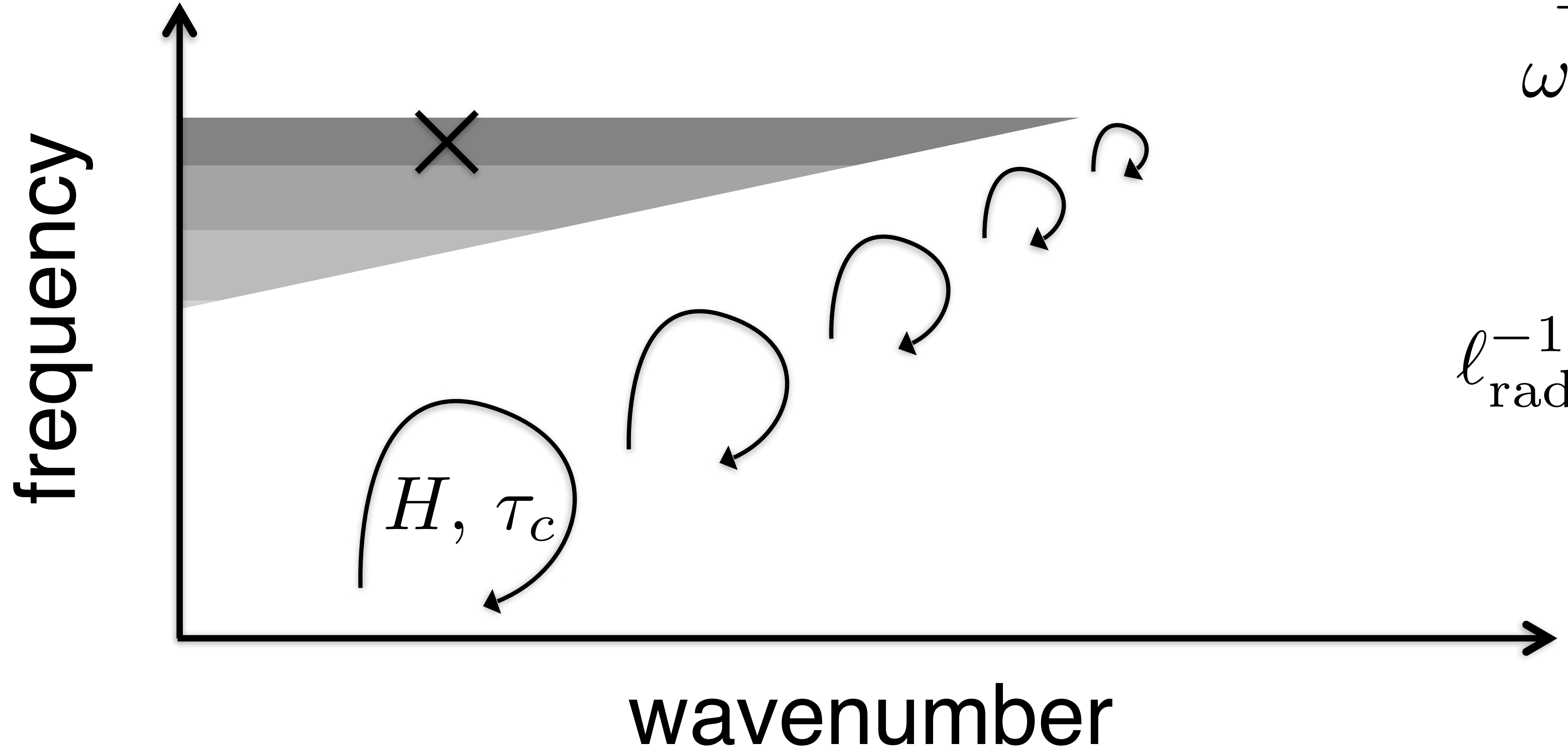

$$\ell_{\rm rad}^{-1} \sim \frac{k_\perp^3}{\omega^4}$$

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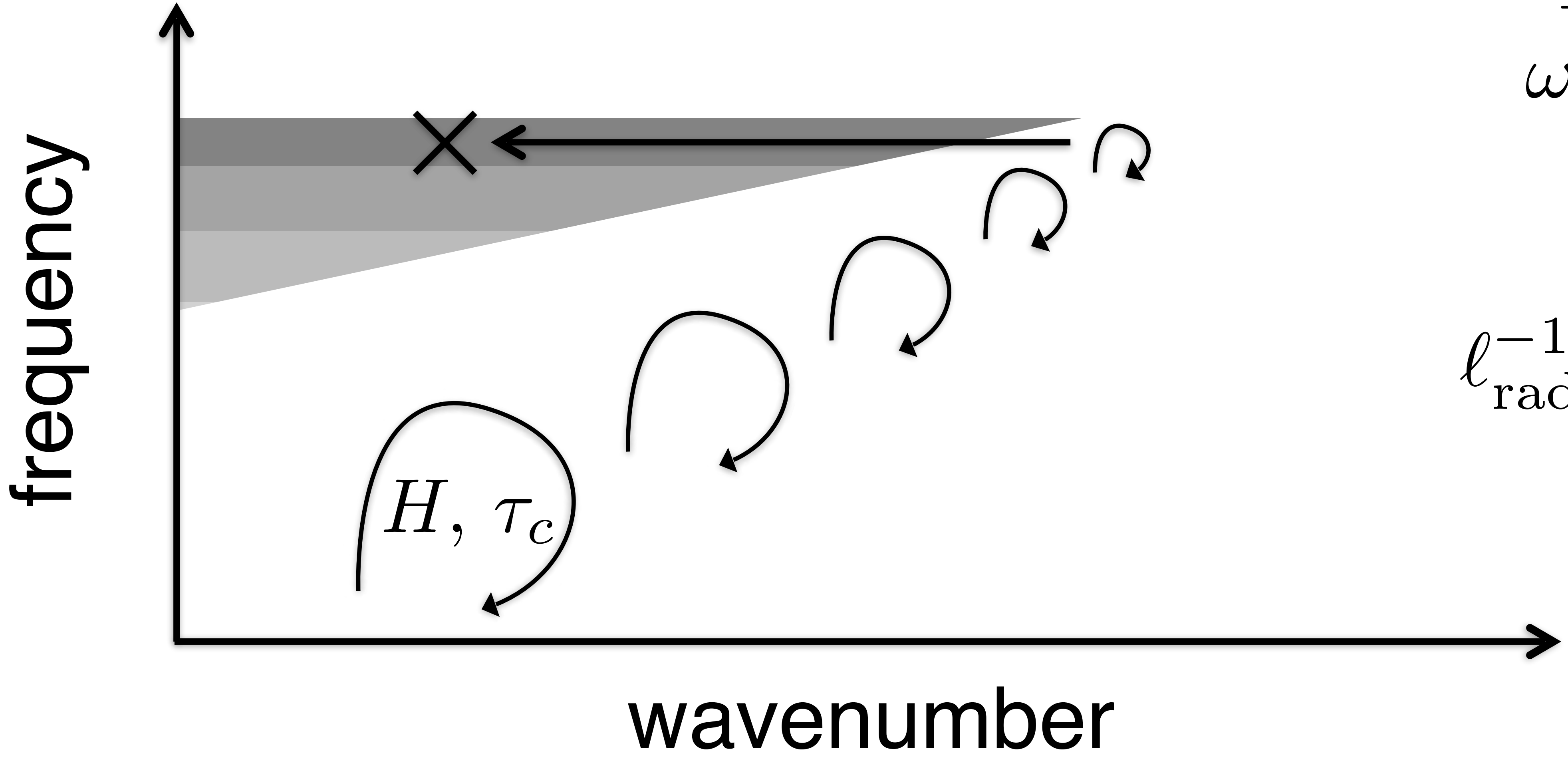

$$\ell_{\rm rad}^{-1} \sim \frac{k_\perp^3}{\omega^4}$$

Spectrum

$$F_w \sim F_c \frac{1}{N\tau_c} \left(\omega\tau_c\right)^{-13/2} \left(k_\perp H\right)^4$$

Spectrum

$$\boxed{F_w \sim F_c \frac{1}{N\tau_c} \left(\omega\tau_c\right)^{-13/2} \left(k_\perp H\right)^4}$$

Key Predictions:

Spectrum

$$\boxed{F_w \sim F_c \frac{1}{N\tau_c} \left(\omega\tau_c\right)^{-13/2} \left(k_\perp H\right)^4}$$

