Nonlinear States in Stratified Shear Flows

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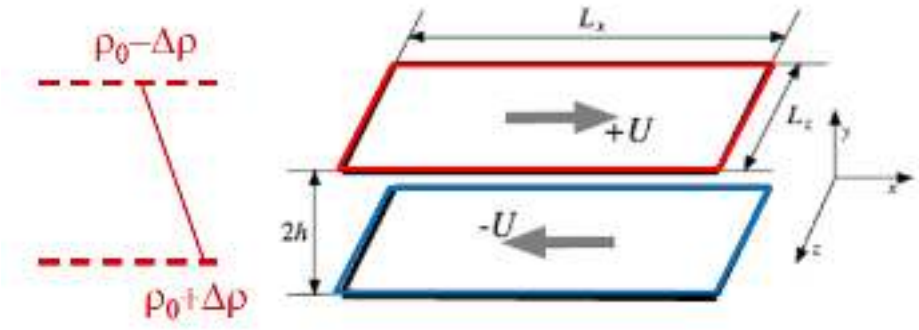
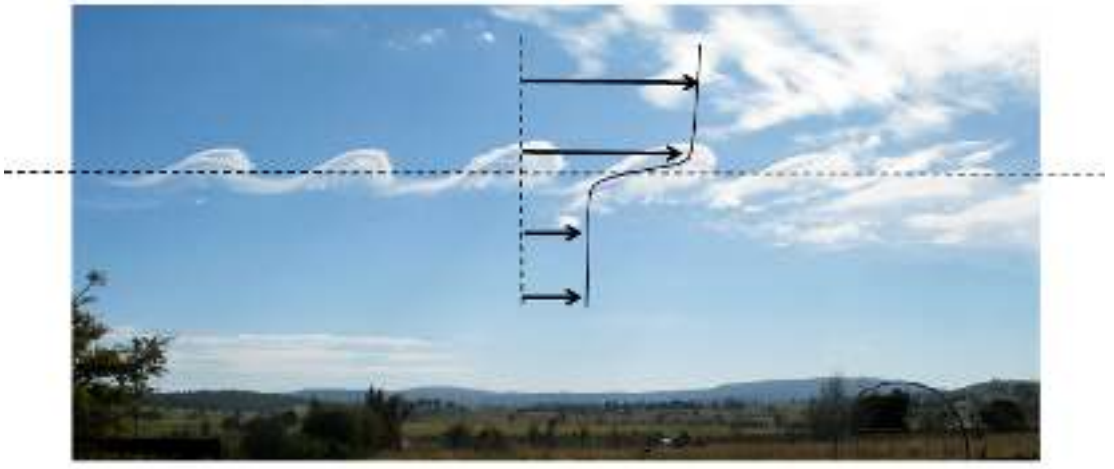
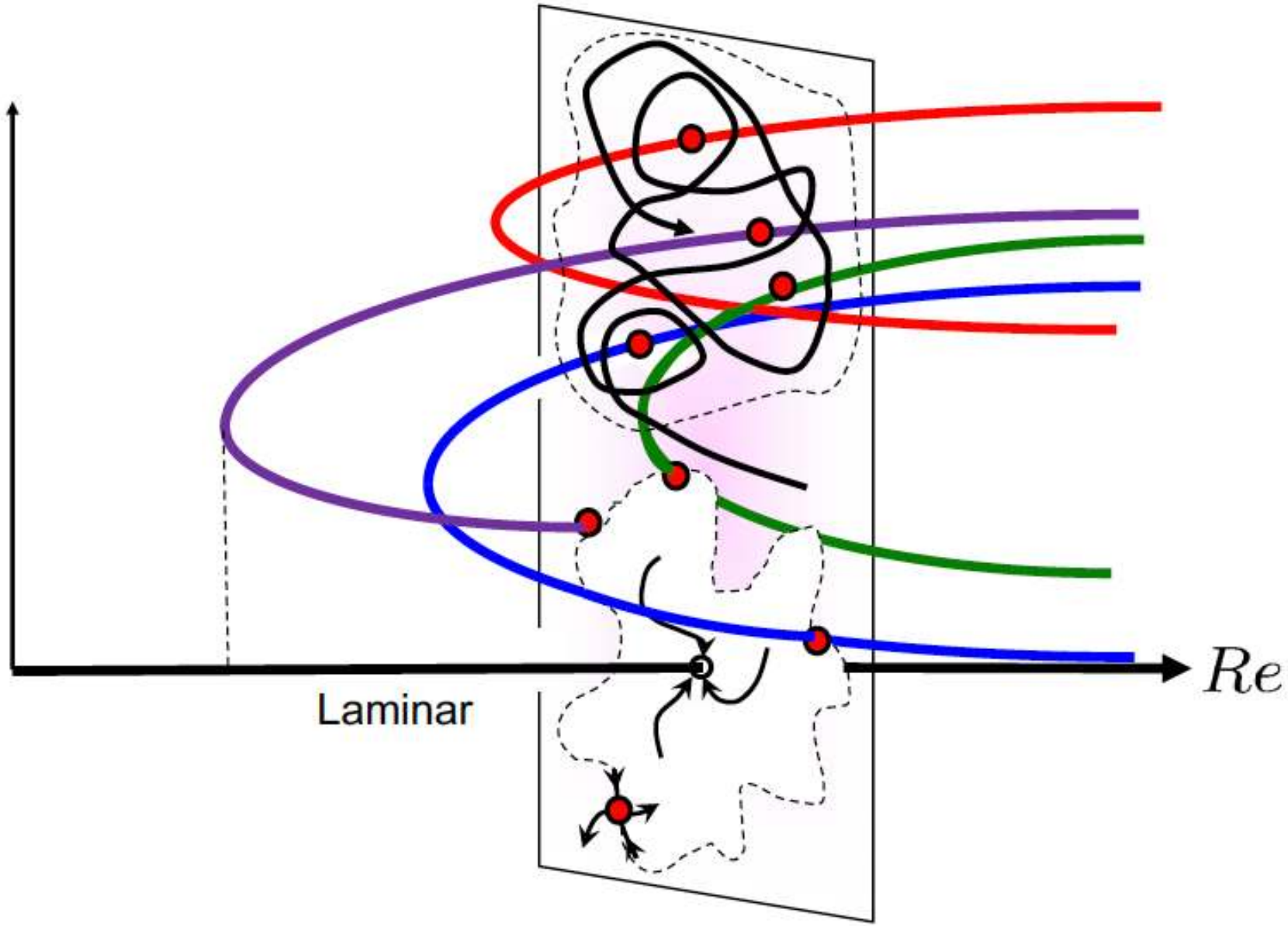


Stratified plane Couette flow

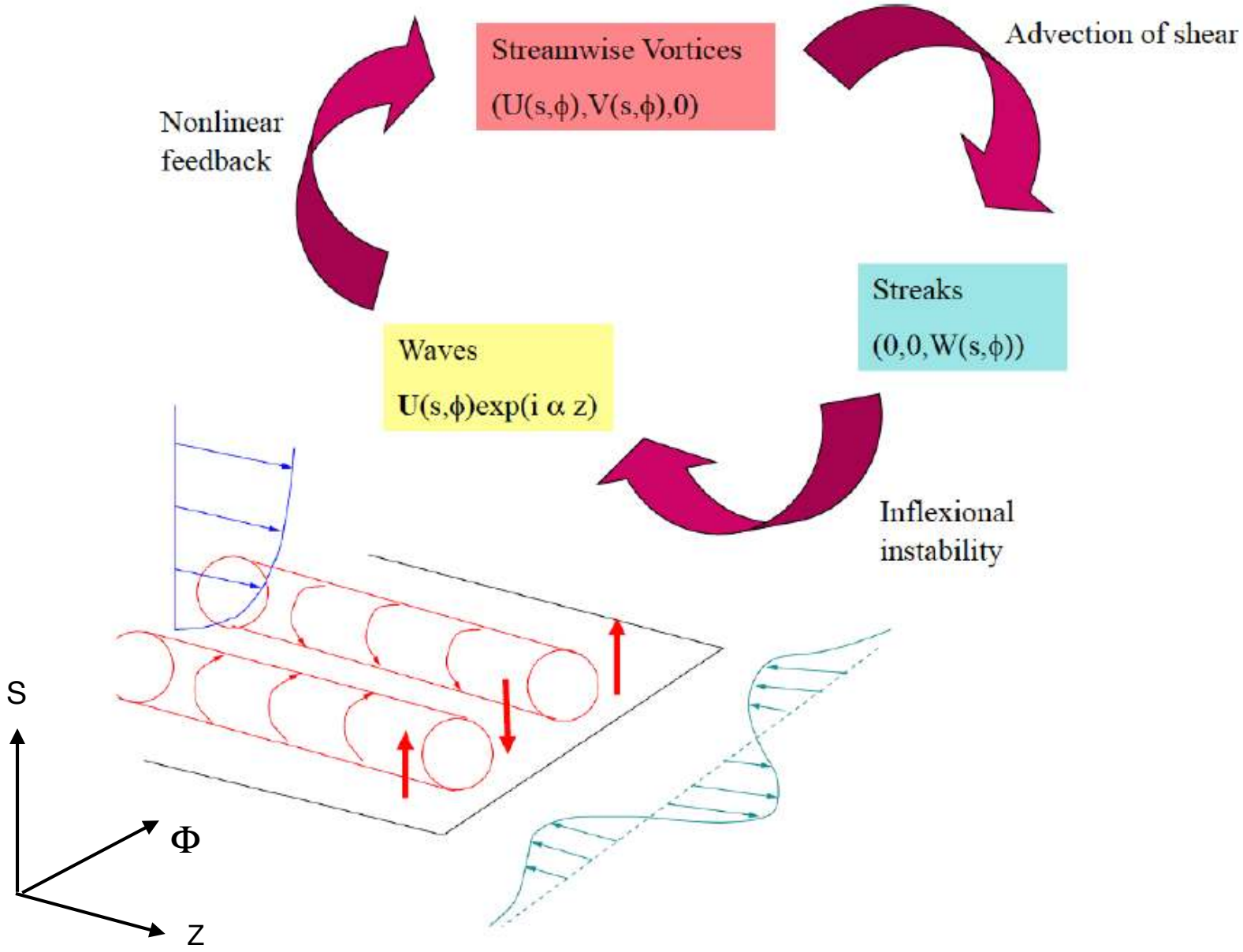


Stratified Kelvin Helmholtz instability

Motivation: Alternative solutions exist in (unstratified) shear flows



SSP/VWI (Waleffe 1997, Hall & Sherwin 2010)



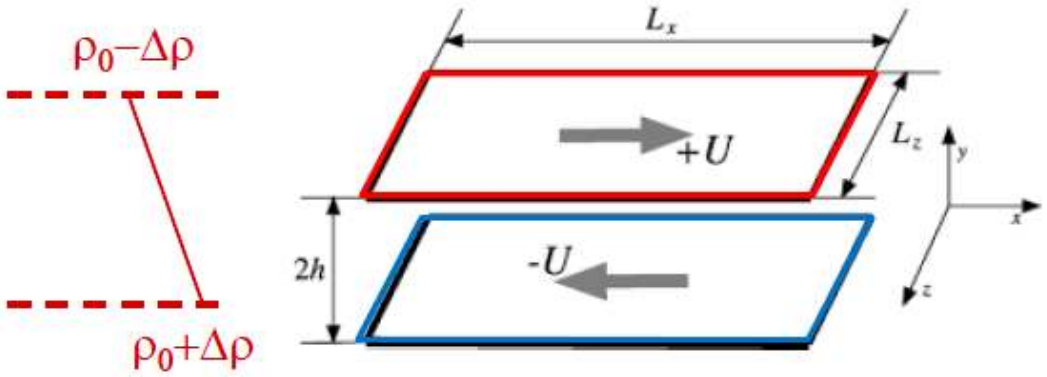
Questions

1. How does SSP/VWI get disrupted by stable stratification?
2. Modified? - new waves exist…
3. Do states exist up to Ri=1/4? (Miles-Howard)
4. Does the structure of the states say anything about stratified turbulence?
5. Stratification triples the number of parameters in the problem

$$Re \rightarrow (Re, Ri, Pr)$$

How important is *Pr*? (usually neglected in deference to *Ri*)

A. Stratified plane Couette flow



$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p - Ri_b \, \rho \, \hat{\mathbf{y}} + \frac{1}{Re} \nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0,$$

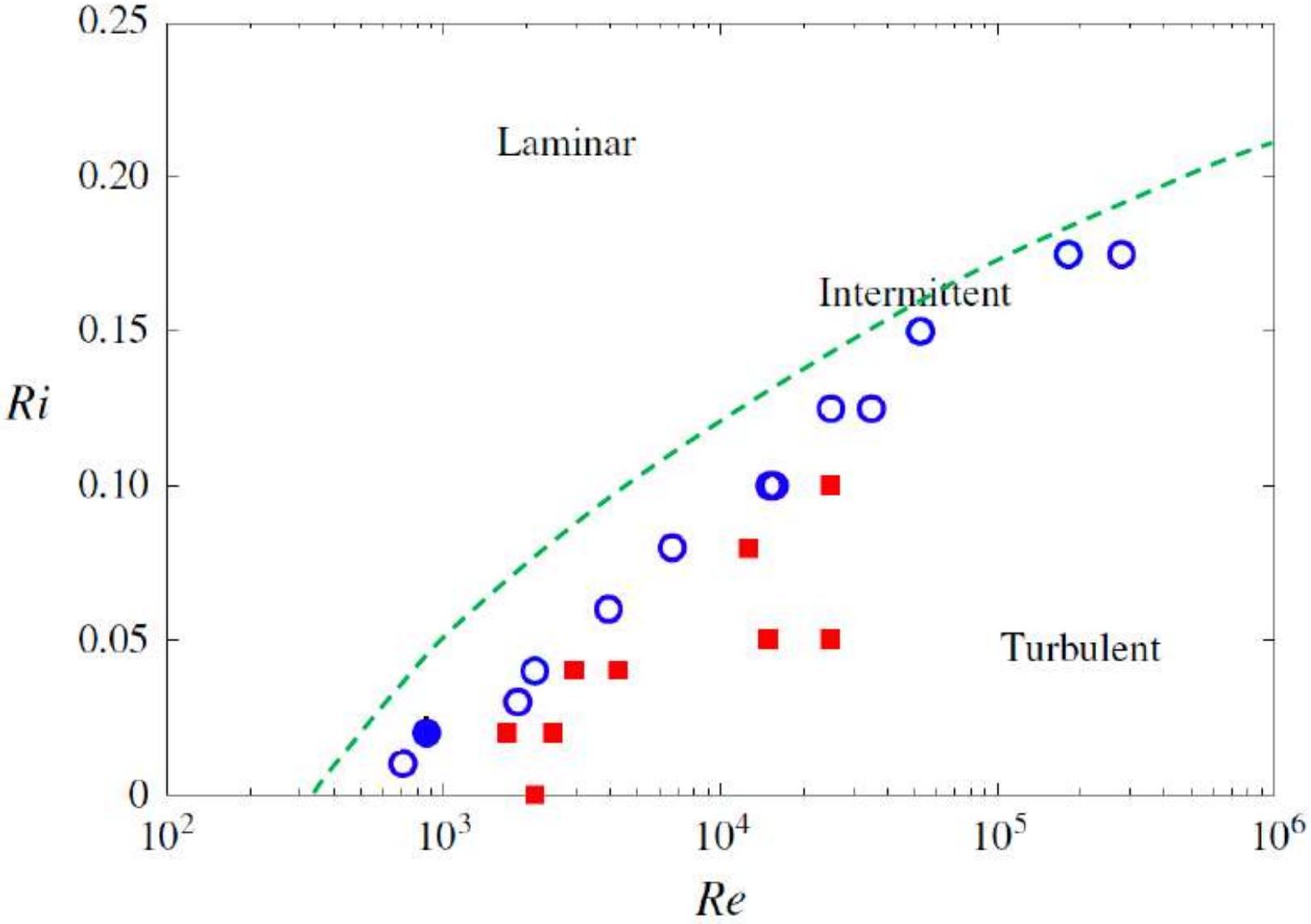
$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = \frac{1}{Re \; Pr} \nabla^2 \rho$$

$$Ri_b := \frac{\Delta\rho \, g \, h}{\rho_0 \, U^2}, \qquad Re := \frac{U \, h}{\nu}, \qquad Pr := \frac{\nu}{\kappa}$$

