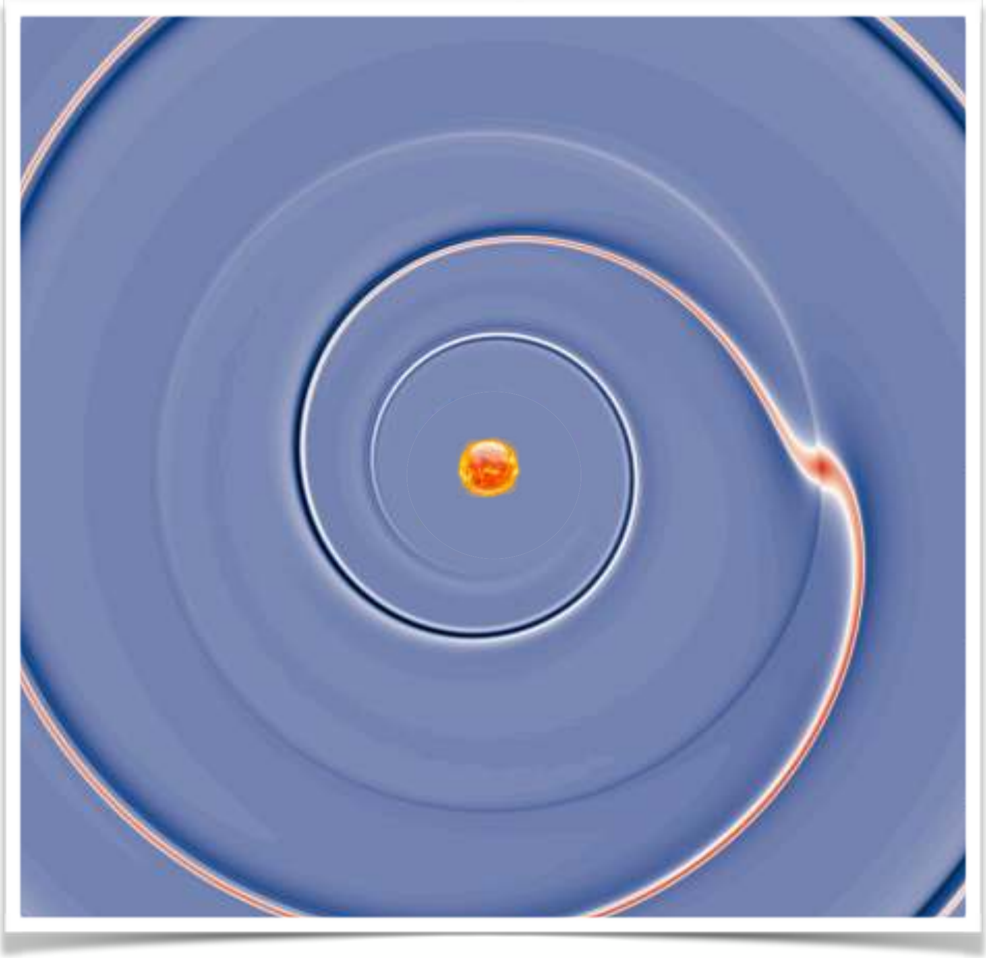
Lecture 4: disk-planet interactions and planetary migration



Suggested references:

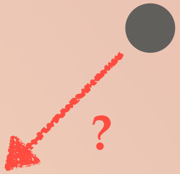
- Baruteau et al. 2016, ***Formation, Orbital and Internal Evolutions of Young Planetary Systems***
  arxiv.org/abs/1604.07558
- Baruteau et al. 2014, ***Planet-Disc Interactions and Early Evolution of Planetary Systems***
  arxiv.org/abs/1312.4293

Planet formation and orbital evolution

planet-disc interactions

change planets semi-major axes (planetary *migration*)