Key Predictions: $F_w \sim \omega^{-13/2}$

Spectrum

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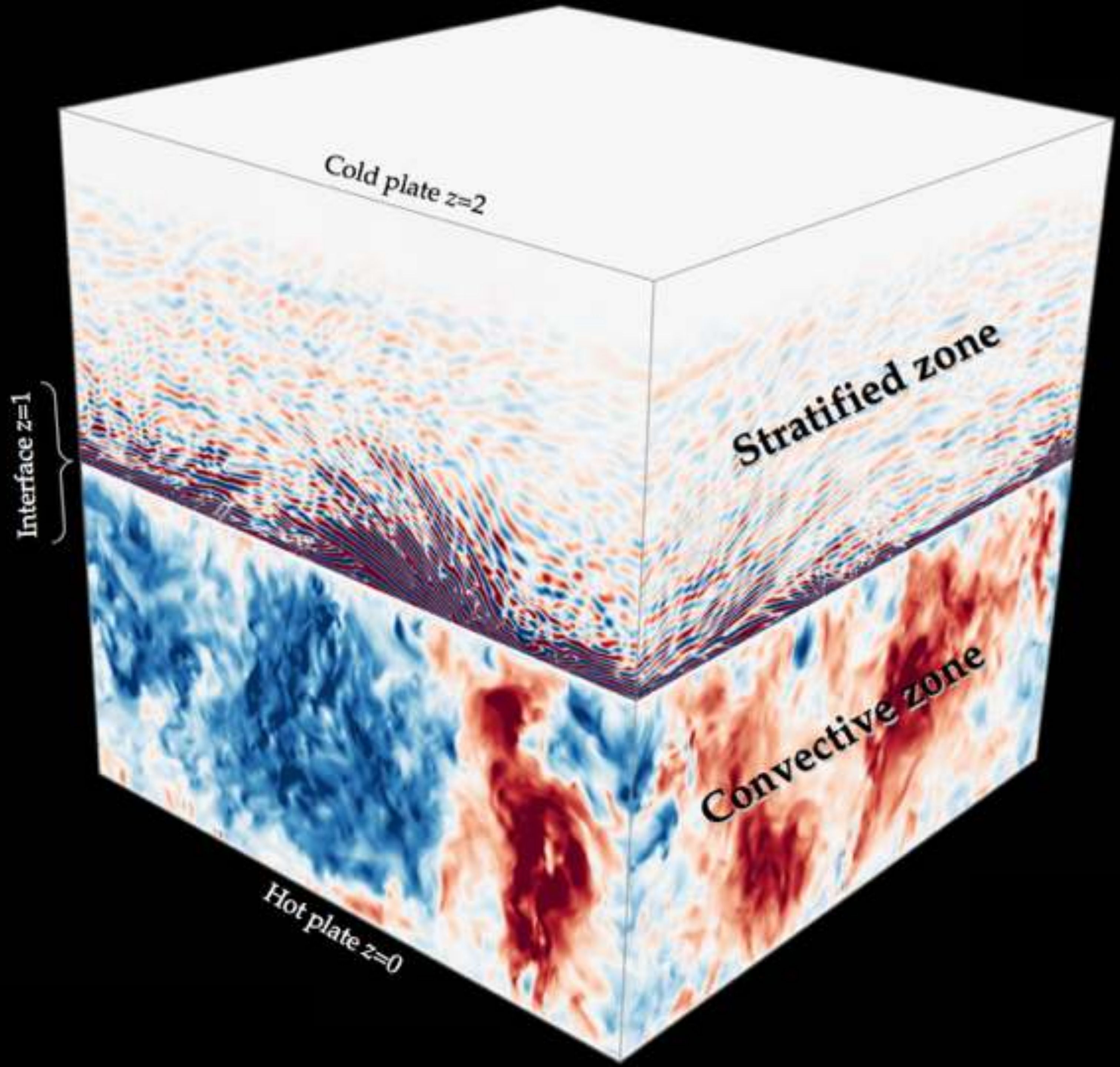
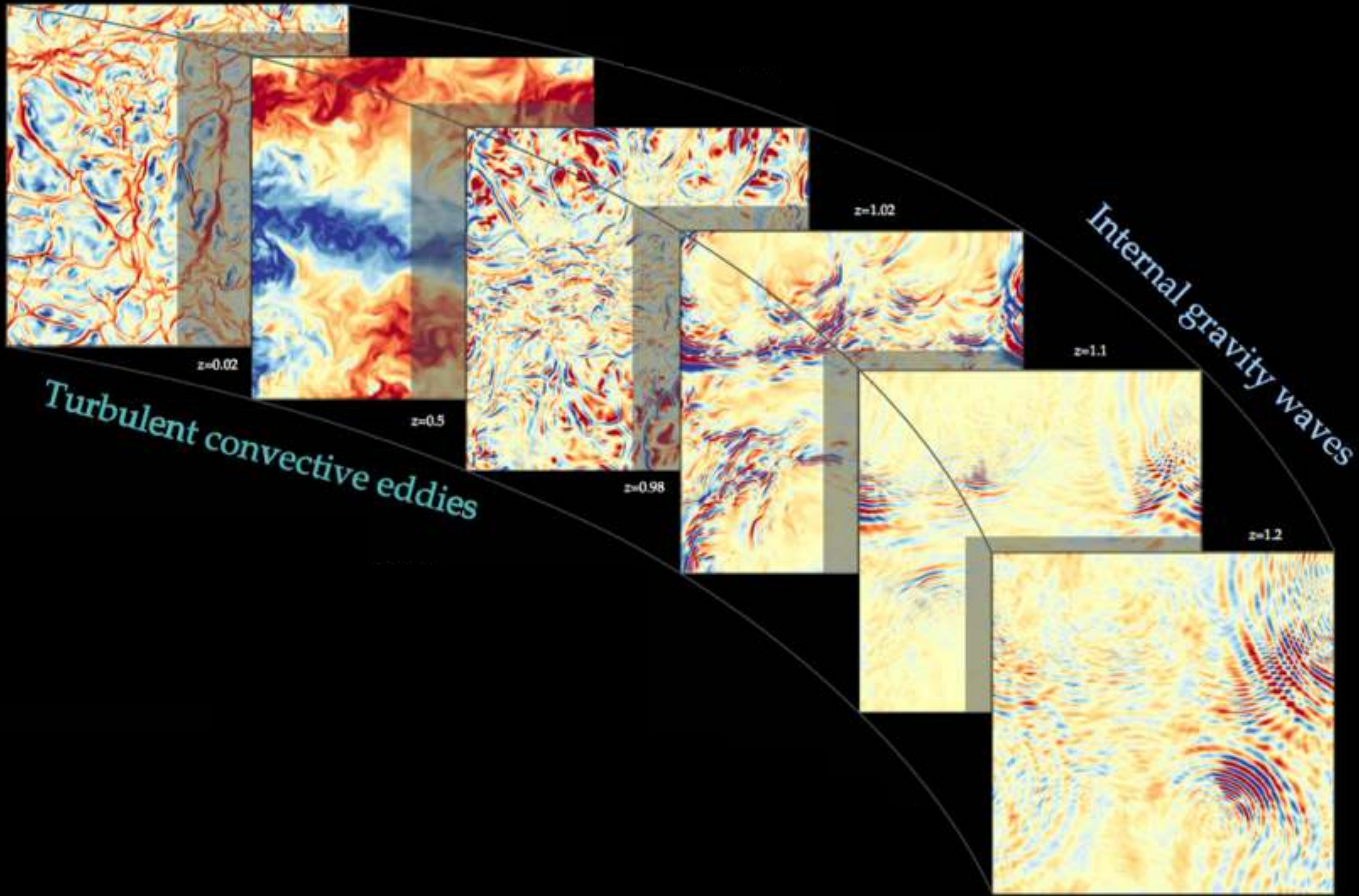
Key Predictions:

$$F_w \sim \omega^{-13/2}$$

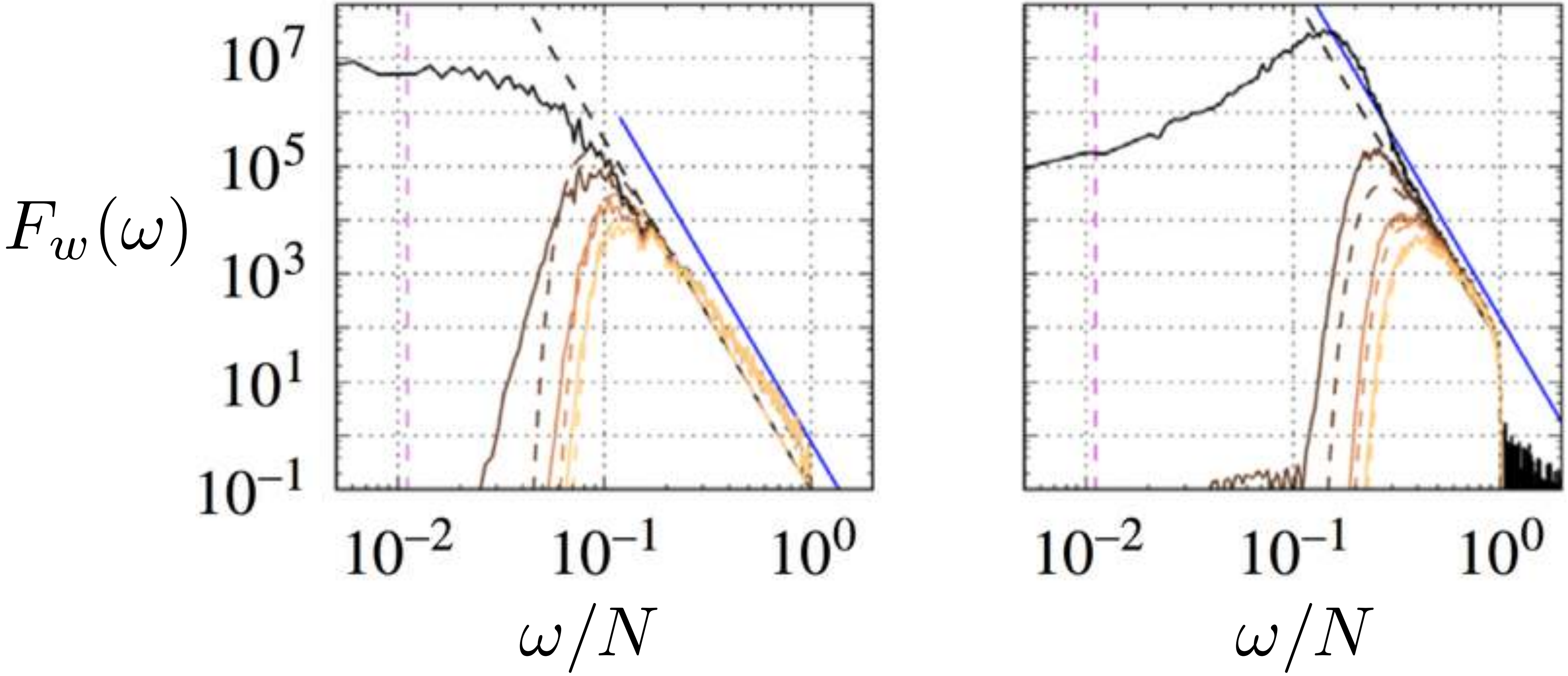
$$F_w \sim k_\perp^4$$



Cold plate z=2
Stratified zone
Interface z=1
Convective zone
Hot plate z=0

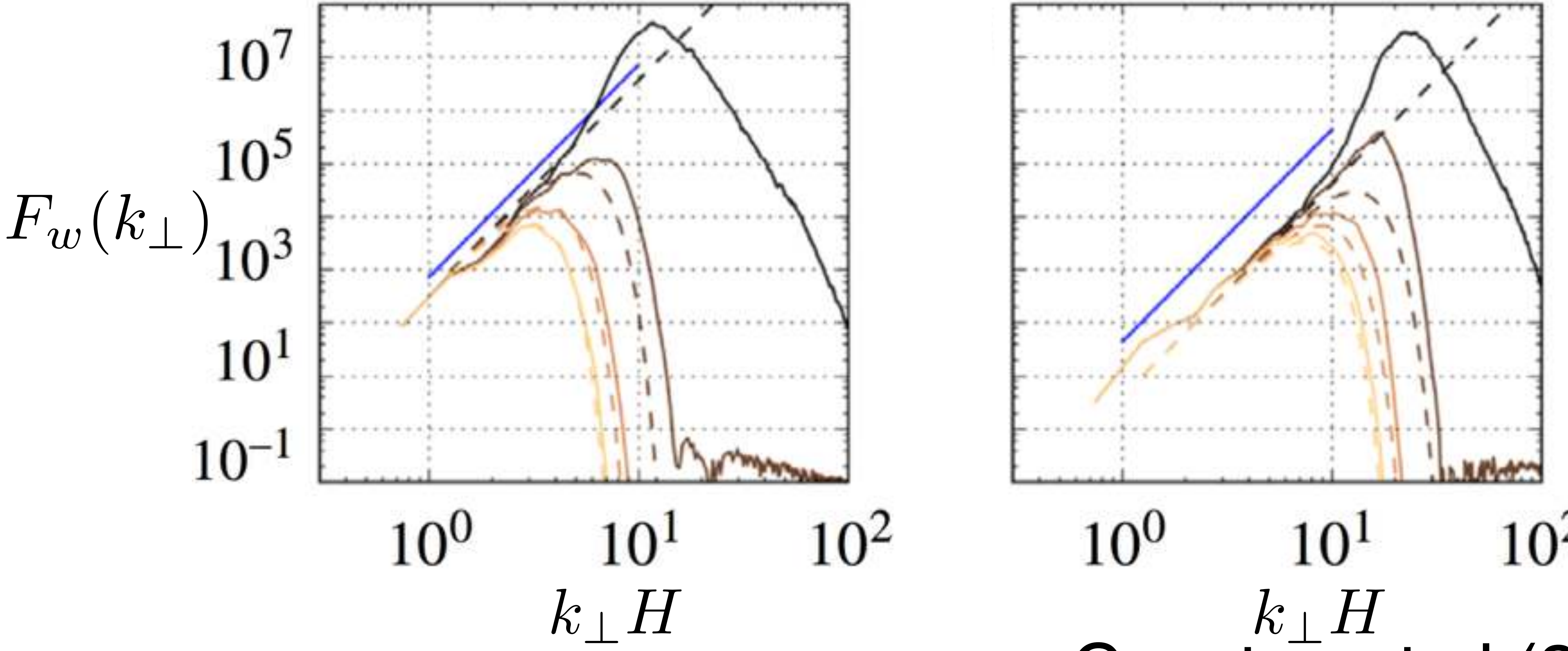

z=0.02
z=0.5
z=0.98
z=1.02
z=1.1
z=1.2
Turbulent convective eddies
Internal gravity waves

$$F_w \sim F_c \frac{1}{N\tau_c} \left(\omega\tau_c\right)^{-13/2} \left(k_\perp H\right)^4$$

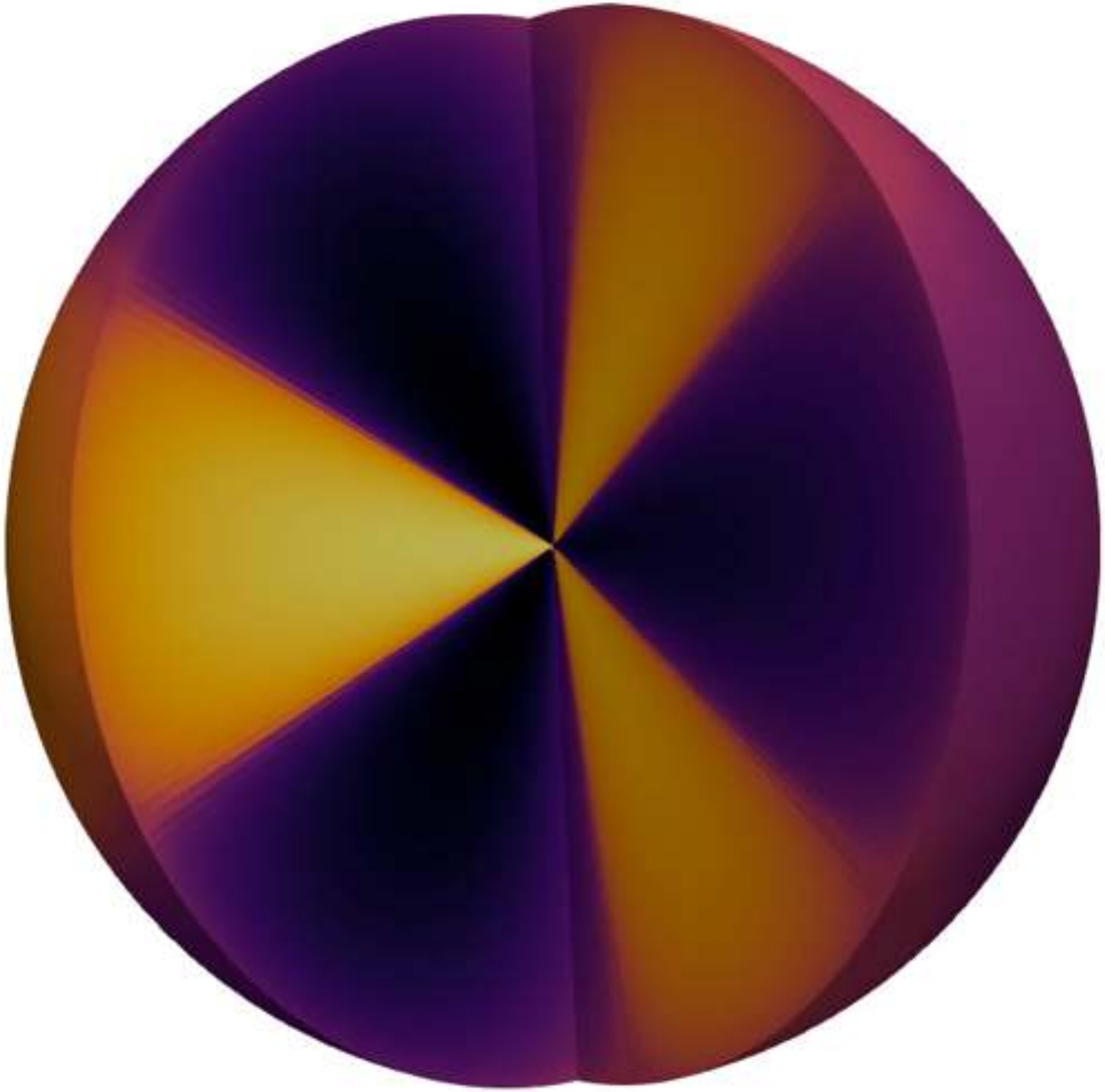


Couston et al (2018)

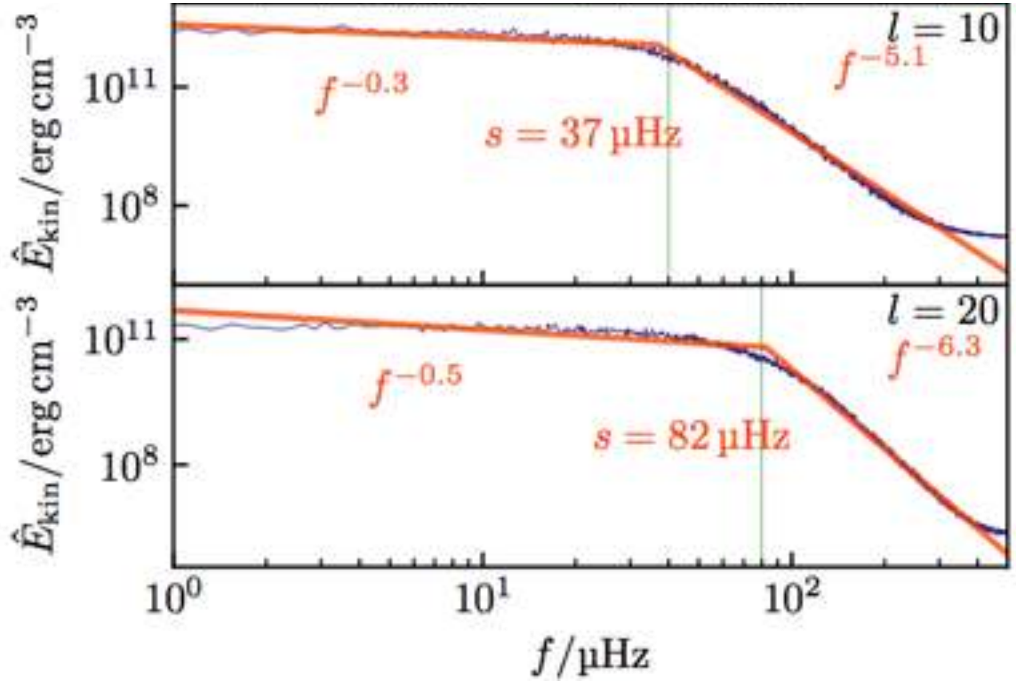
$$F_w \sim F_c \frac{1}{N\tau_c} (\omega\tau_c)^{-13/2} (k_\perp H)^4$$