($Ri_b < 0$ --> Rayleigh-Benard convection + imposed shear $Ra = -Re^2 \, Ri_b \, Pr$)

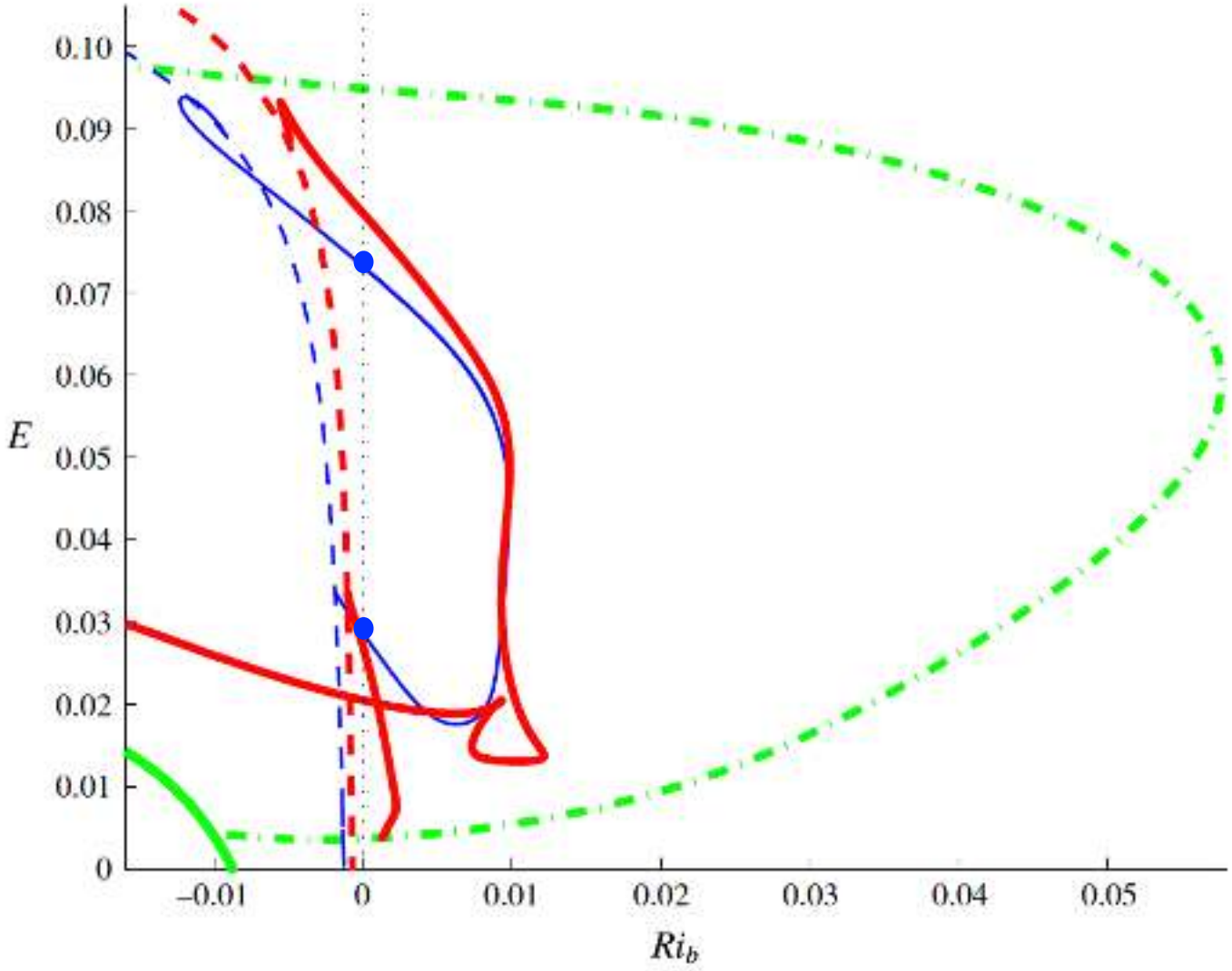
Pr = 0.7 (heated air) – 700 (salty water)

Pr=1



Brethouwer et al. (2012), Deusebio et al. (2015)

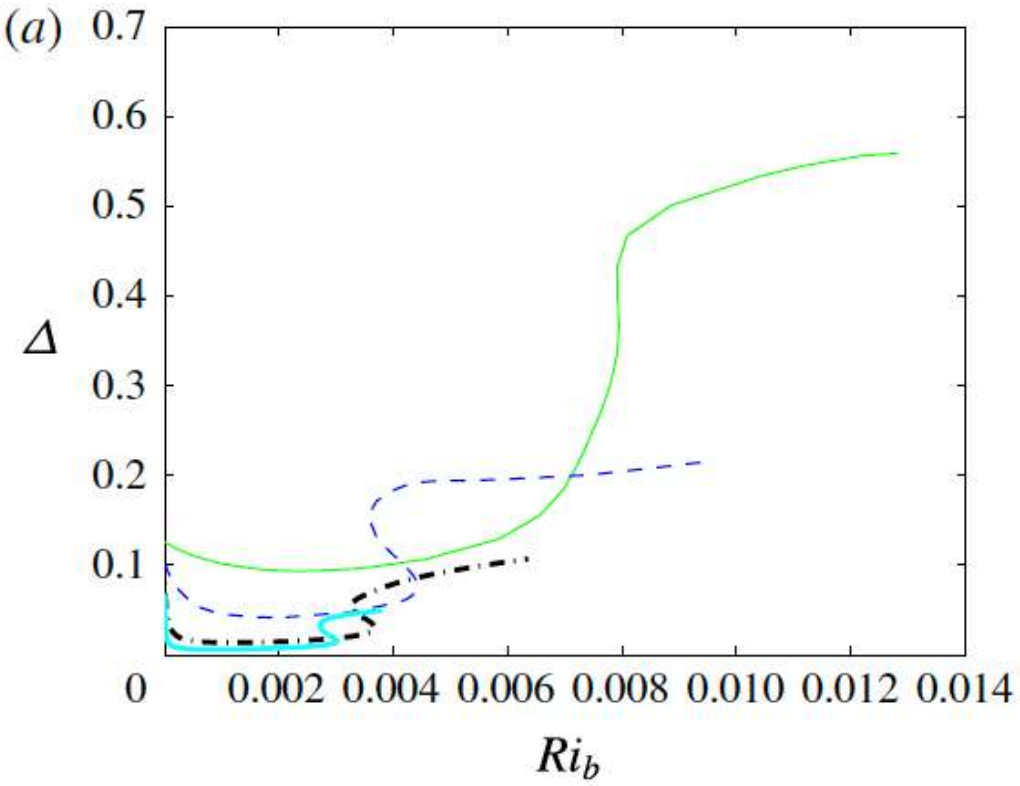
RB+shear —— pCf —— pCf+stable stratification