damp eccentricities and inclinations

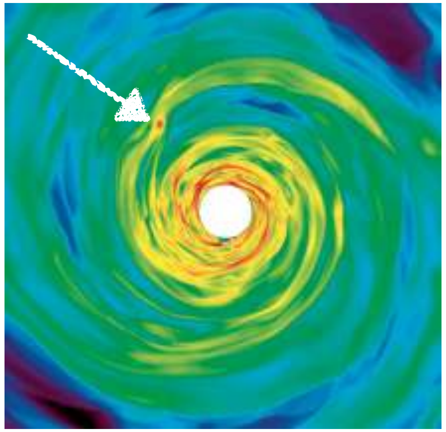


protoplanetary disc

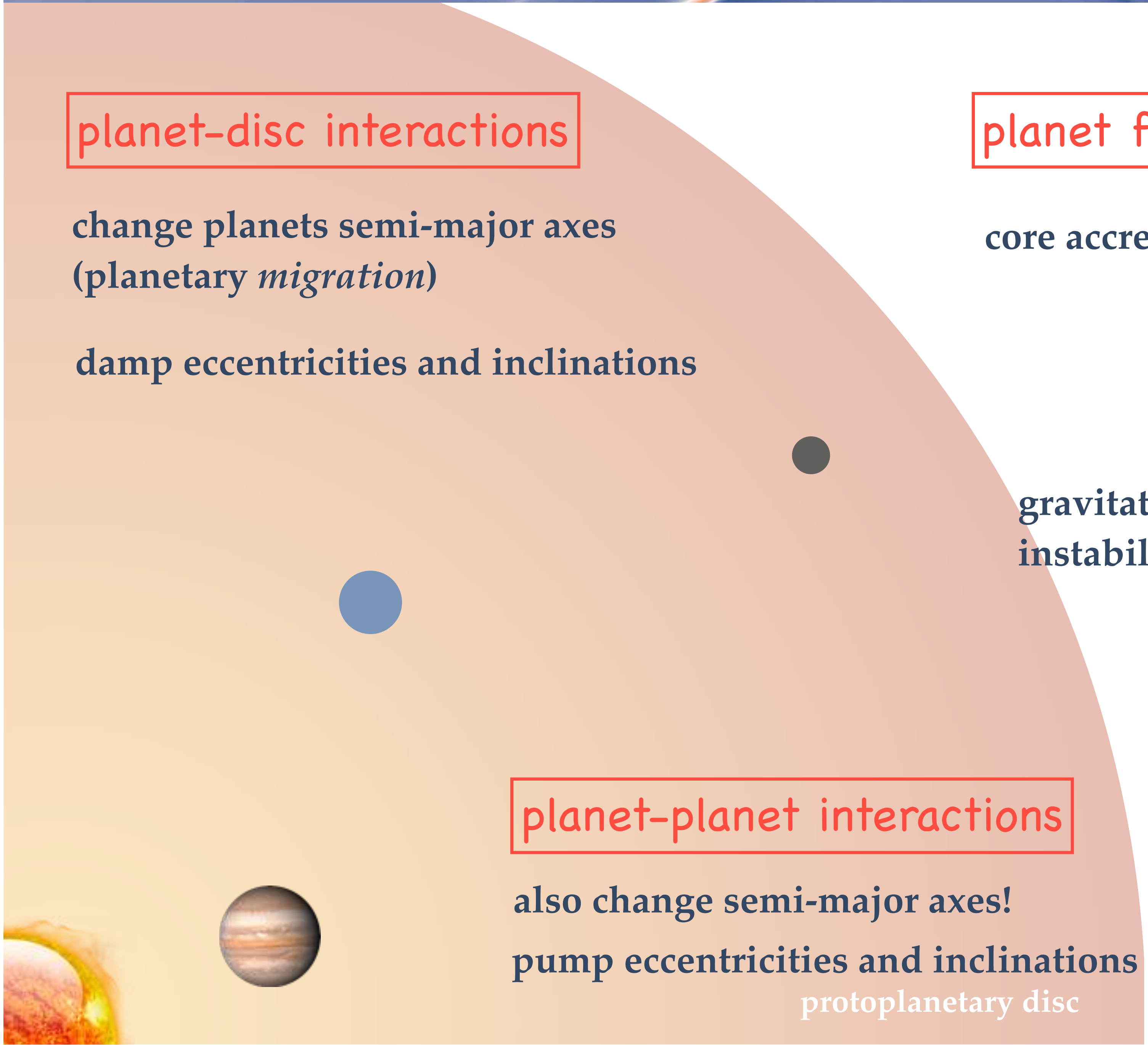
planet formation

core accretion?

gravitational instability?