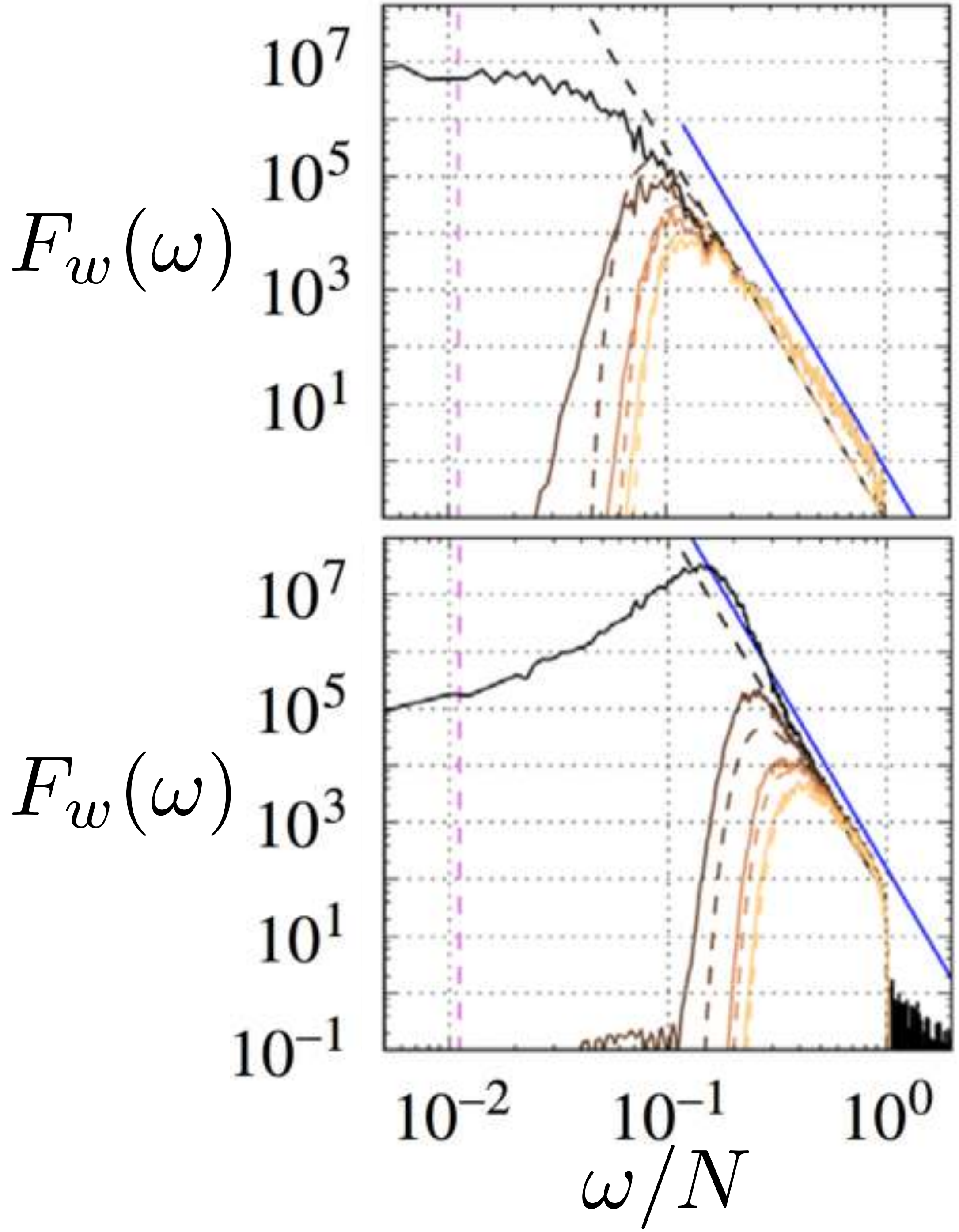


Couston et al (2018)



Edelmann et al (2019)





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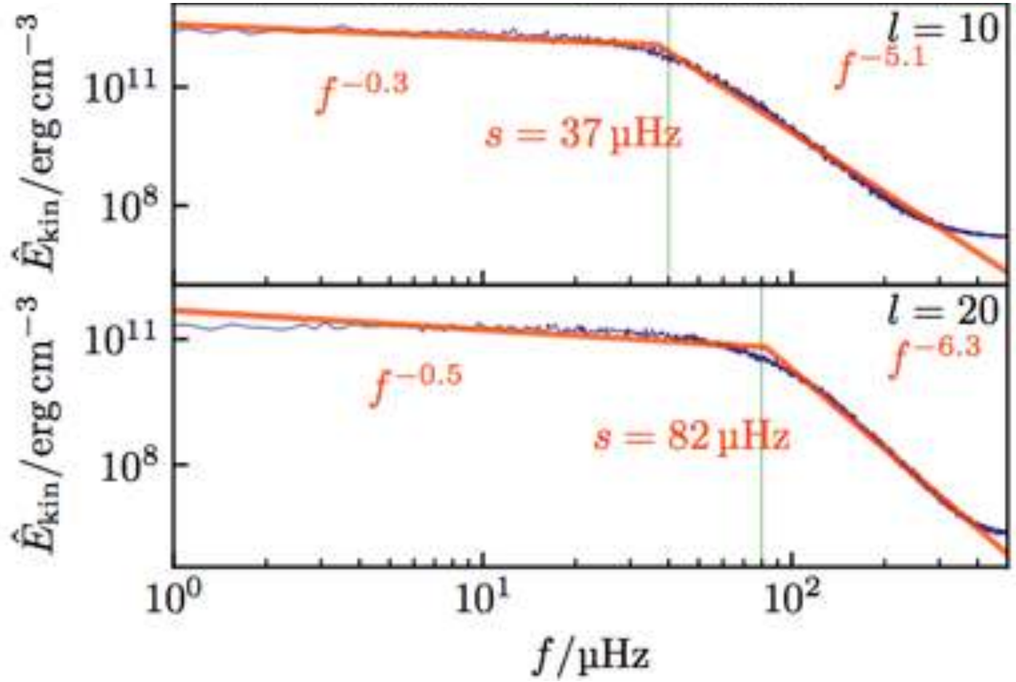
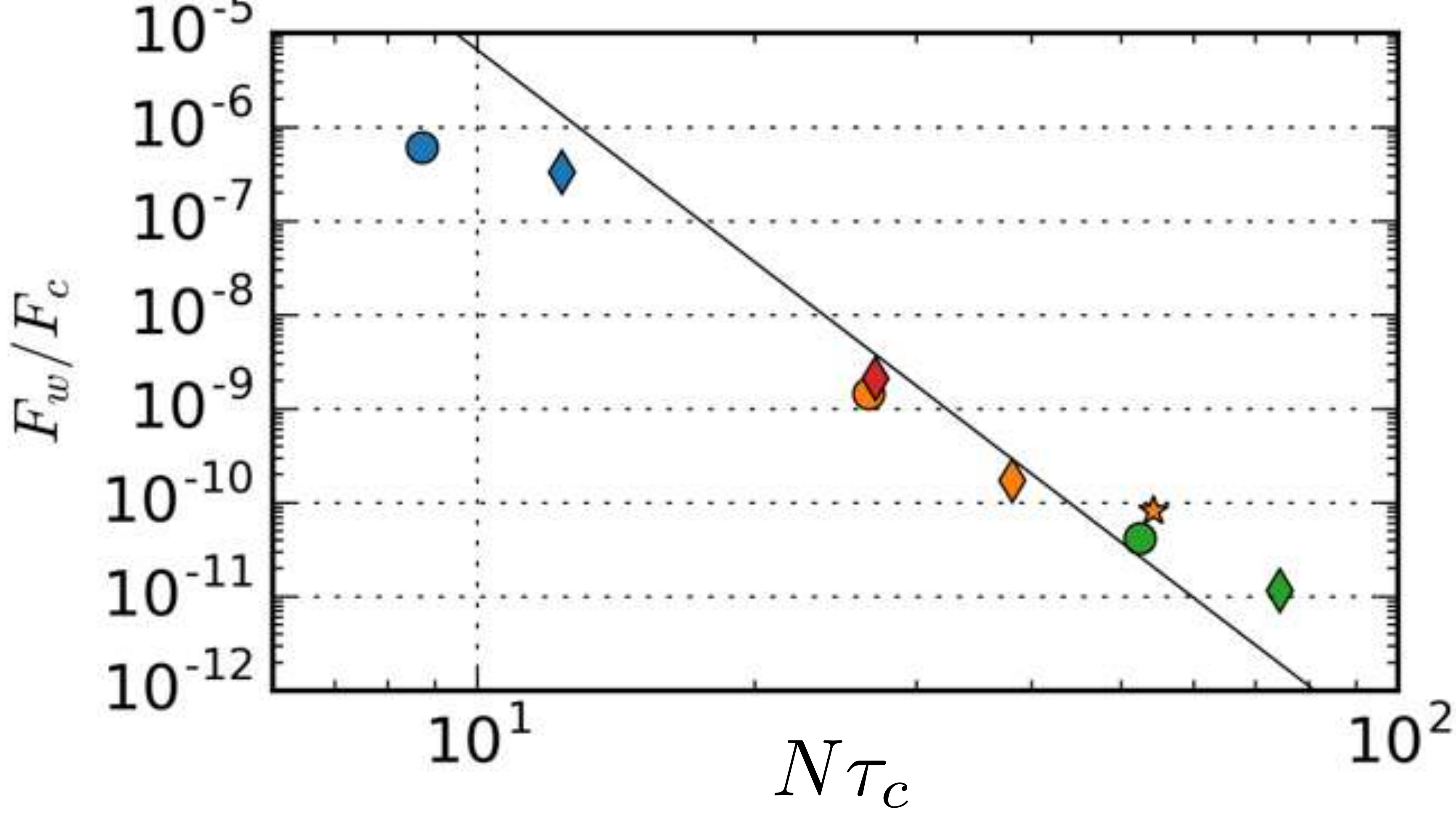


I: How?