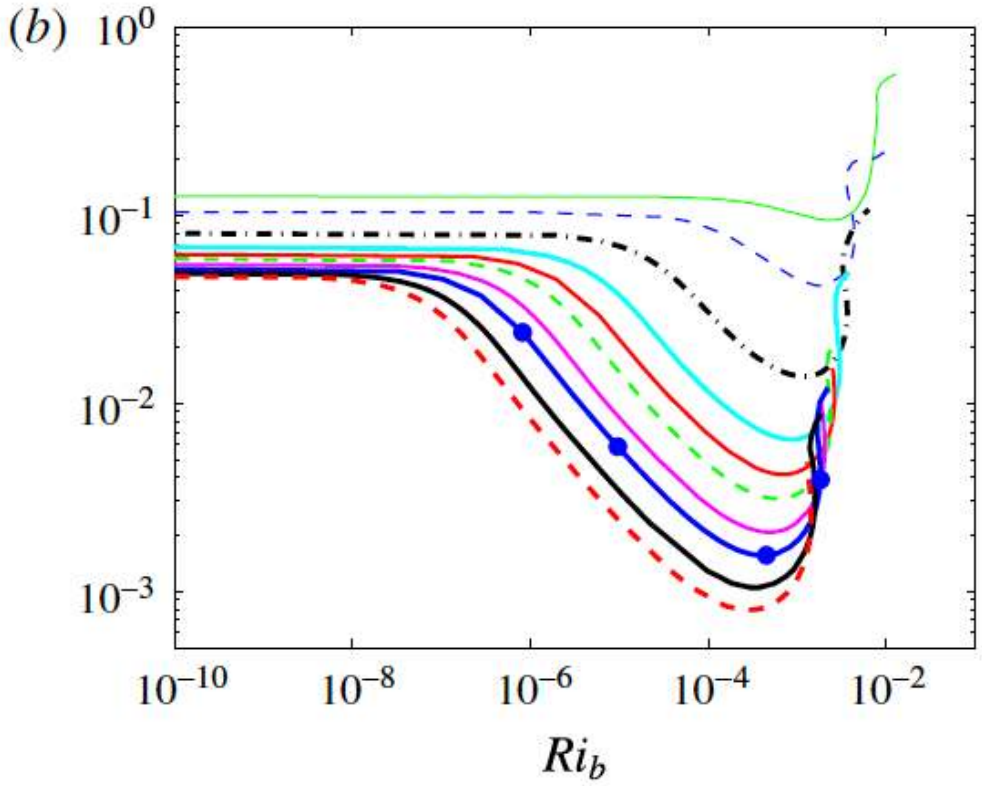


Blue Re=300
Red & Green Re=400

Olvera & K JFM 2017

Fix Pr=1 but increase Re…

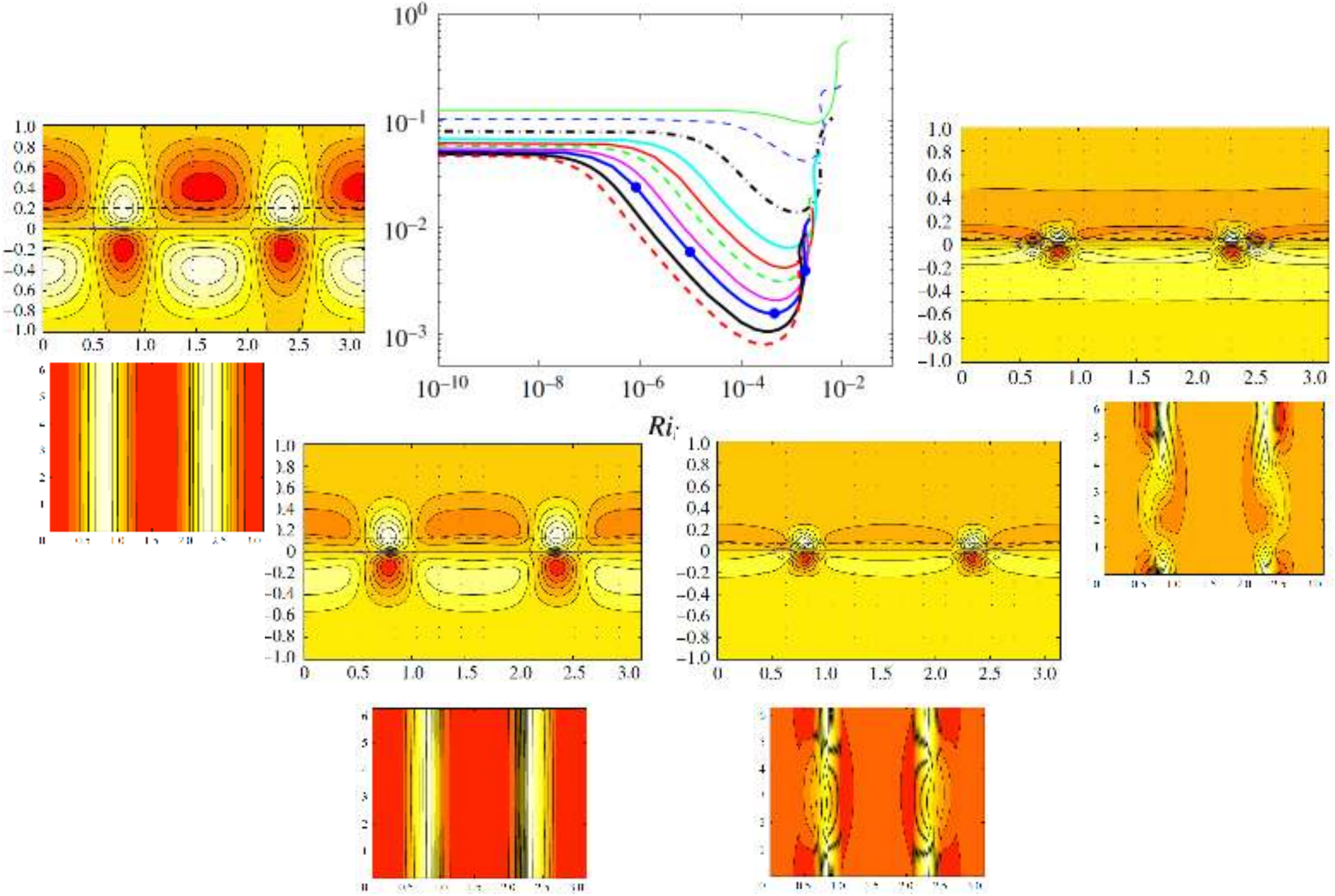




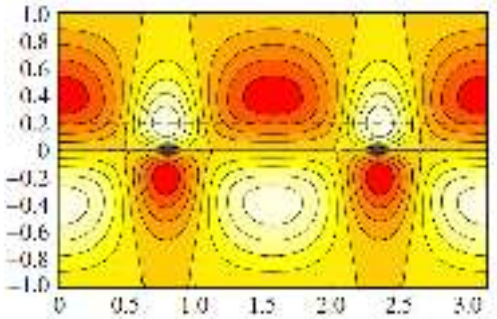
Re=1000, 2000, 5000, 10k

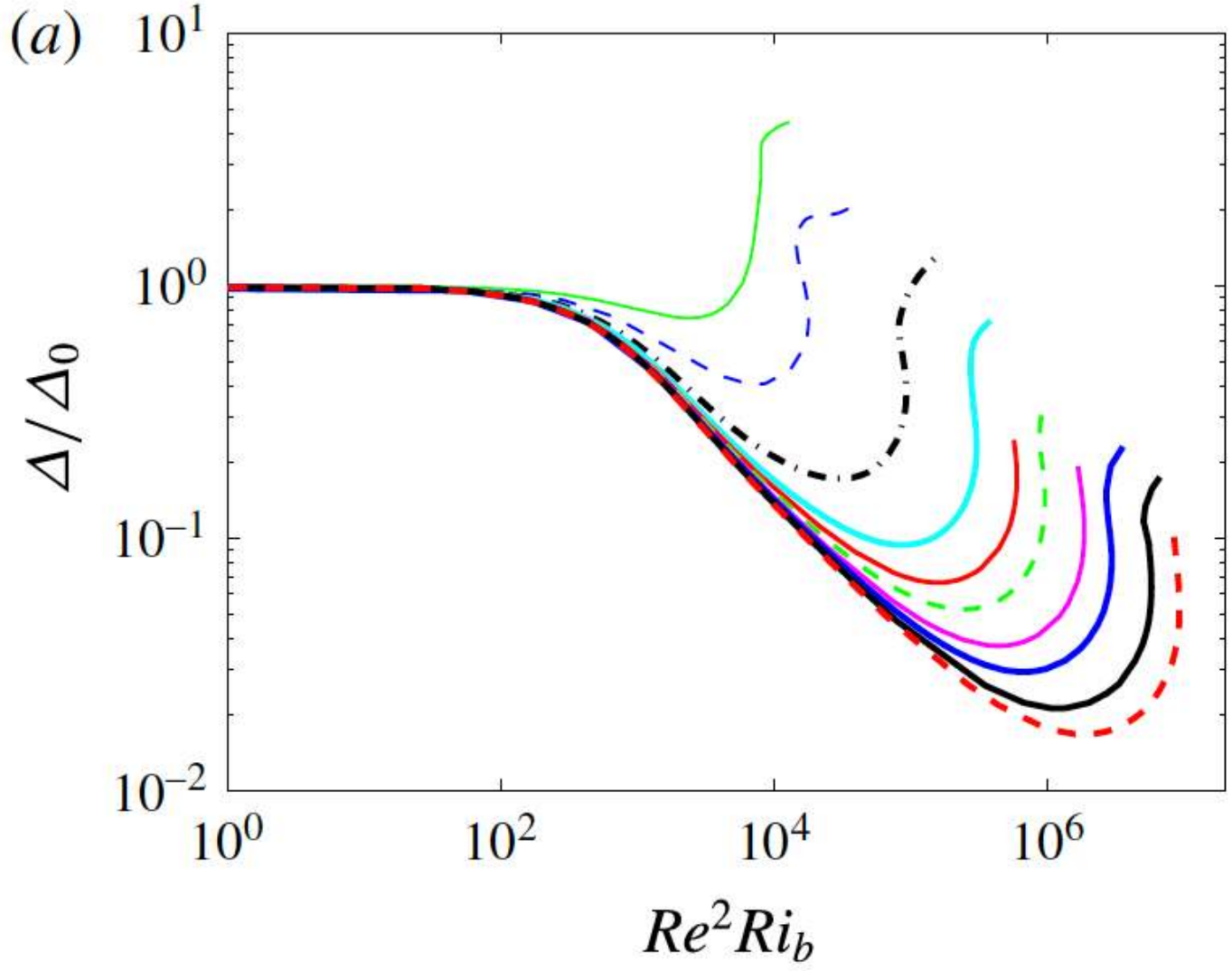
Re=1000, 2000, 5000, 10k, 15k, 20k, 30k, **40k**, **60k**, 80k

Re=1000, 2000, 5000, 10k, 15k, 20k, 30k, **40k**, **60k**, 80k



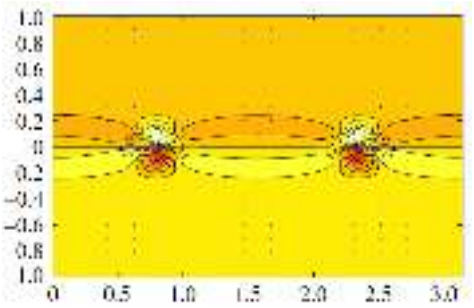
Regime I Suppression of streamwise rolls $Ri_b = O(Pr^{-1}Re^{-2})$

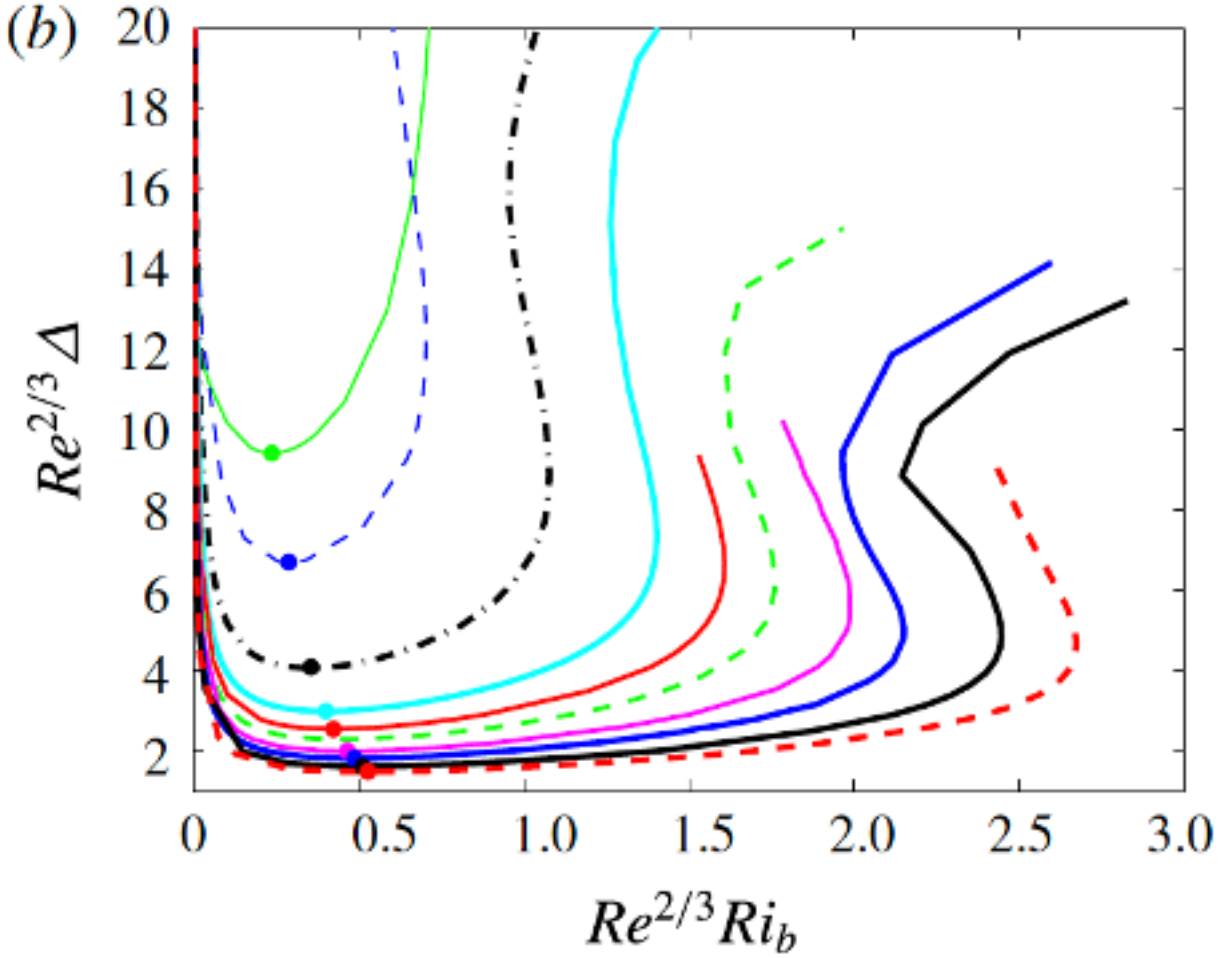




Eaves & Caulfield 2015, Olvera & K 2017, Deguchi 2017

Regime II spanwise and cross-stream localisation $Ri_b = O(Re^{-2/3})$





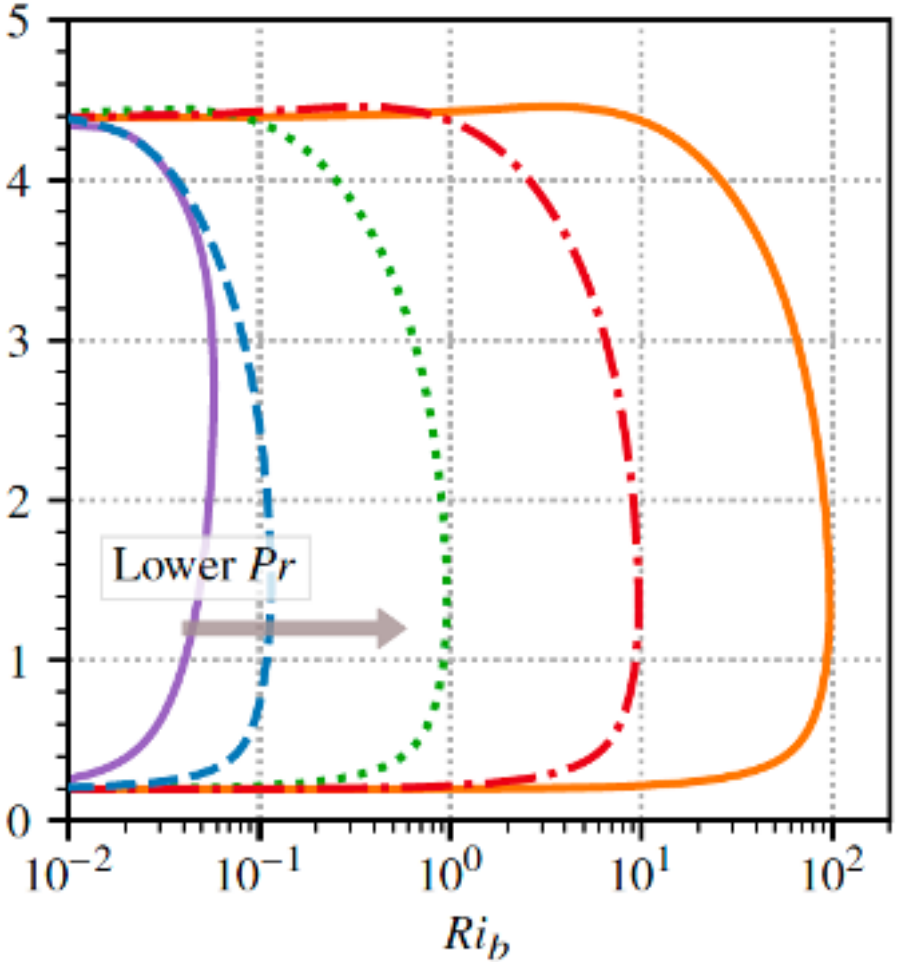
Regime III Full localisation in 3D $Ri_b = O(1)$

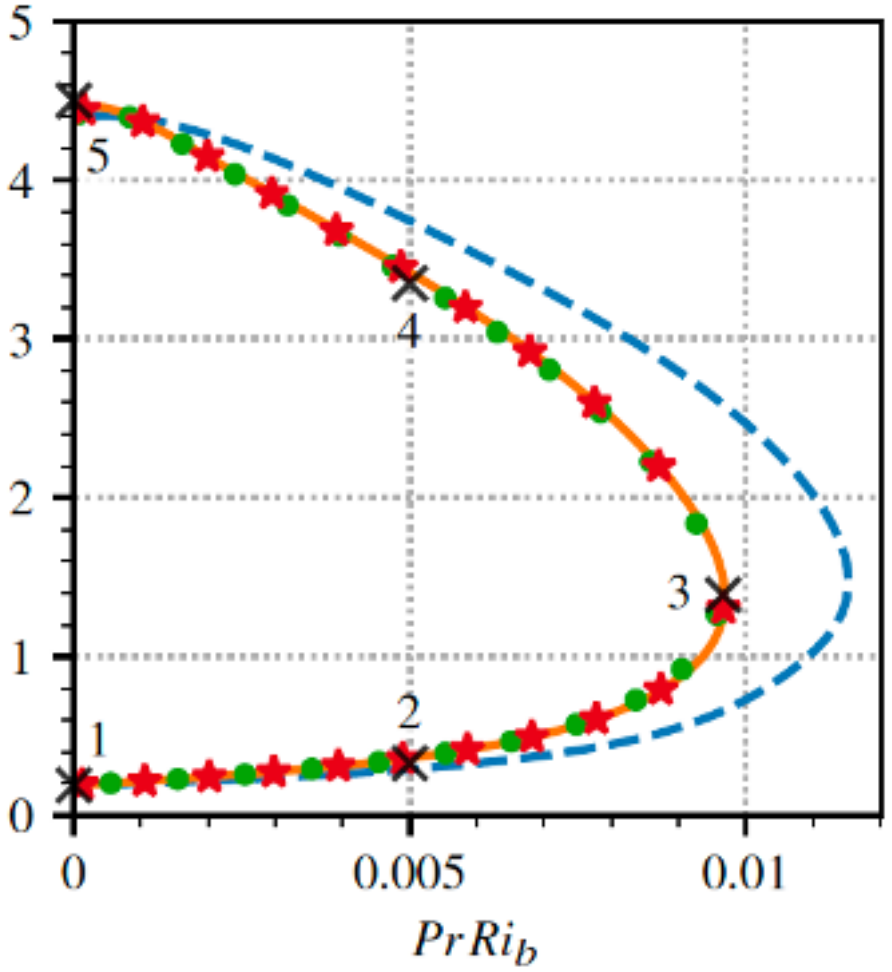
$(\partial_x, \partial_y, \partial_z) \rightarrow Re^{1/2}(\partial_x, \partial_y, \partial_z)$ and $(u, v, w, \rho, p) \rightarrow (Re^{-1/2}[u, v, w, \rho], Re^{-1}p)$