Planet formation and orbital evolution



planet-disc interactions

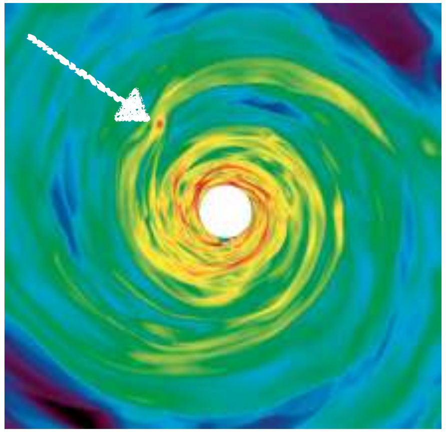
change planets semi-major axes
(planetary *migration*)

damp eccentricities and inclinations

planet formation

core accretion?

gravitational
instability?



planet-planet interactions

also change semi-major axes!

pump eccentricities and inclinations

Planet formation and orbital evolution

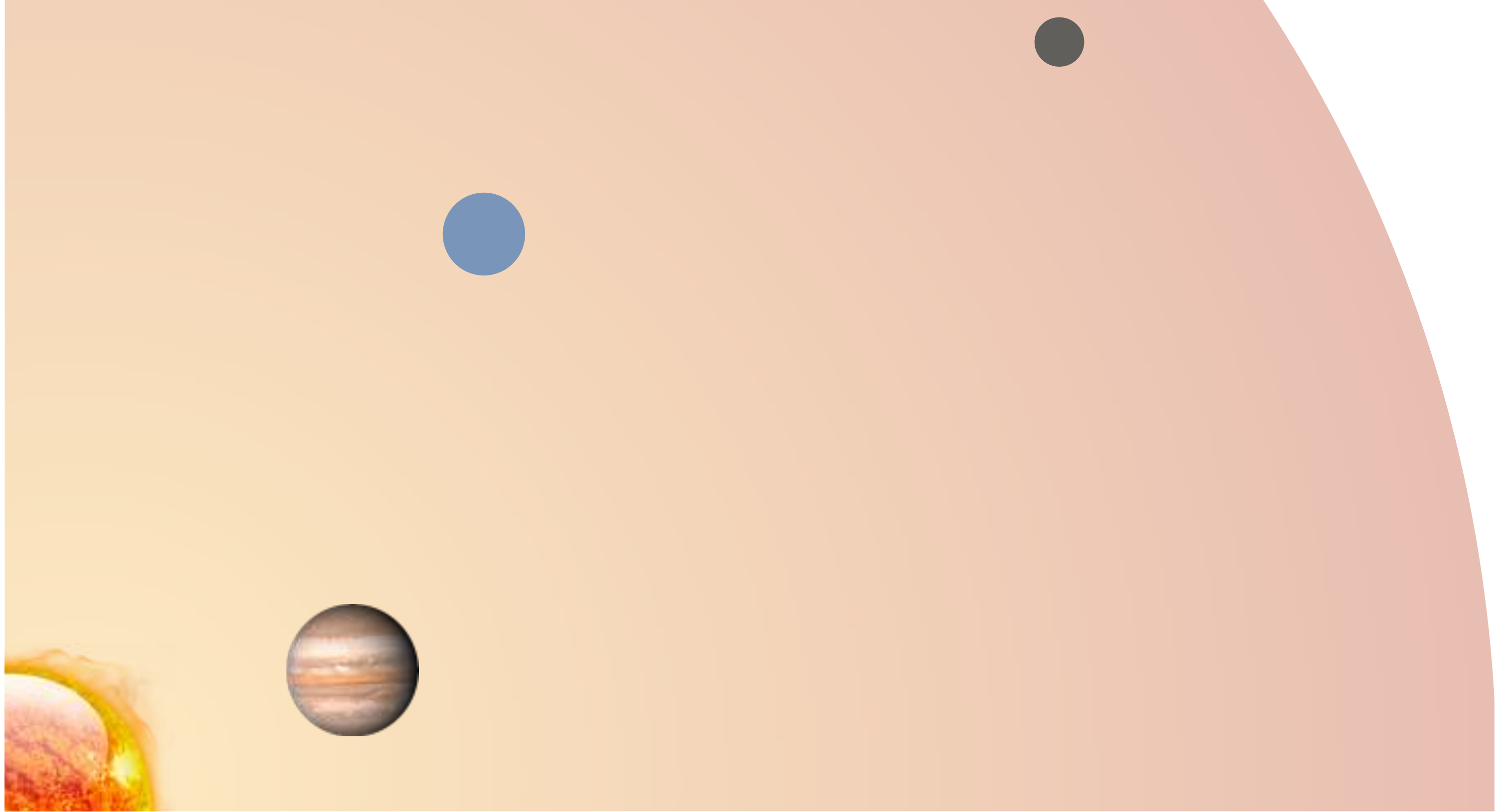
disc dispersal

(after 1-10 Myr)