II: Which?

III: What?



$$F_w = 2.2 F_c \frac{1}{N\tau_c} \left(\omega\tau_c\right)^{-13/2} \left(k_\perp H\right)^4$$

$$\tau_c^{-1} = 4\pi(F_c/\rho)^{1/3}/H$$

Wave amplitude

Energy injection

$$F_w$$

Energy removed

$$\frac{E_w}{\tau_d}$$

Wave amplitude

Energy injection

$$F_w$$

Energy removed

$$\frac{E_w}{\tau_d}$$

$$E_w = F_w \tau_d$$

Wave amplitude

Energy injection

$$F_w$$

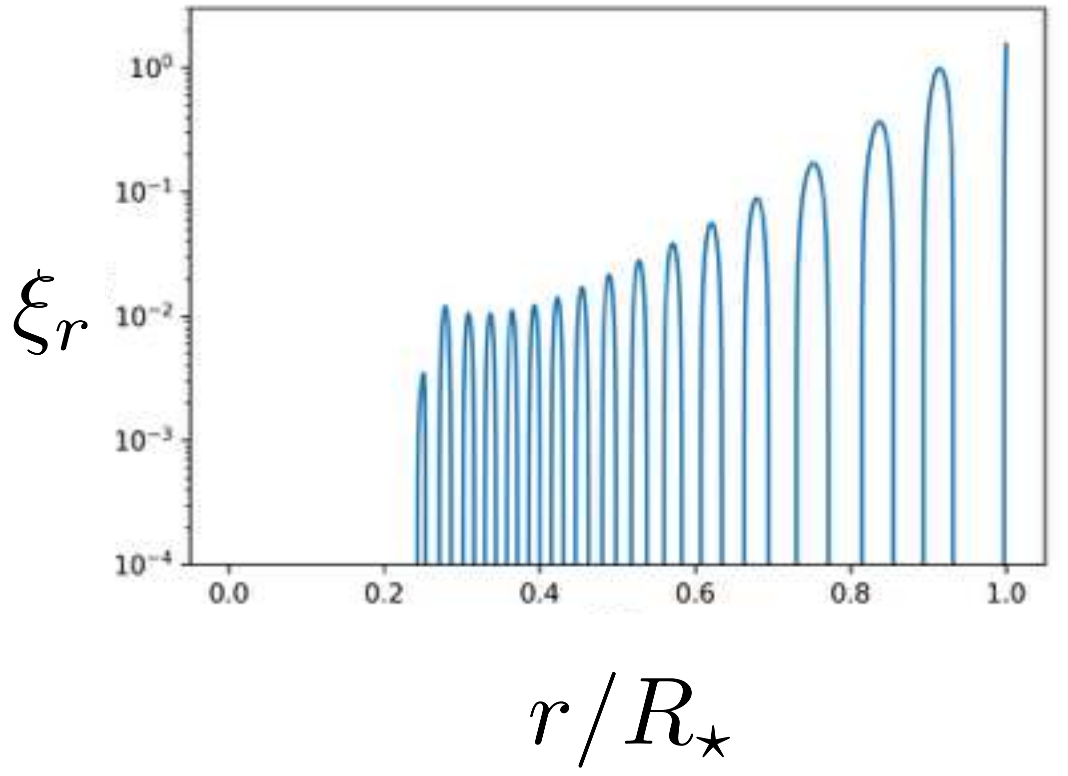
Energy removed

$$\frac{E_w}{\tau_d}$$

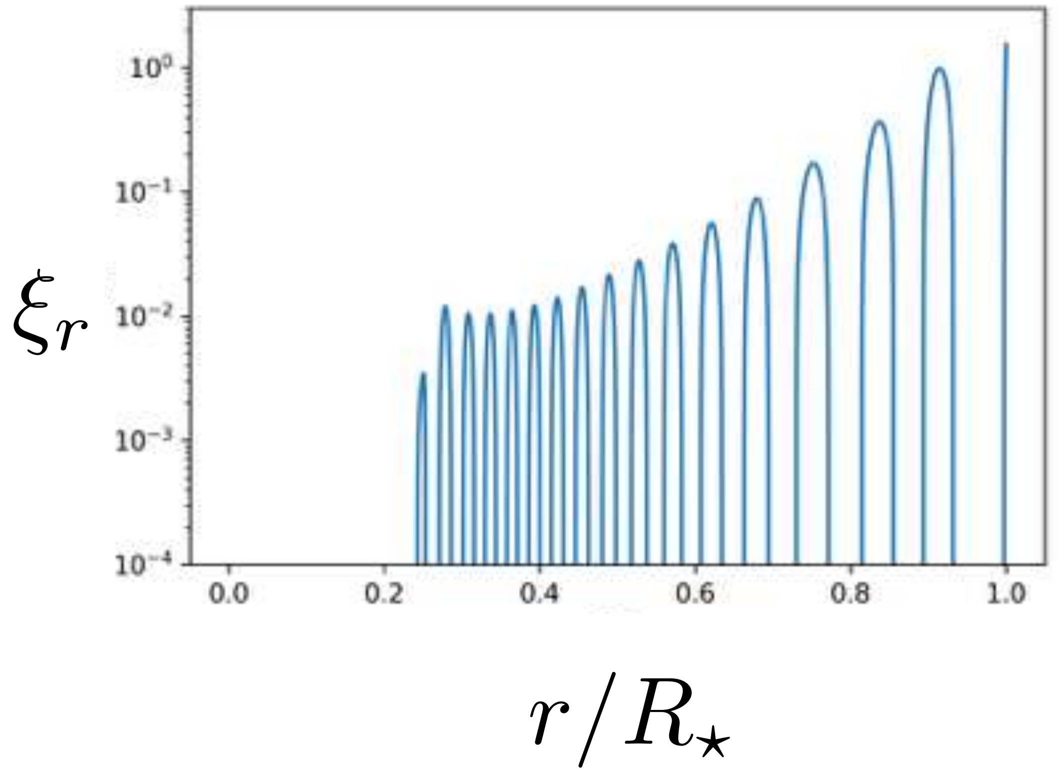
$$E_w = F_w \tau_d$$

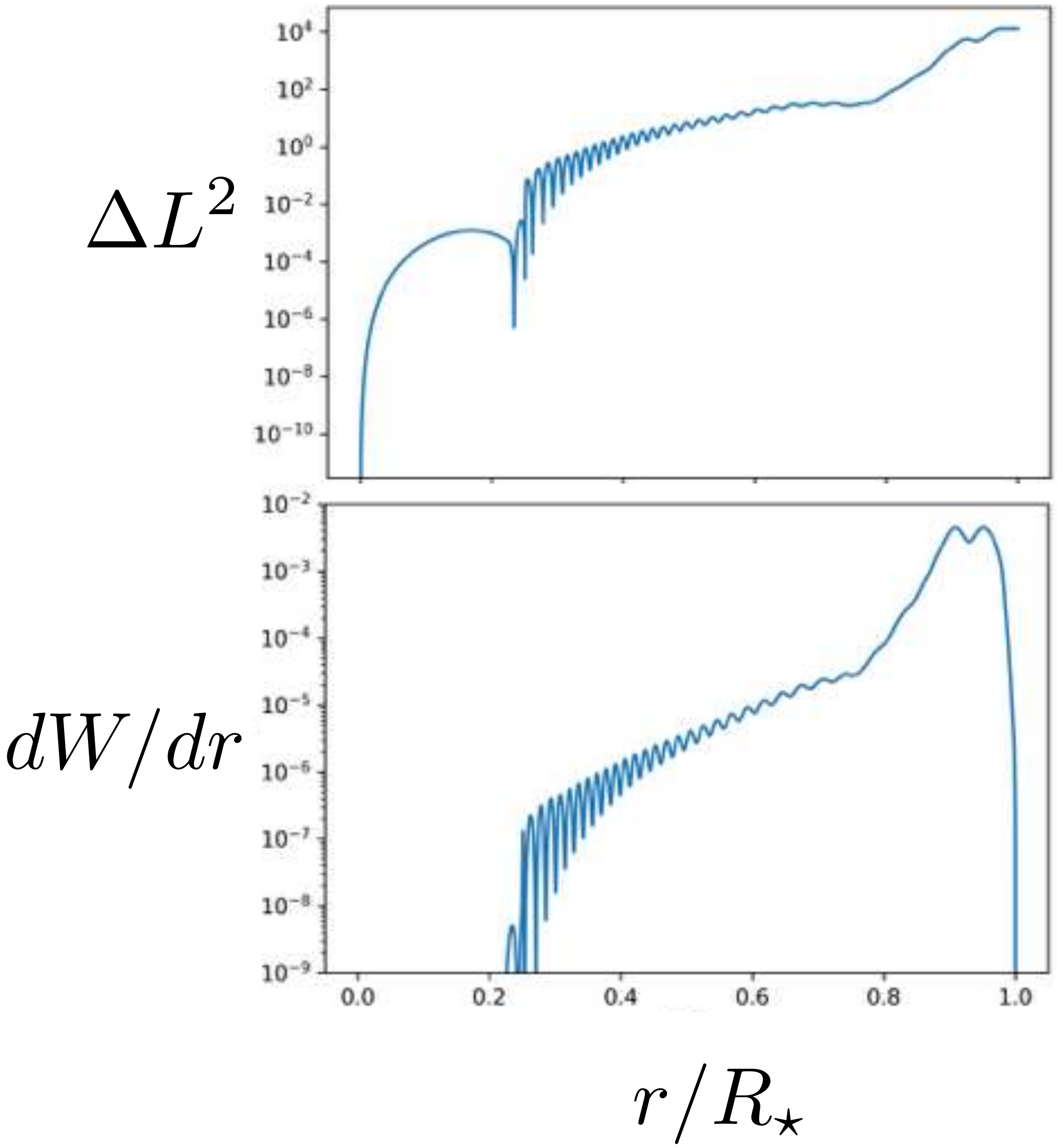
$$\Delta L^2 = E_w \, L(r = R_\star)^2$$