Now fix Re=400 & vary Pr

Langham, Eaves & K 2019

a) $Pr \to 0$



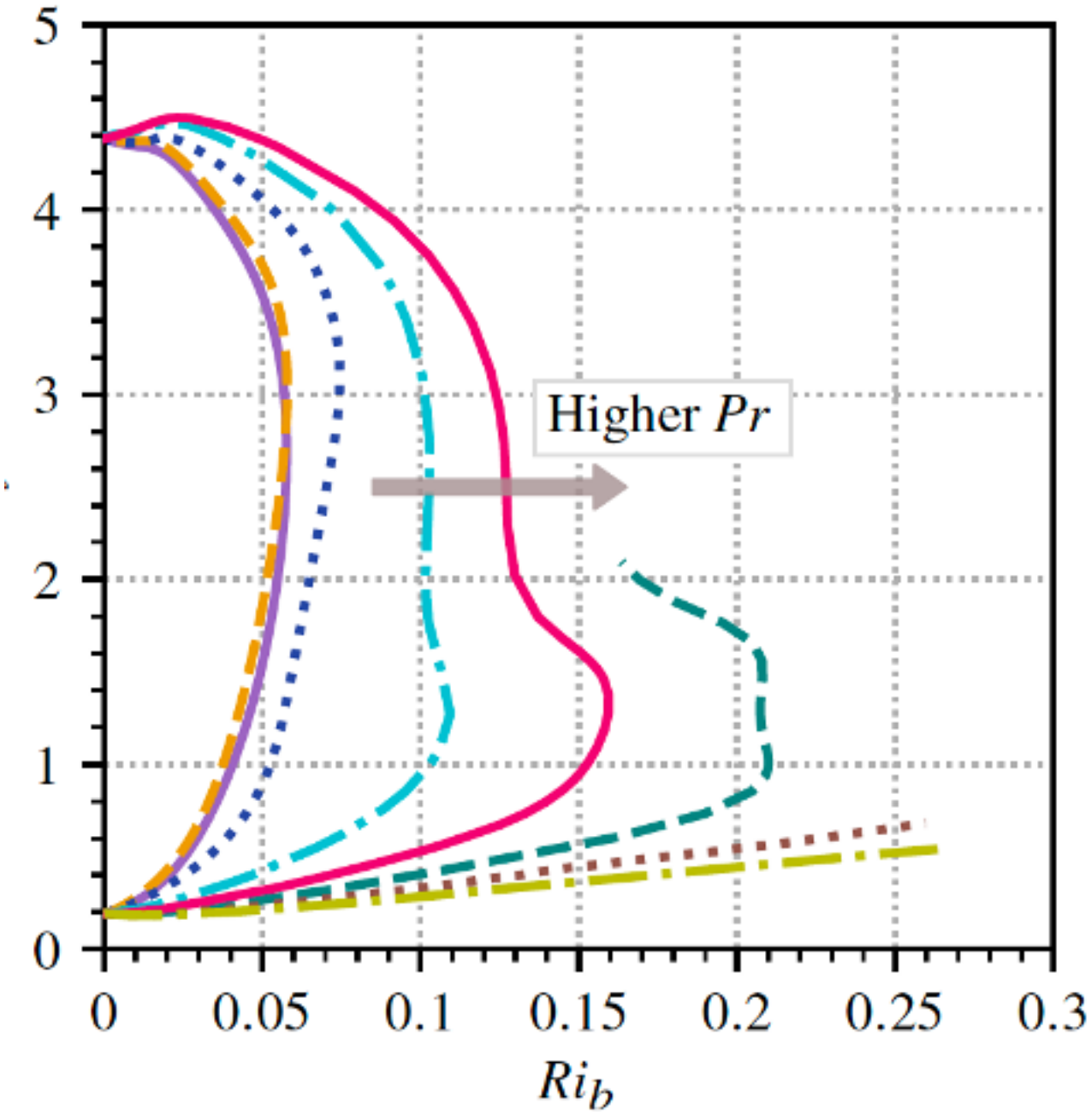


$$\rho_0 = -y; \quad -v \approx \frac{1}{RePr}\nabla^2\rho$$

Pr=1, 0.1, 0.01, 0.001, 1e-4

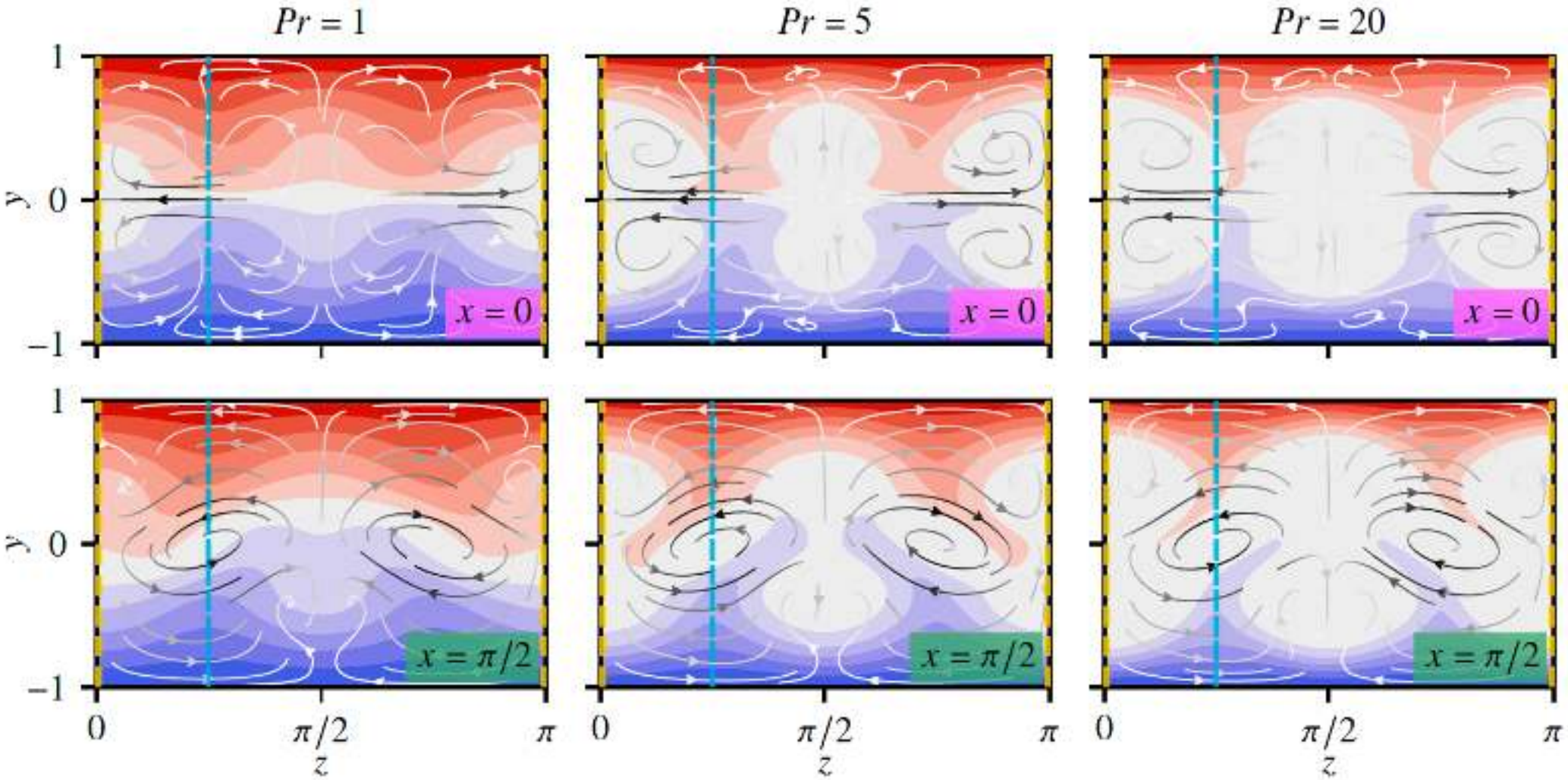
$$\frac{1}{Re}\nabla^2 v - Ri\rho \to \frac{1}{Re}\left[\nabla^2 + Ri_b Pr Re^2 \nabla^{-2}\right] v$$

b) $Pr \to \infty$

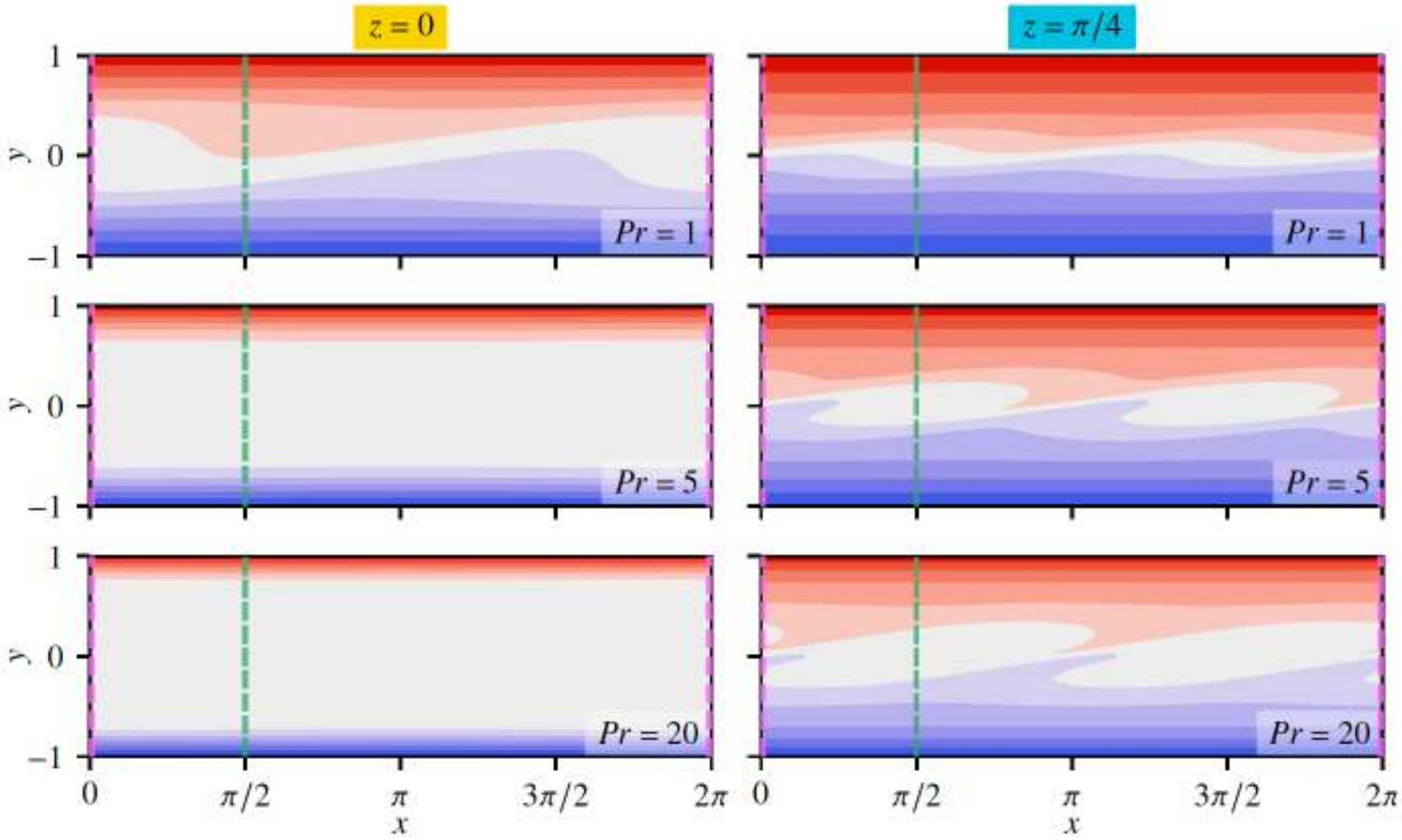


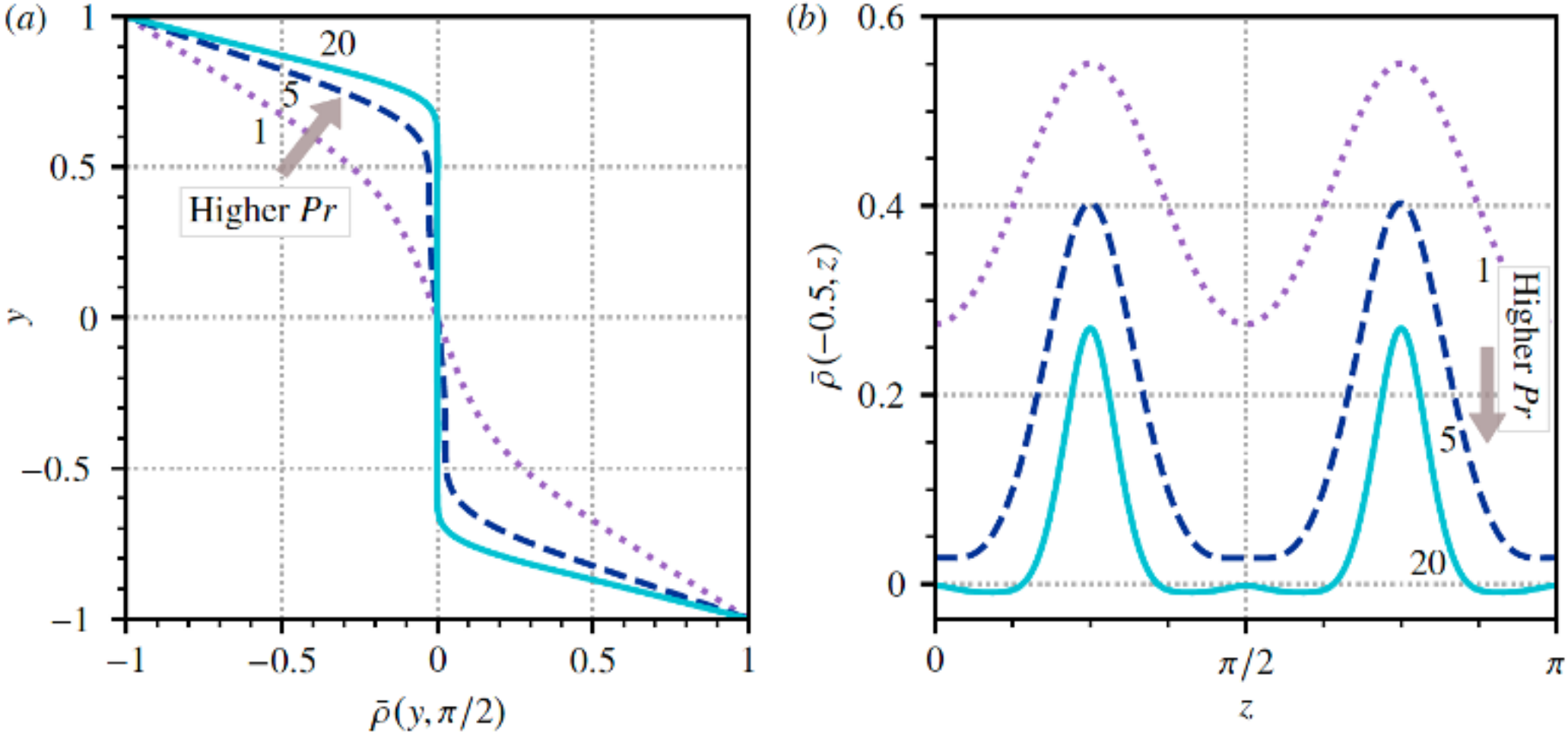
Pr=1, 2, 7, 20, 40, 70, 120, 200

Re=400, Ri=0.01



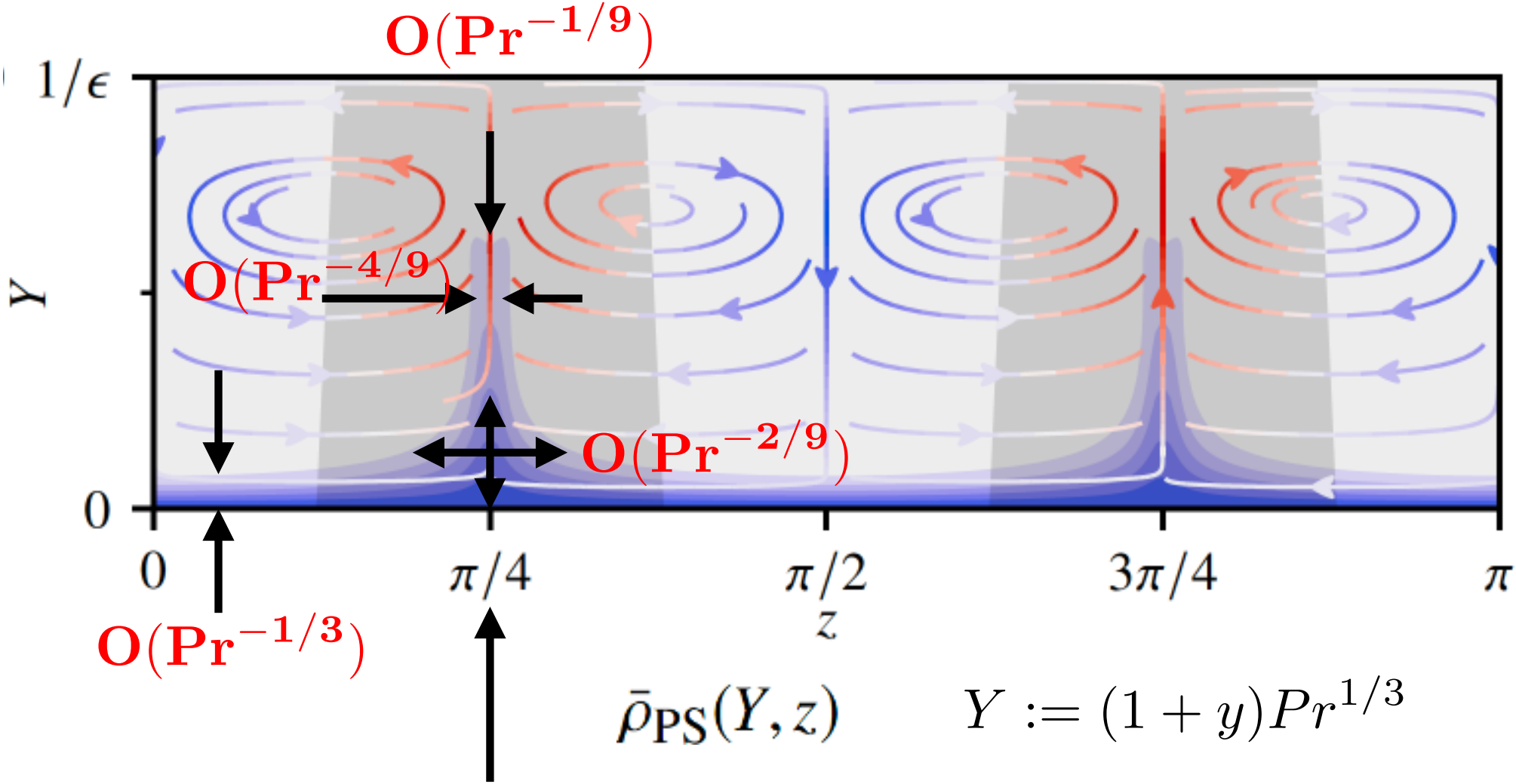
Re=400, Ri=0.01




(a)
1
0.5
0
−0.5
−1
y
−1
−0.5
0
0.5
1
$\bar{\rho}(y, \pi/2)$
20
5
1
Higher Pr
(b)
0.6
0.4
0.2
0
$\bar{\rho}(-0.5, z)$
0
π/2
π
z
1
5
20
Higher Pr

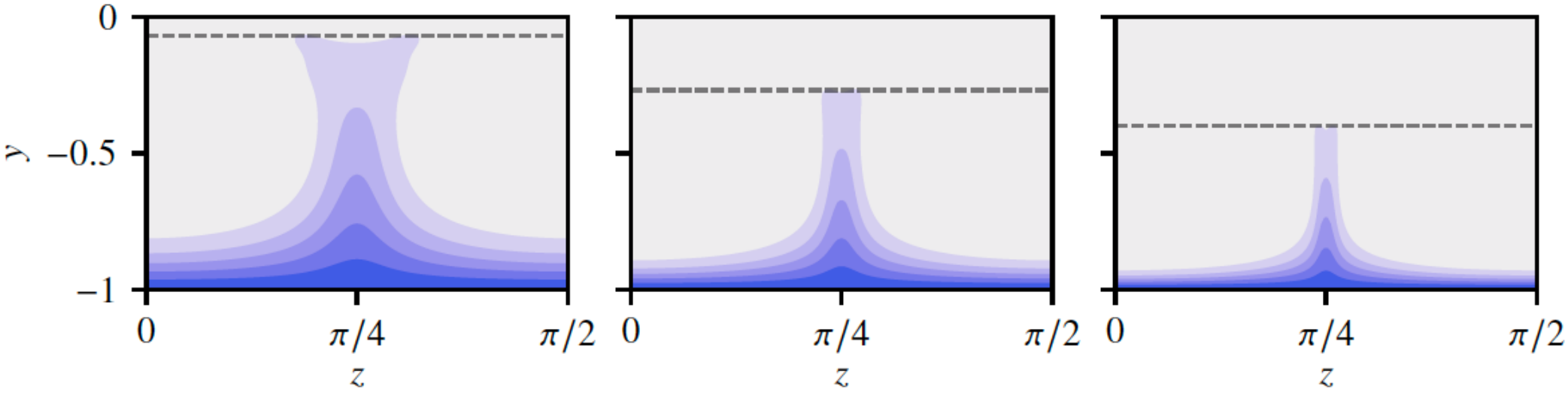
Passive scalar limit (numerically $Ri \to 0$)