Wave amplitude

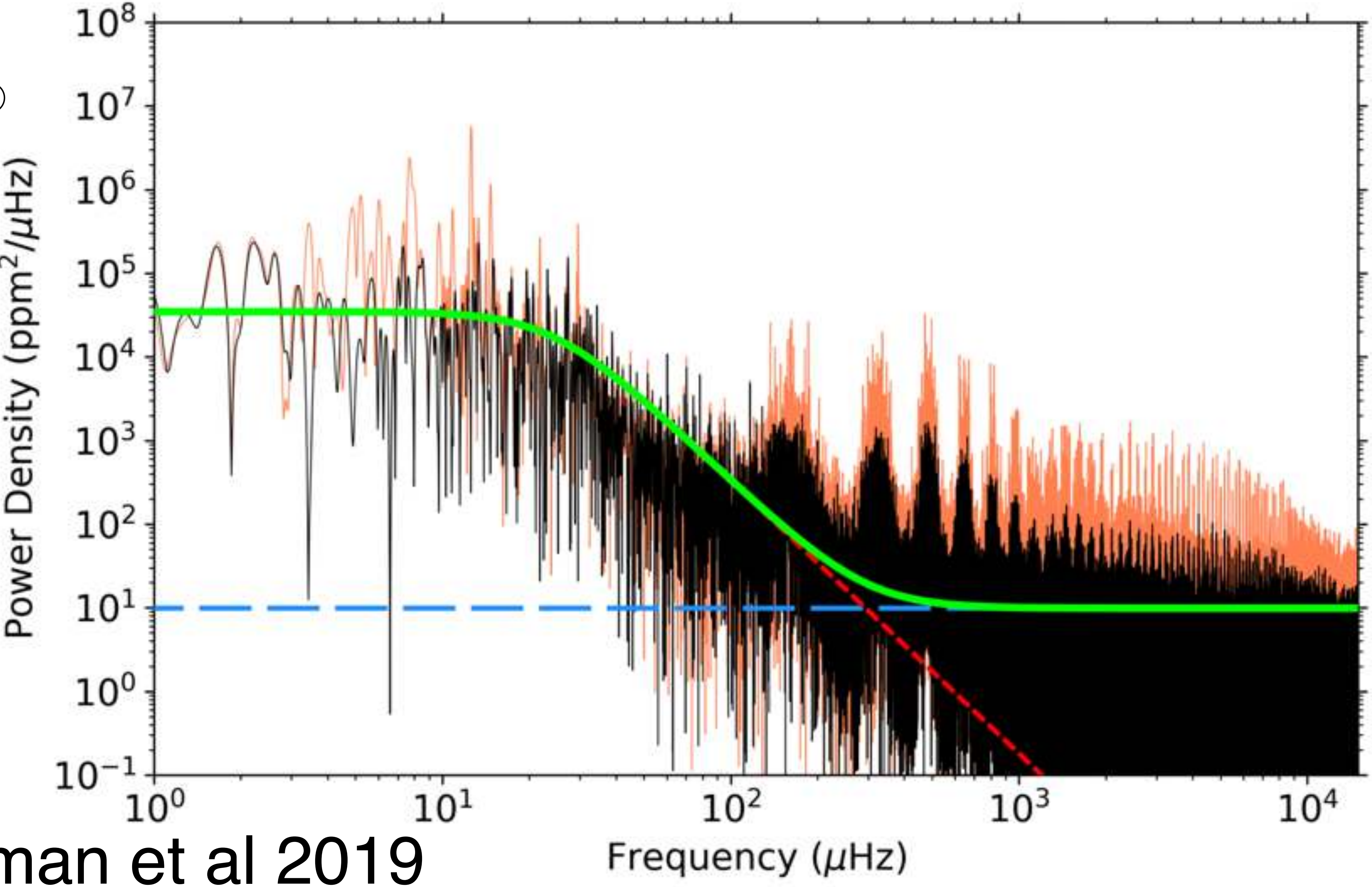


Wave amplitude

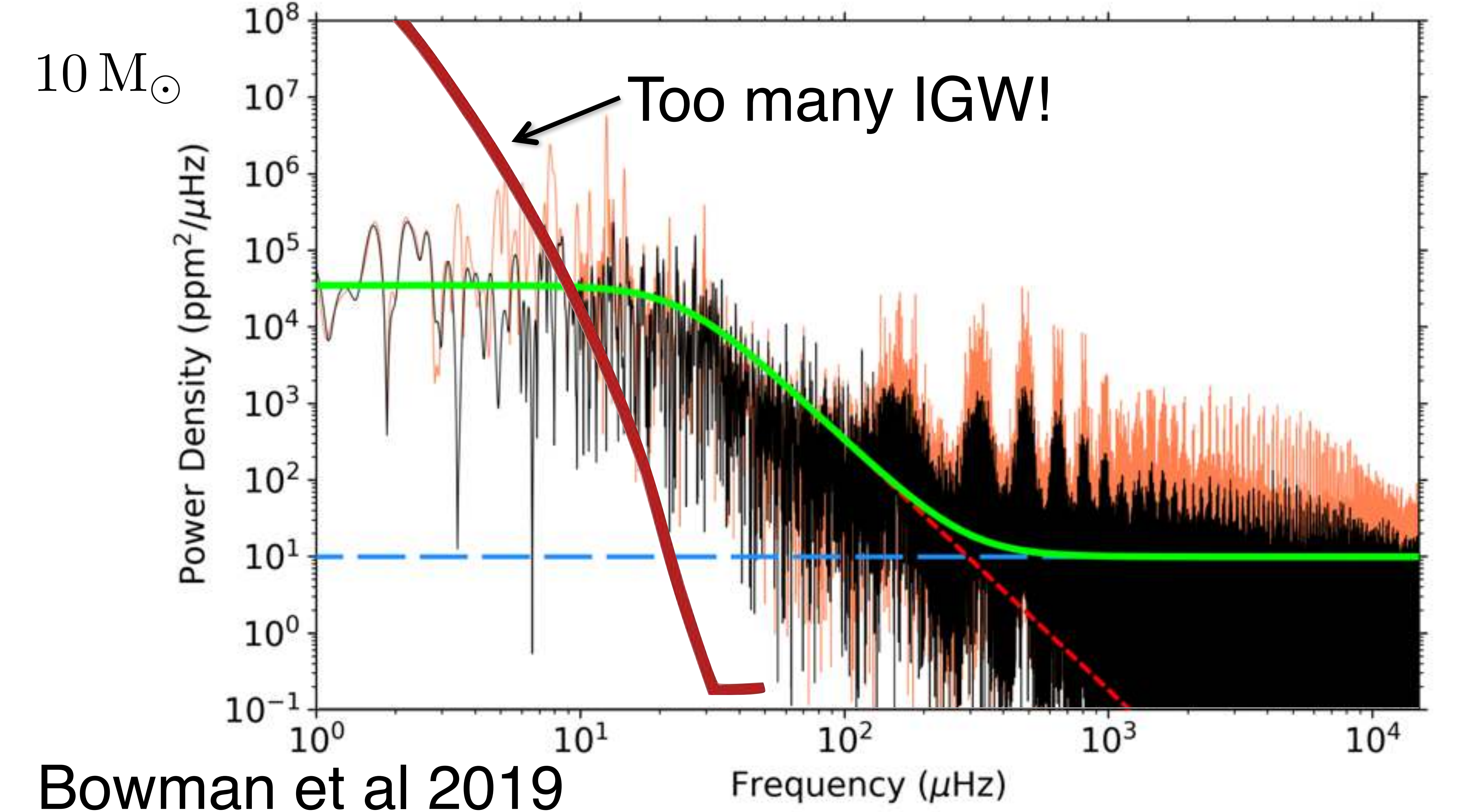




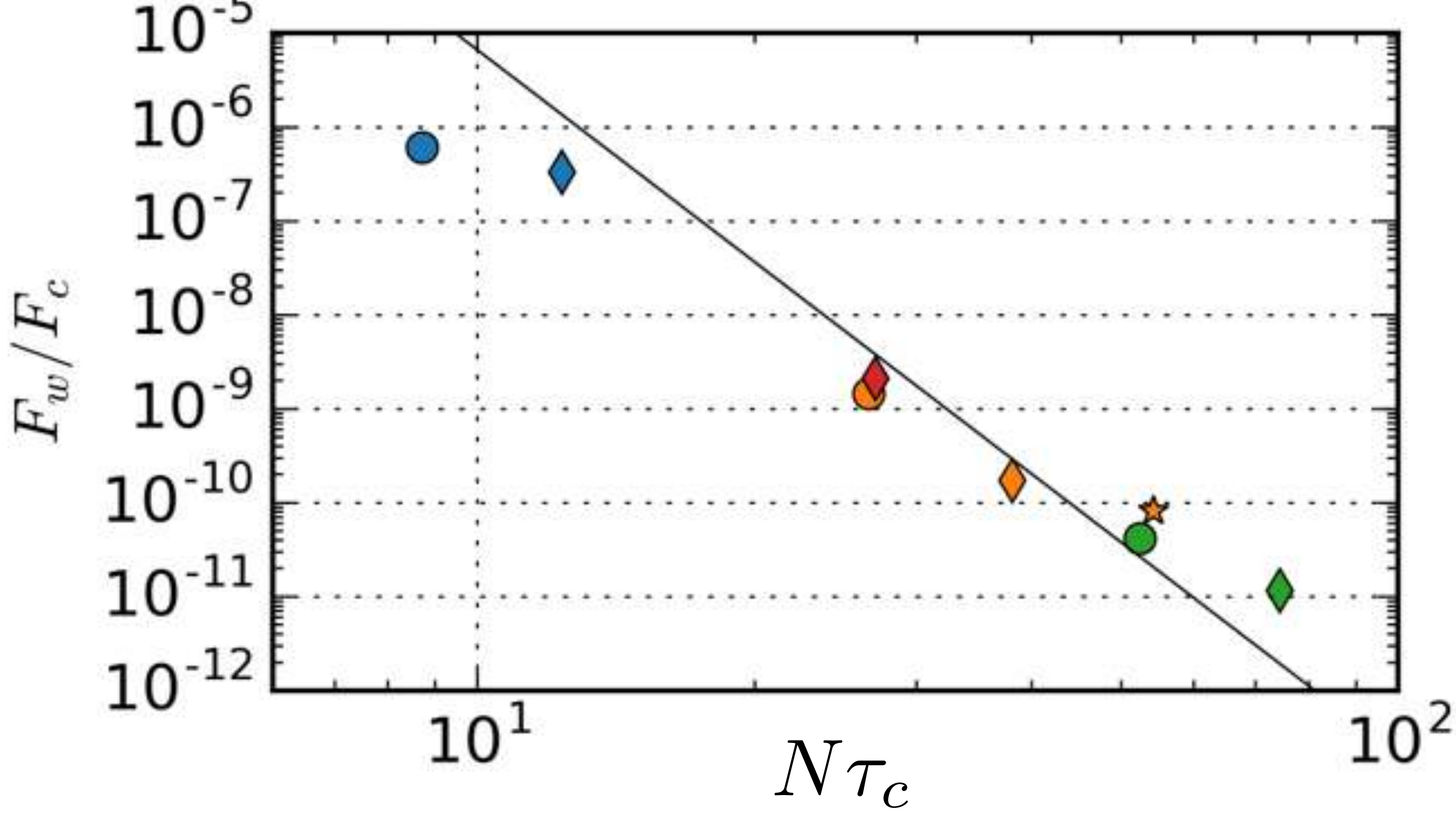
$10\,\mathrm{M}_\odot$



Bowman et al 2019

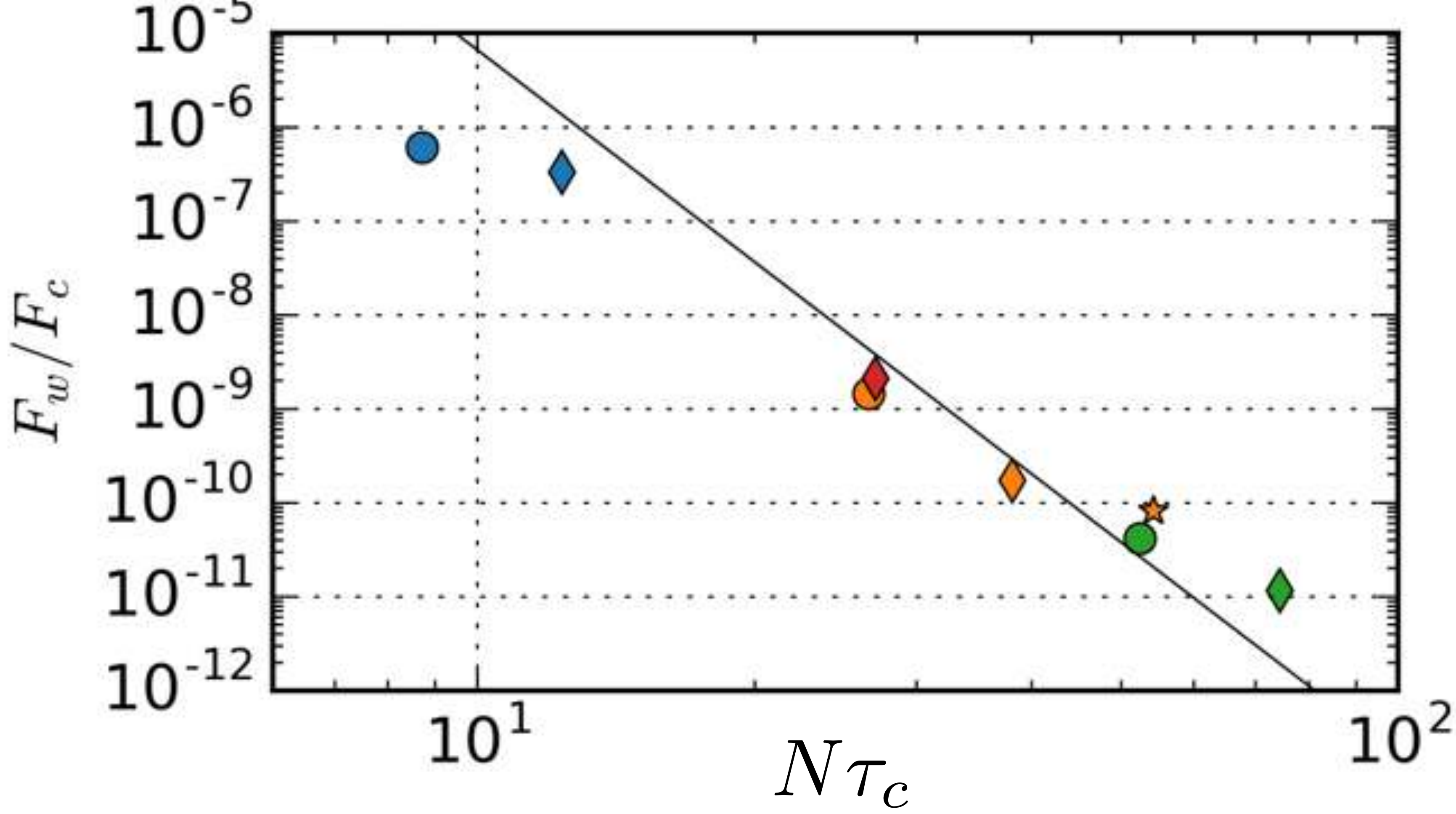


Bowman et al 2019



$$F_w = 2.2 F_c \frac{1}{N\tau_c} (\omega\tau_c)^{-13/2} (k_\perp H)^4$$

$$\tau_c^{-1} = 4\pi (F_c/\rho)^{1/3}/H$$



$$\frac{\Delta\tau_c}{\tau_c} = 2\pi$$

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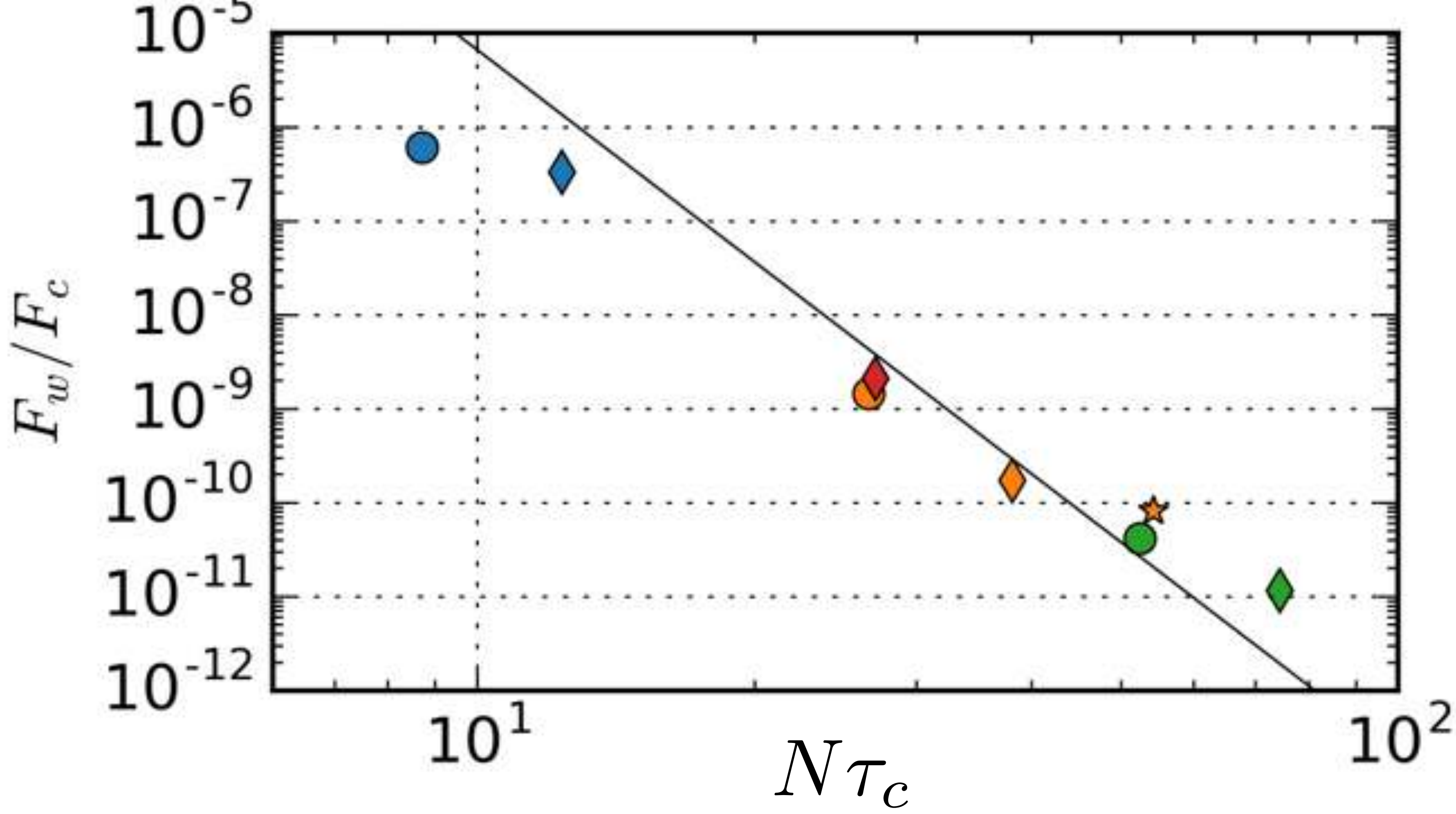