$$\mathbf{u}.\nabla\rho = \frac{1}{RePr}\nabla^2\rho$$



So fingers should vanish as Pr increases…

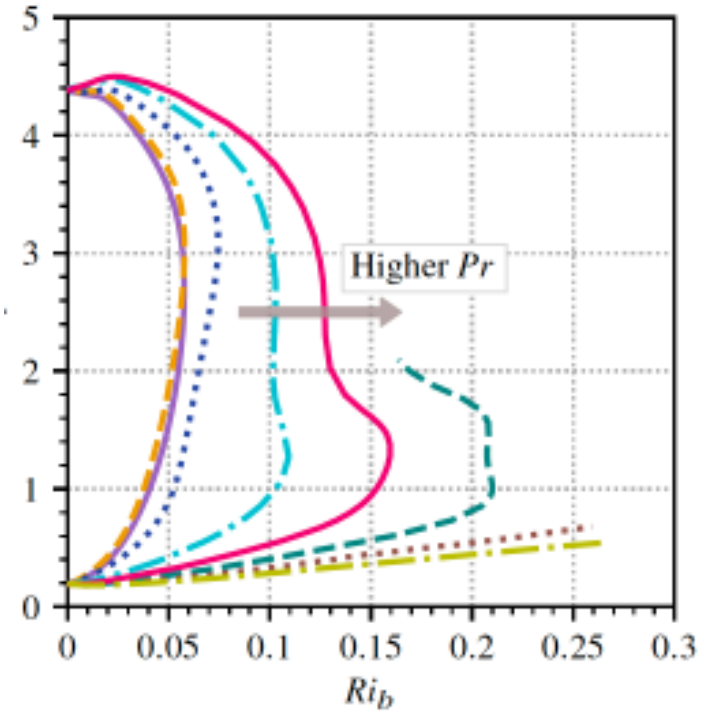
$$\bar{\rho}_{PS}(y,z)$$



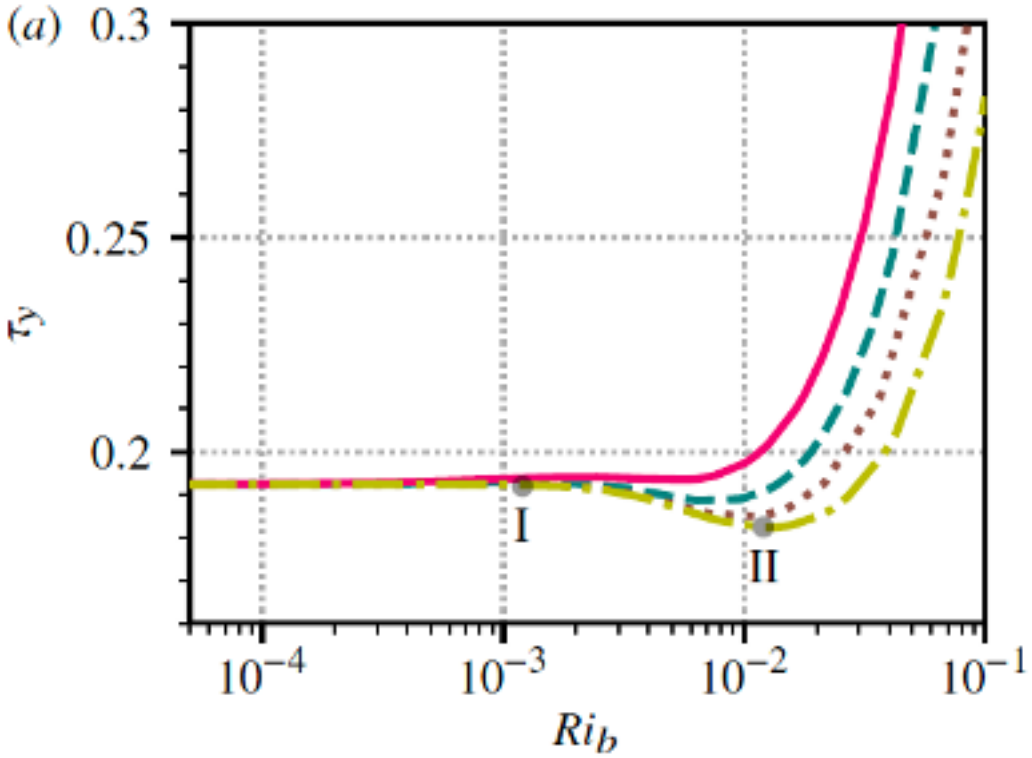
Pr = 20 100 400

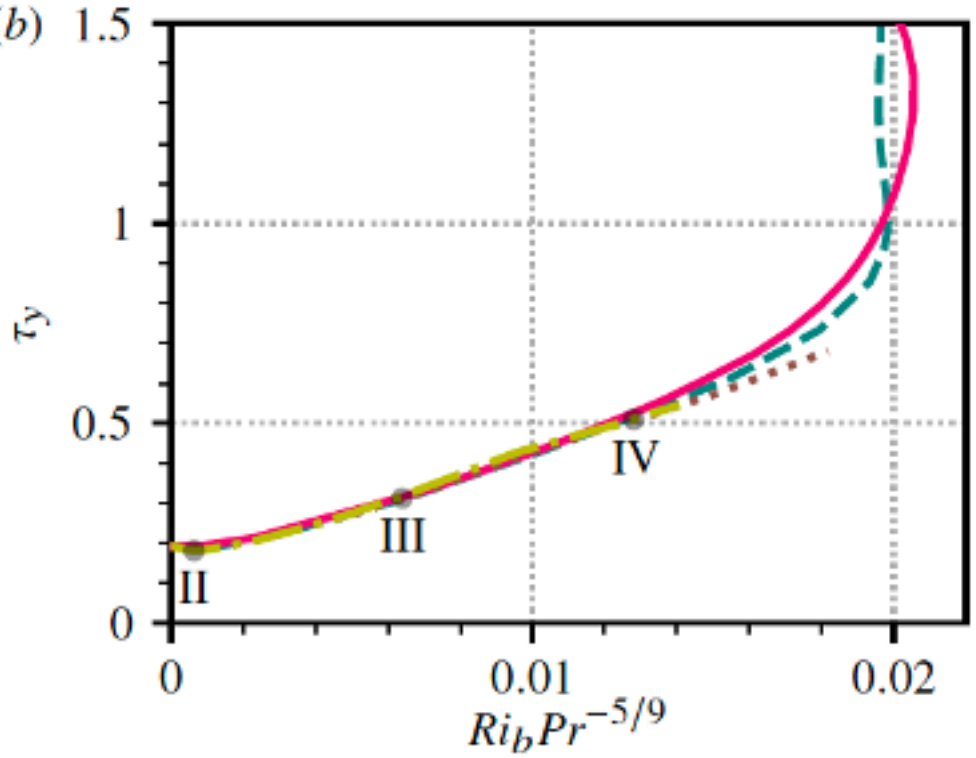
$Pr \to \infty$

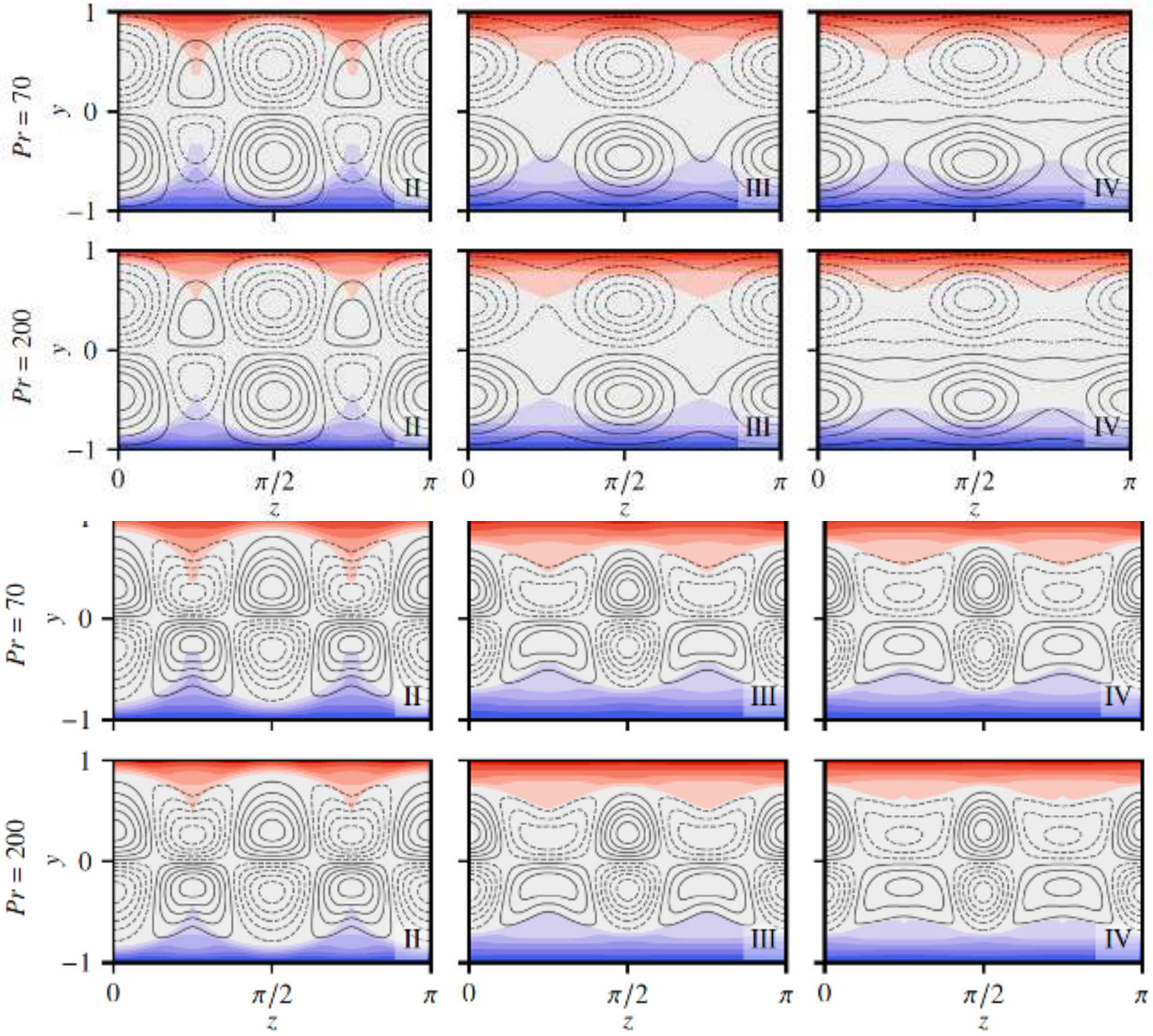
Ri > 0



Pr = 40, 70, 120, 200

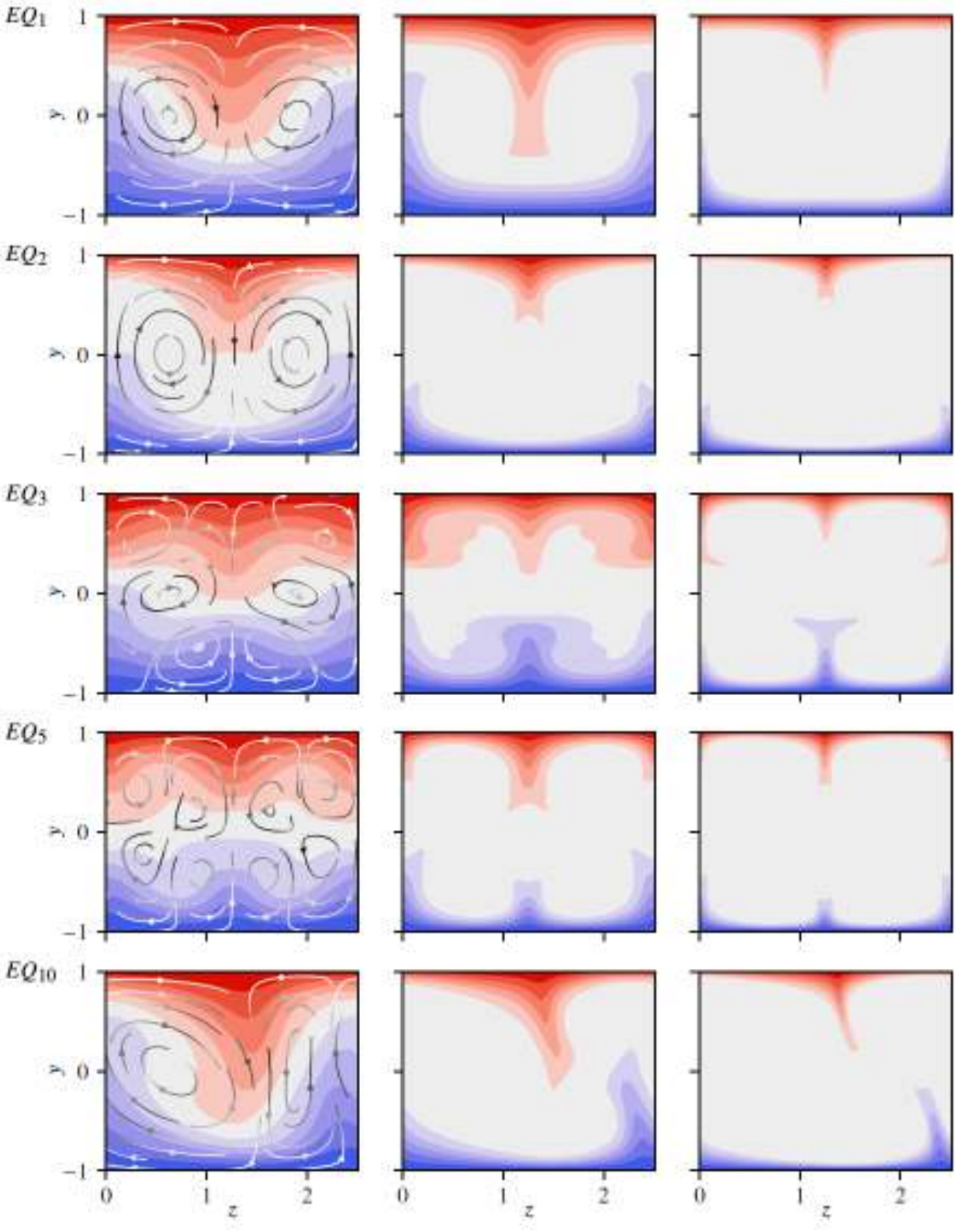





Pr = 70
Pr = 200
y
z
0
π/2
π
1
−1
II
III
IV
u
v

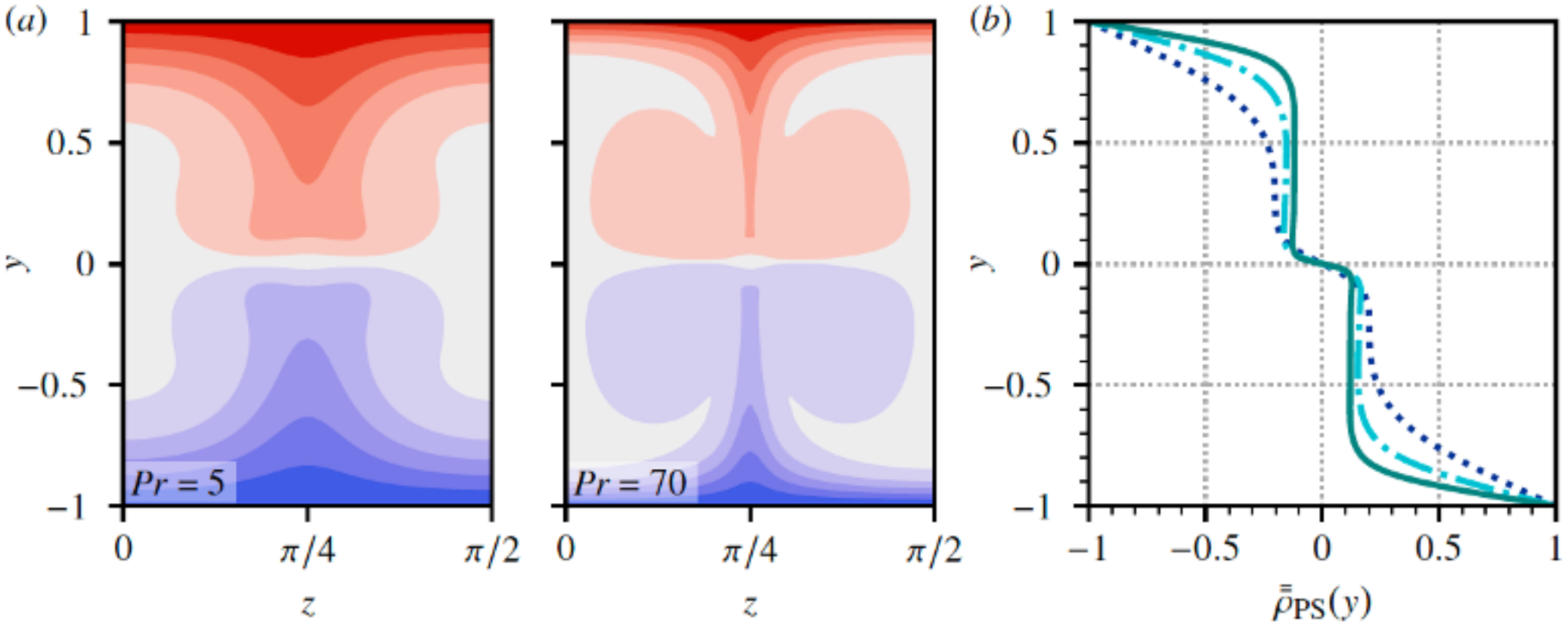
$\bar{\rho}(y,z)$

Pr=1,10 & 70



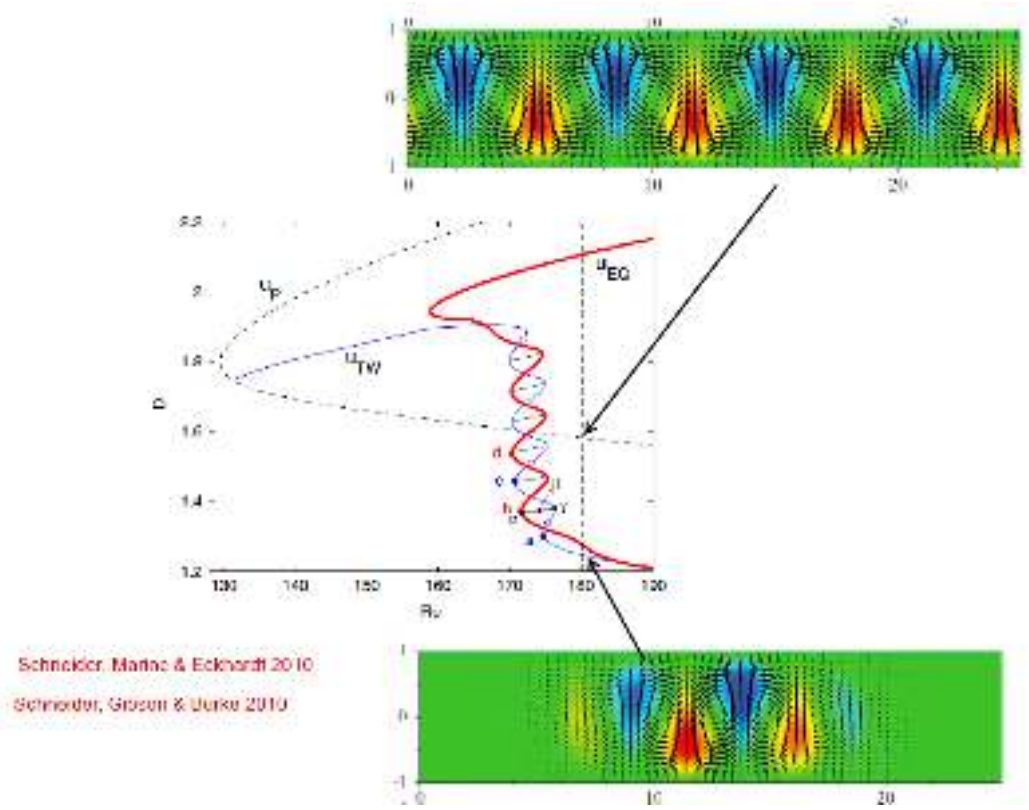
High Re and Pr?

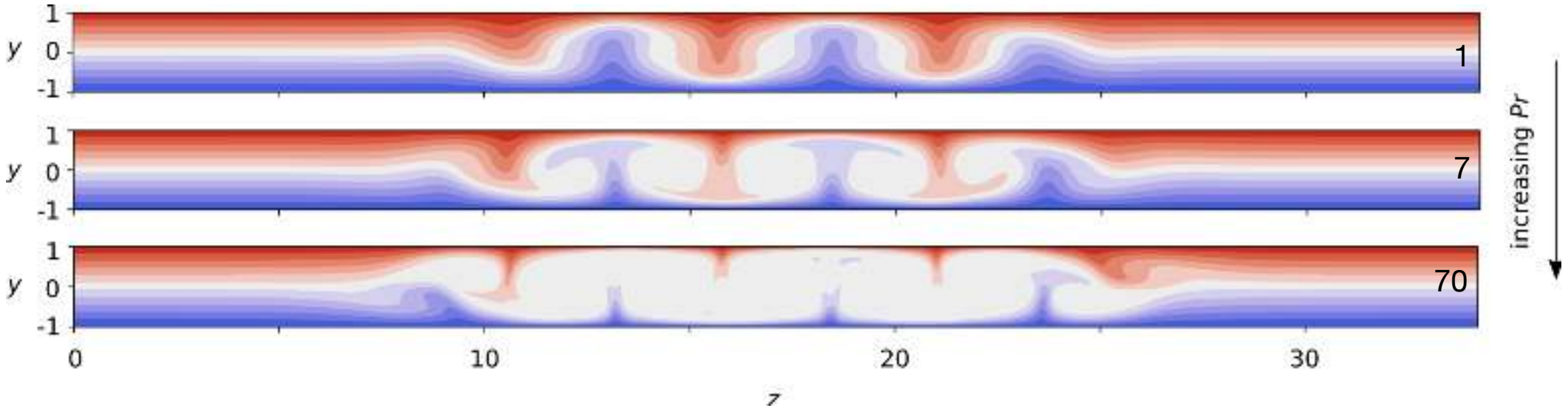
Why? VWI scalings and symmetry of state have v -> 0 at midplane



$Re = 10^5$

Passive scalar limit using `snake' velocity field





A. Take Home Messages

1. How does SSP/VWI get disrupted by stable stratification?

   - *hampers streamwise rolls first, then forces localisation*

2. Modified? - new waves exist… - *not found yet!*

3. Do states exist up to Ri=1/4? (Miles-Howard) - *yes, at least for large or small Pr*

4. Does the structure of the states say anything about stratified turbulence?

   - high Pr shows layering, mixed regions, boundary layer scaling

5. Stratification triples the number of parameters in the problem

How important is *Pr*? (usually neglected in deference to *Ri*)

- looks very important - we need high Pr DNS

Langham, Eaves & K 2019 (arxiv:1904.04853)
Olvera & Kerswell JFM 2017

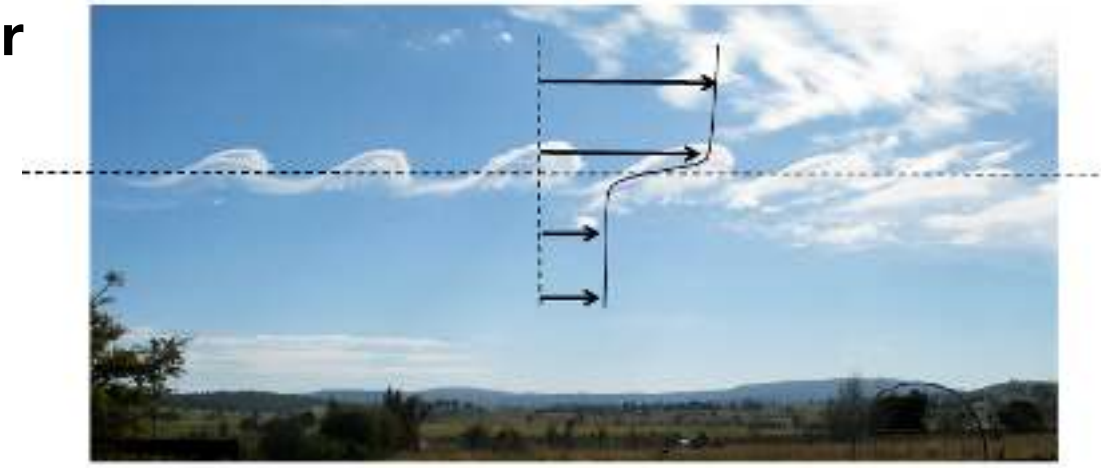
B. Forced 2D stratified mixing layer

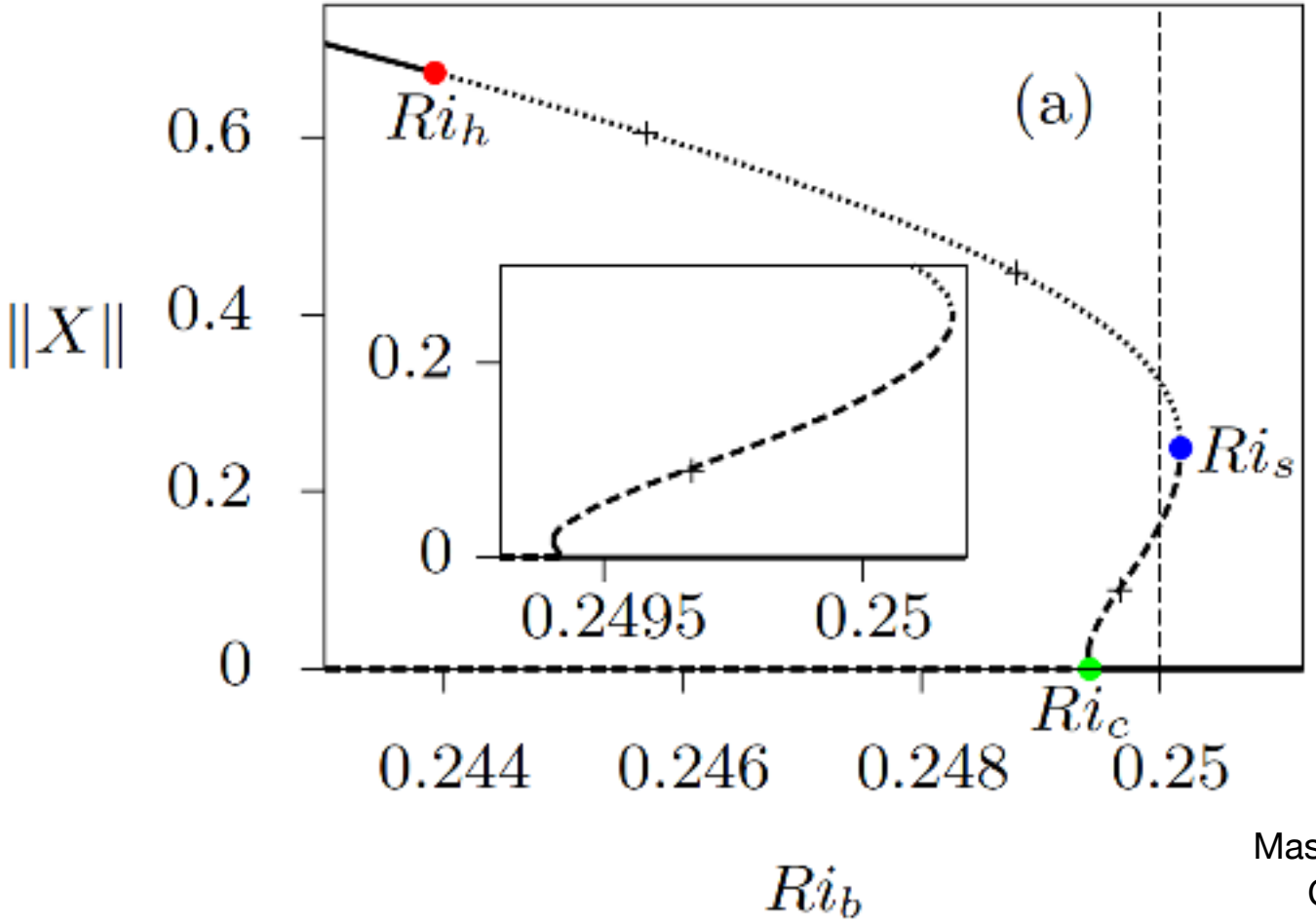
$$Pr = 1, Re = 4000$$

$$u_0 = \tanh z, \quad \rho_0 = -\tanh z$$

$$L_x = 4\pi, L_z = 5, 10$$

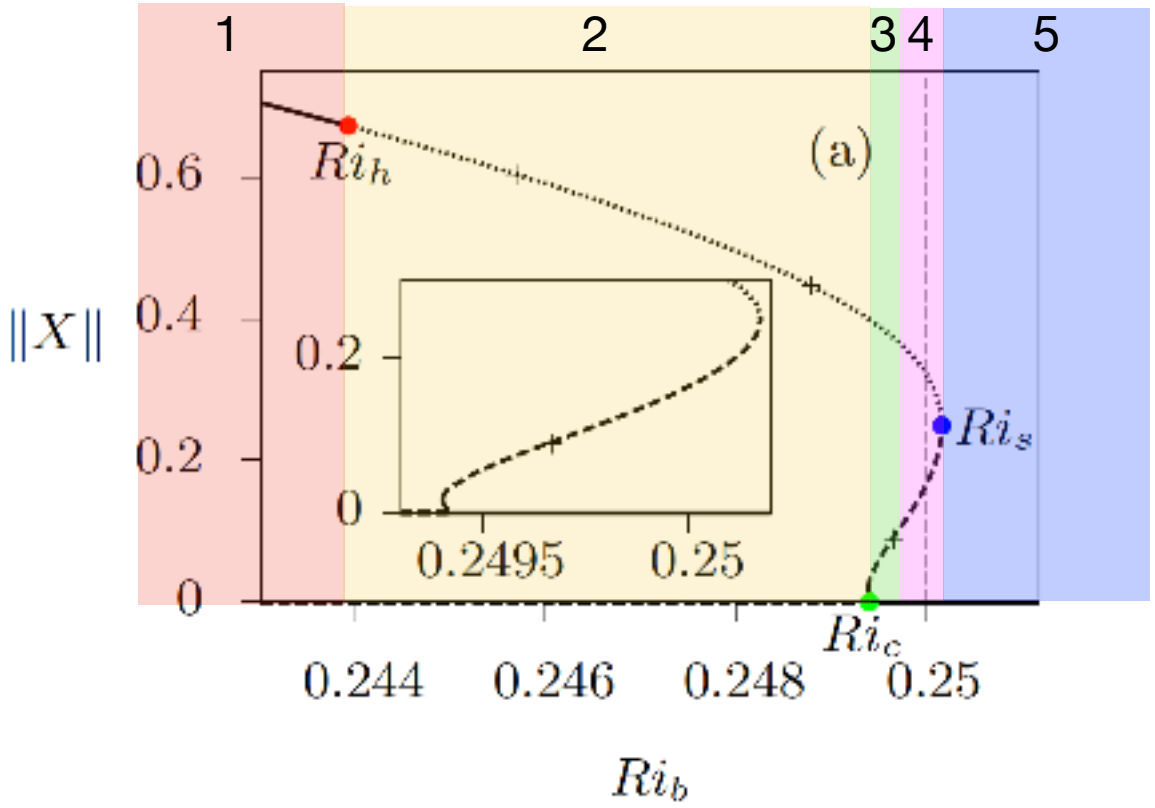
(res = 256 x 512 & 384 x 768)