$$\frac{\Delta\tau_c}{\tau_c} = 2\pi$$

$$\downarrow$$

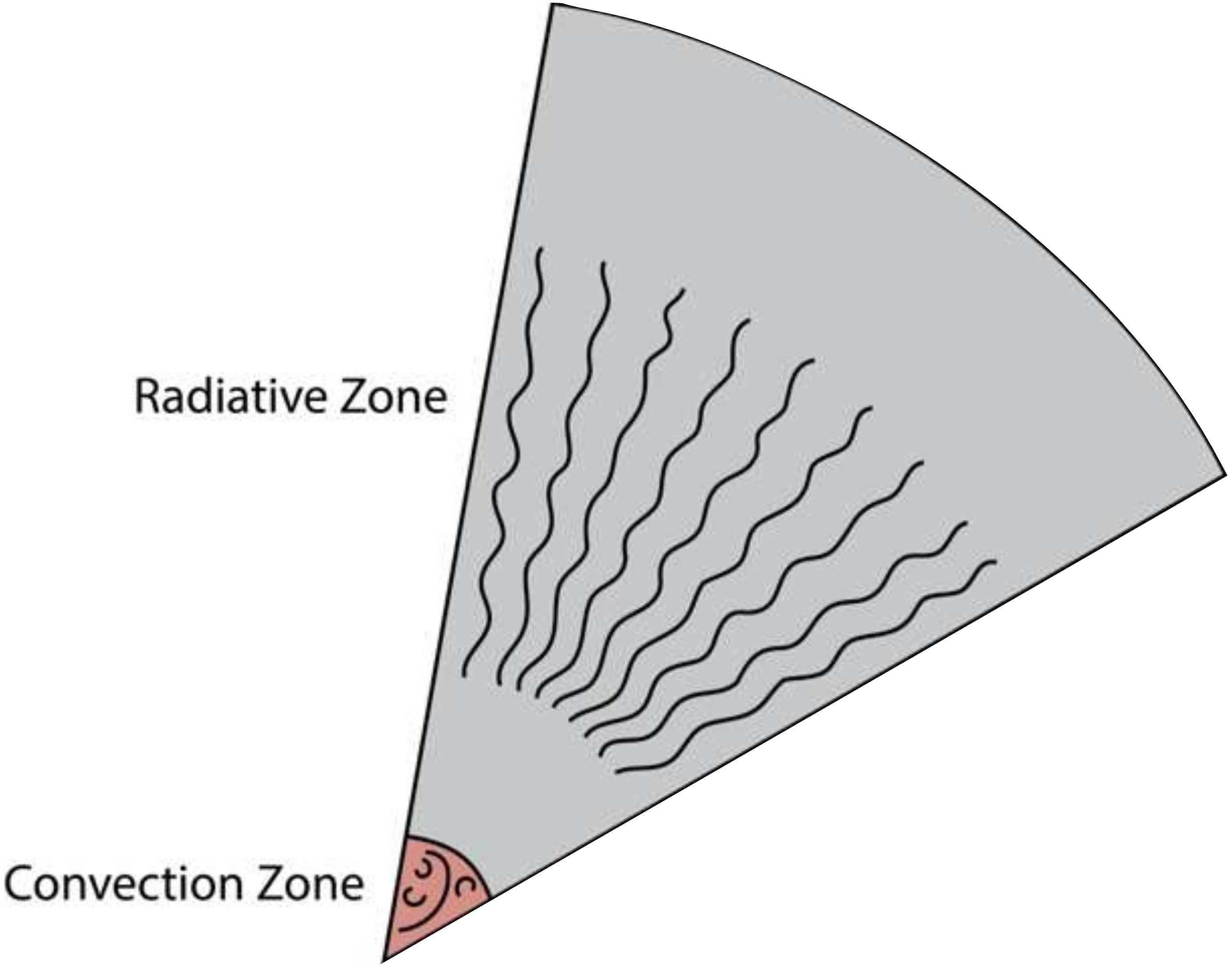
$$\frac{\Delta F_w}{F_w} = 10^6$$

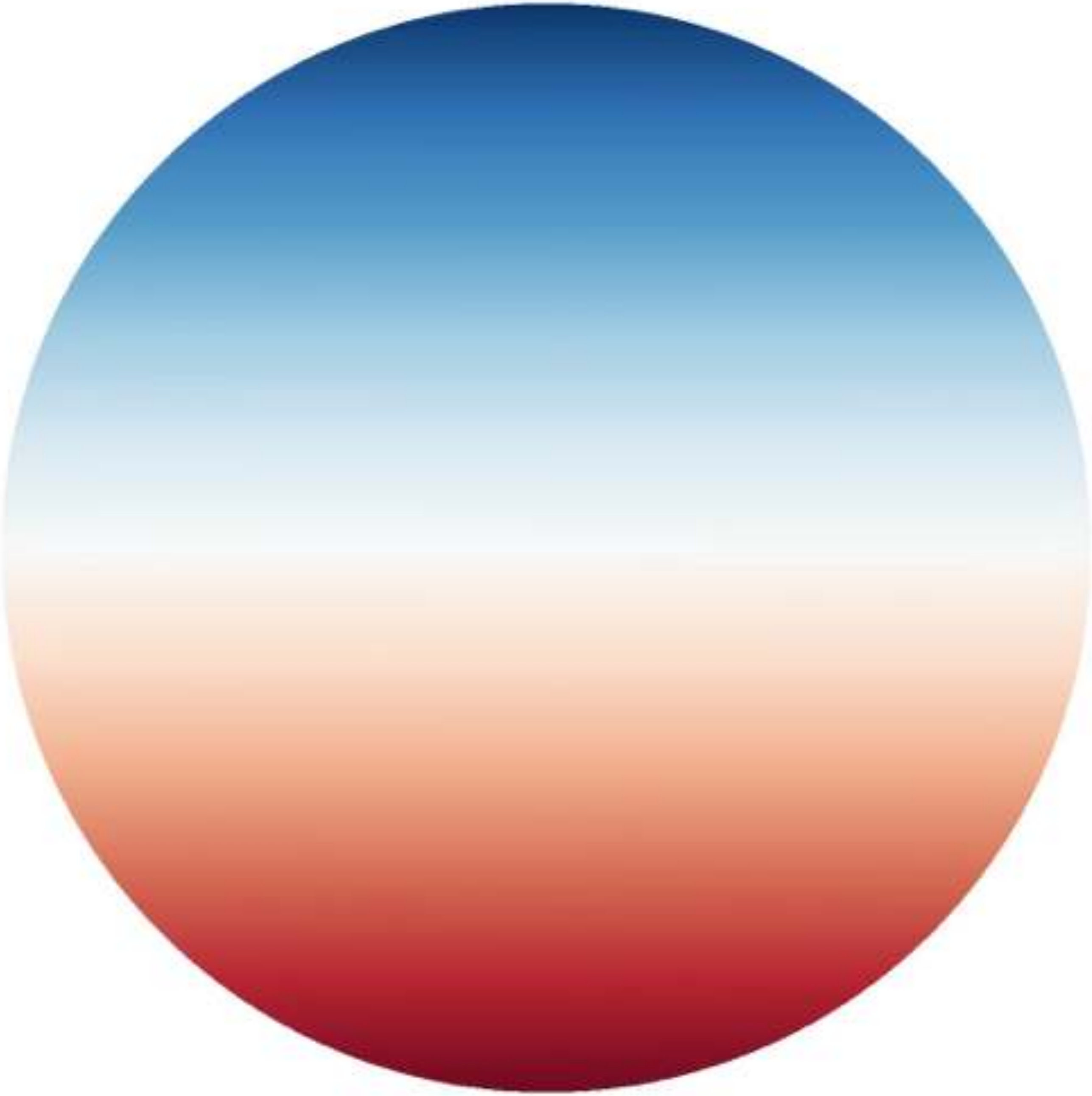
$$F_w = 2.2 F_c \frac{1}{N\tau_c} \left(\omega\tau_c\right)^{-13/2} \left(k_\perp H\right)^4$$

$$\tau_c^{-1} = 4\pi (F_c/\rho)^{1/3}/H$$

Possible differences

- Rotation
- Magnetic Fields
- Geometry





I: How?

II: Which?

III: What?

I: How? Reynolds stresses by conv

II: Which?

III: What?

I: How? Reynolds stresses by conv

II: Which? $F \sim k^4 \omega^{-6.5}$

III: What?

I: How? Reynolds stresses by conv

II: Which? $F \sim k^4 \omega^{-6.5}$

III: What? ?????