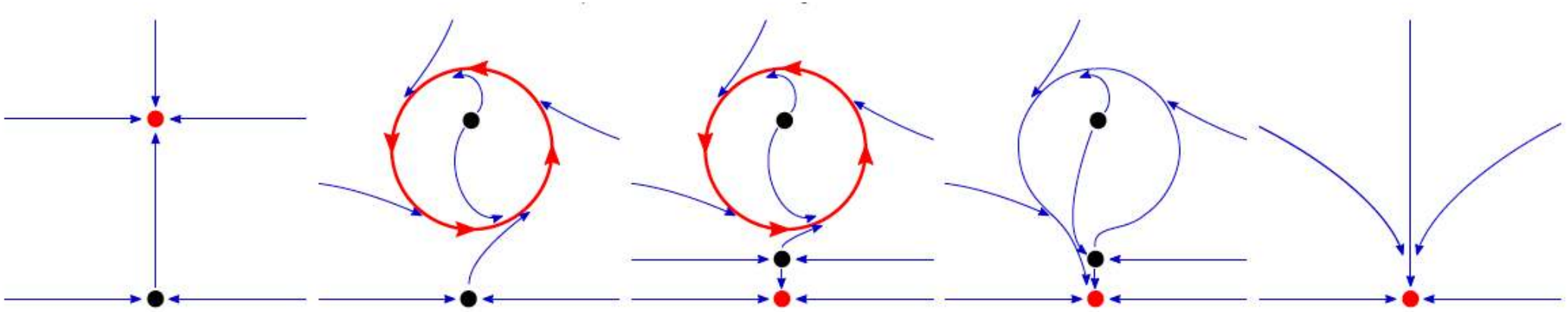




Howland et al. 2018
Kaminski et al. 2017

Maslowe 1977, Brown et al. 1981,
Churilov & Shukhman 1987

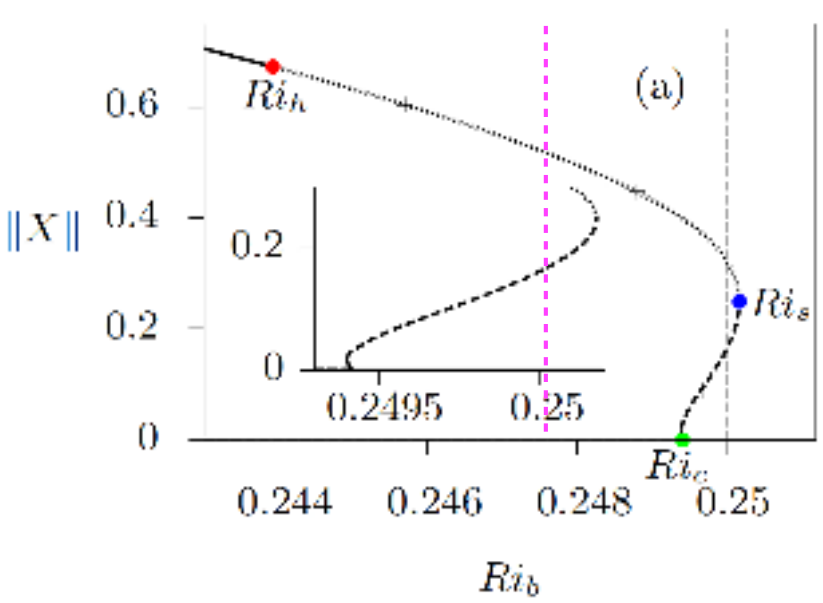


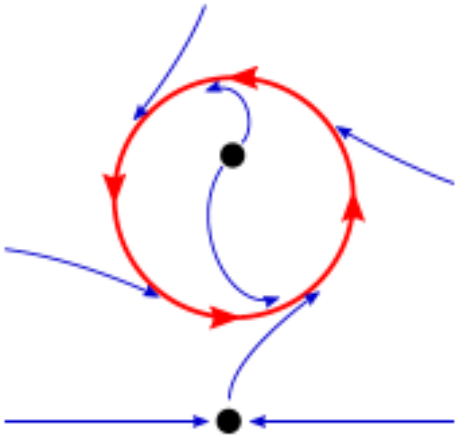


(a) Region 1 (b) Region 2 (c) Region 3 (d) Region 4 (e) Region 5

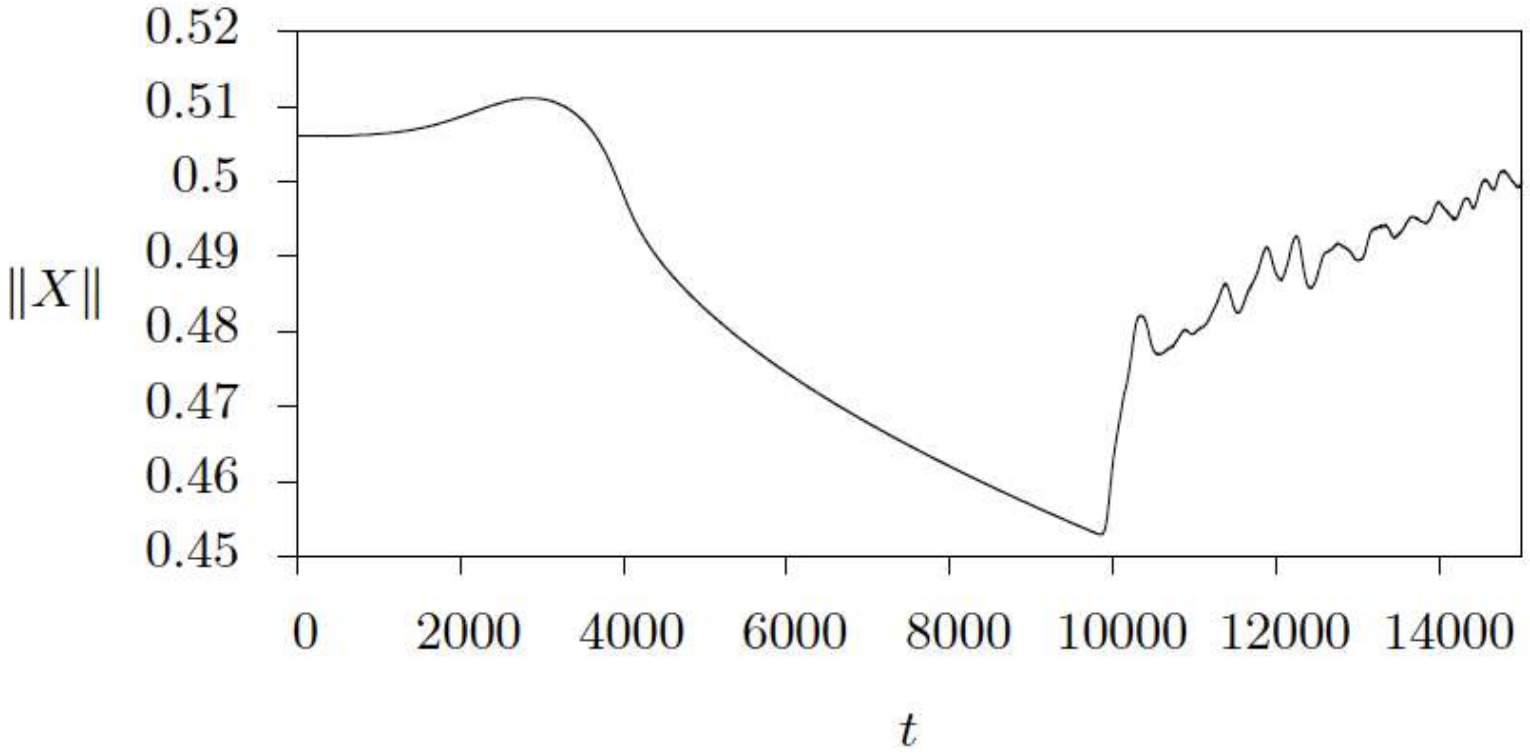
$Ri_b = 0.2478$

i.c near periodic orbit

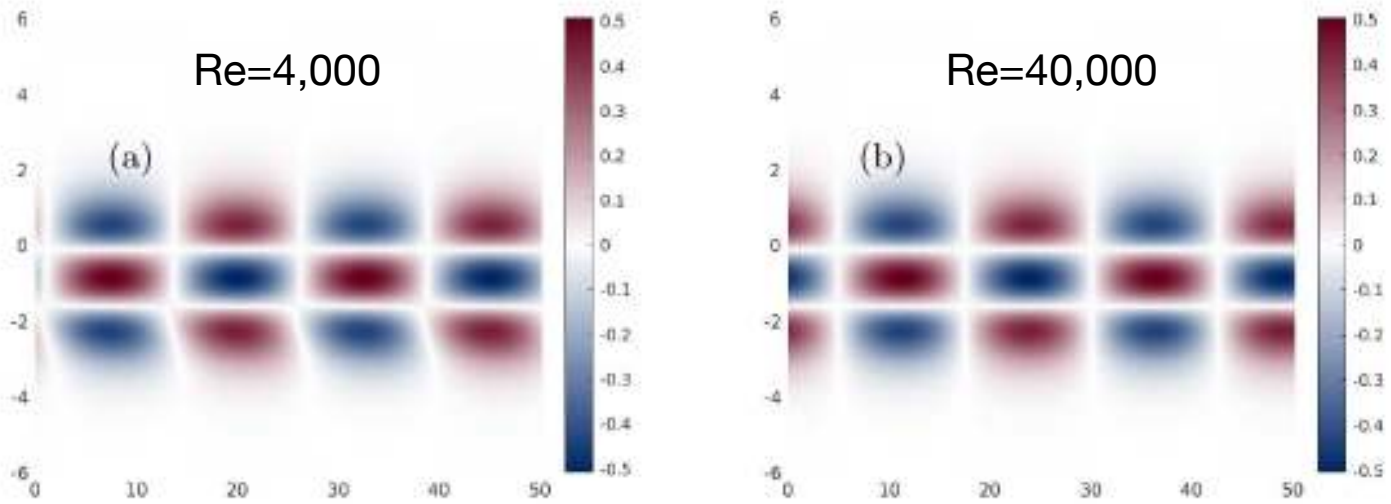


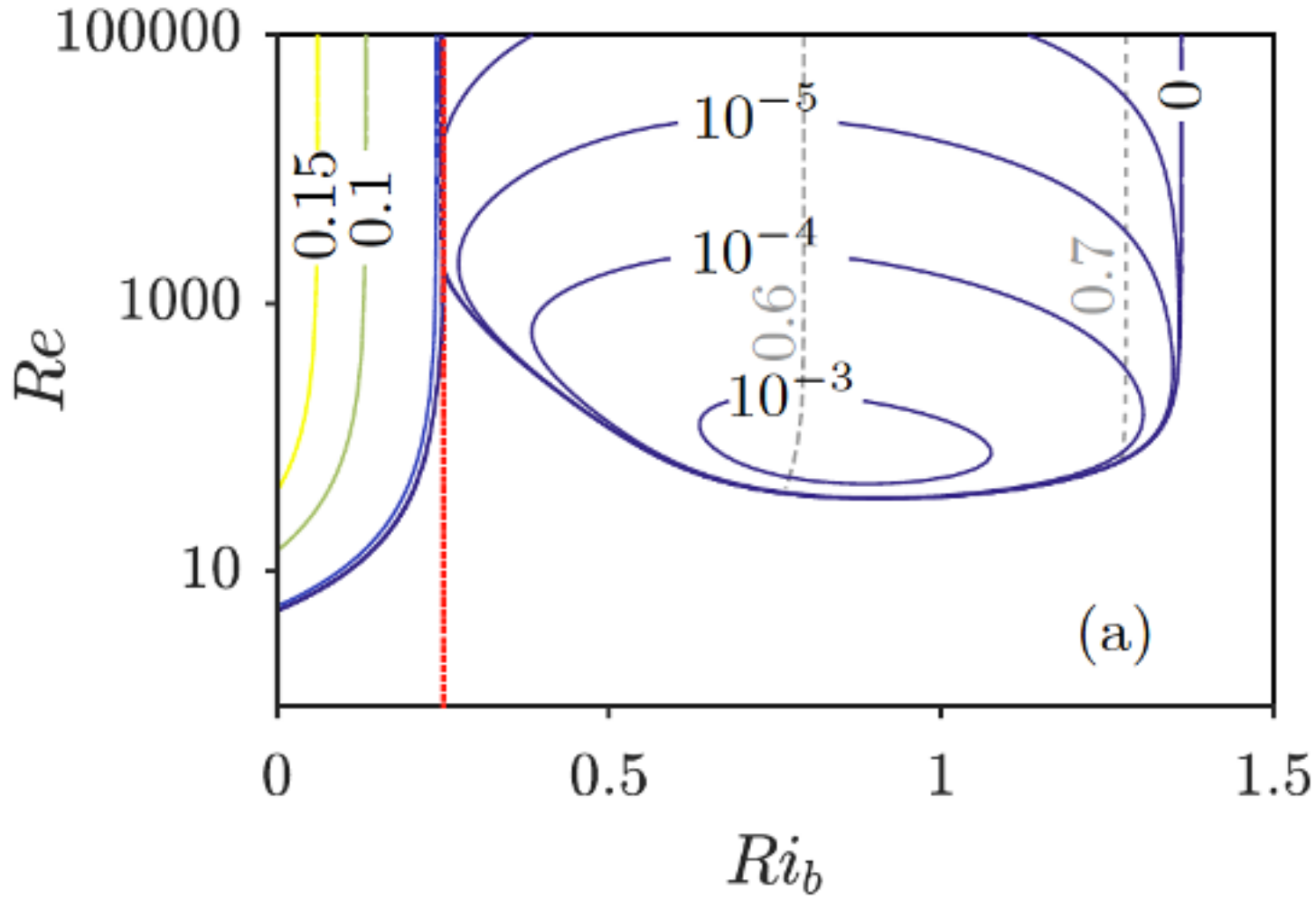


(b) Region 2



New weak instability…



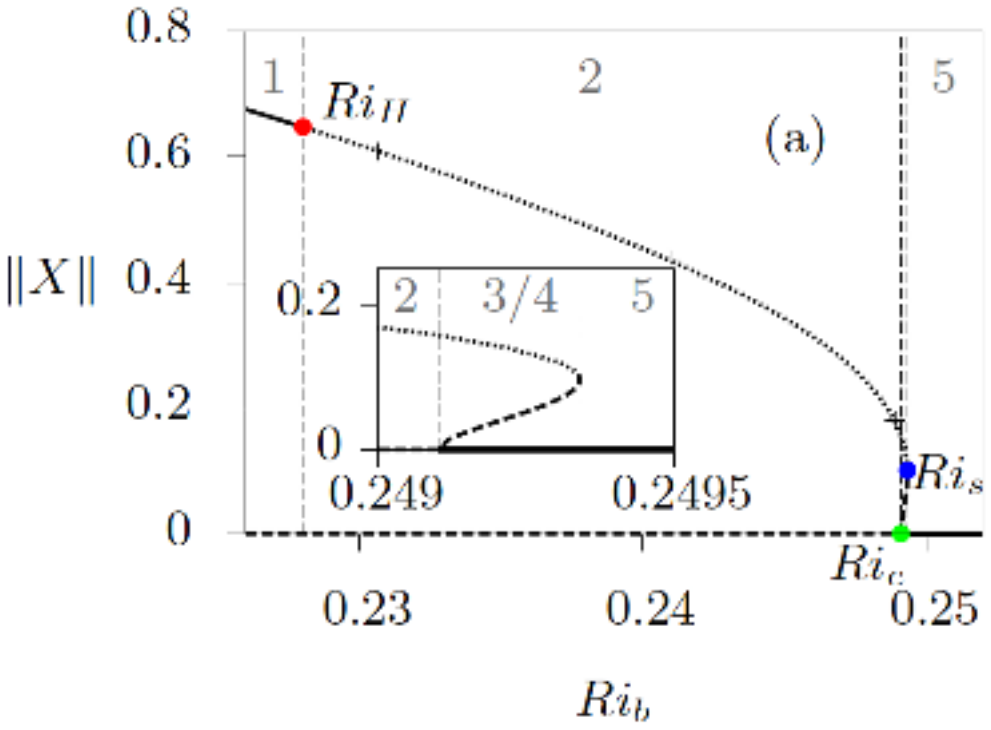


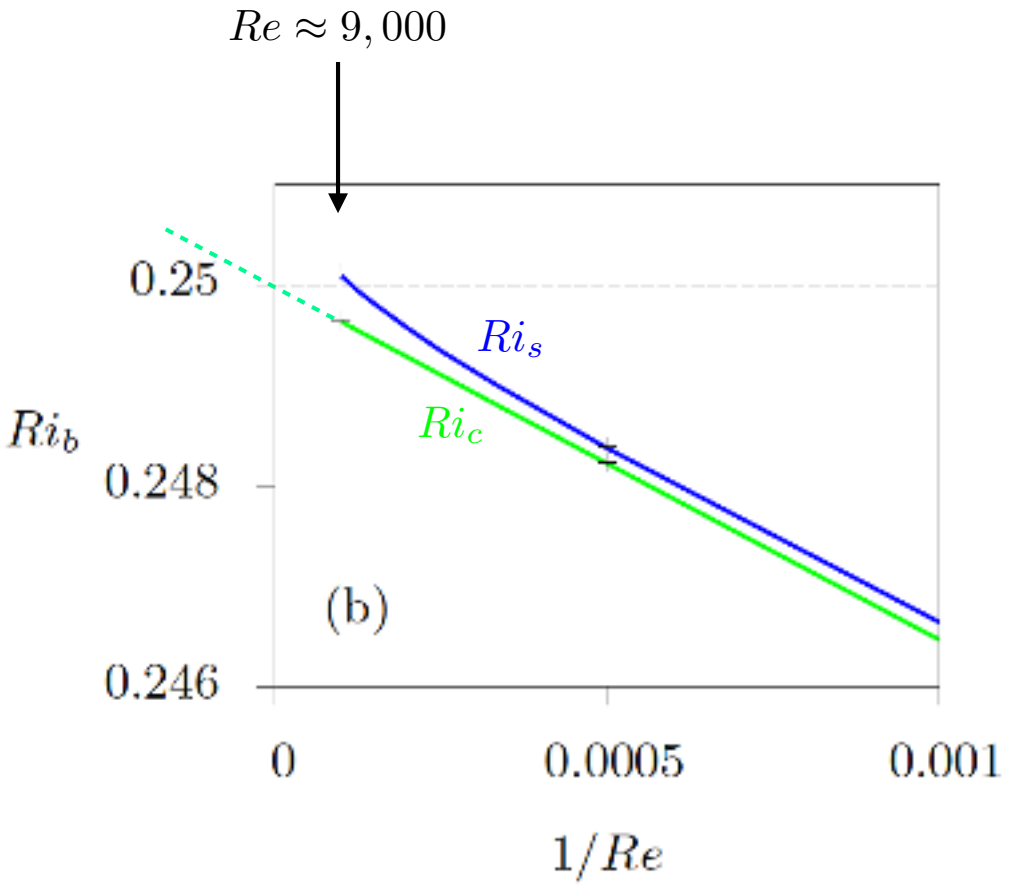
Viscous instability

related to inviscid Holmboe instability?

Change stratification to remove instability….

$$u_0 = \tanh z, \quad \rho_0 = -z$$ (Drazin model)





(+ are done at 256 x 1024)

B. Take Home Messages

6. 2D finite-amplitude Kelvin-Helmholtz billows exist beyond Ri=1/4

- confirming anecdotal evidence (Kaminski et al. 2017, Howland et al. 2018)

- need to explore Pr dependence…

7. Very weak viscous instability found

- too weak to be physically interesting but why unreported?

- is it Holmboe or not? (& who cares?)

Parker, Caulfield & Kerswell (arxiv:1905.04009)

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