. **interactions with the central star** (tides, stellar evolution) **or with nearby stars**

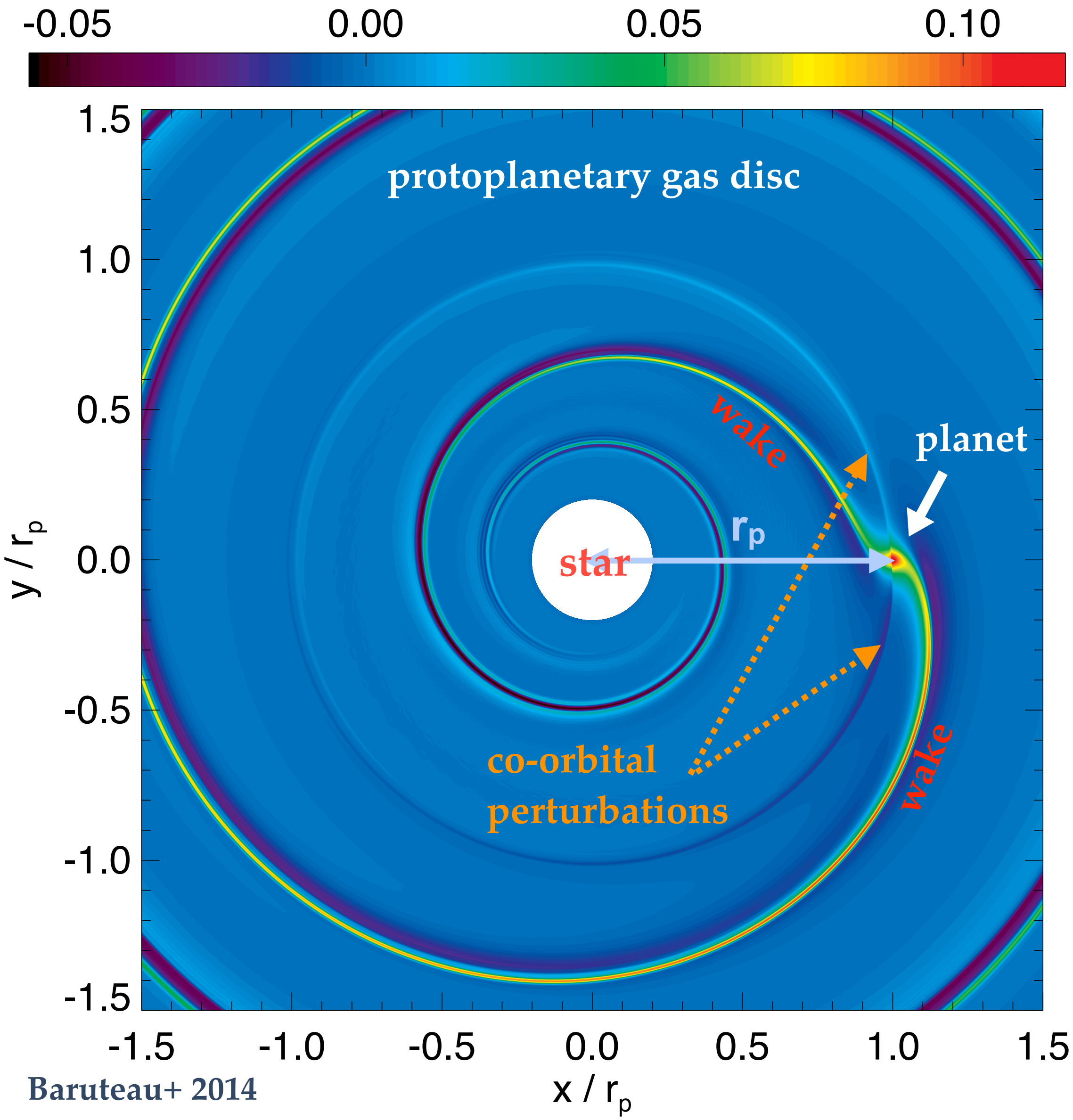
. **planet-planet interactions**

. **planets-debris disc interactions**



Disc migration of low-mass planets

(typically up to ~ 10 Earth masses)



← gas density perturbation by a 5 Earth-mass planet

Baruteau+ 2014

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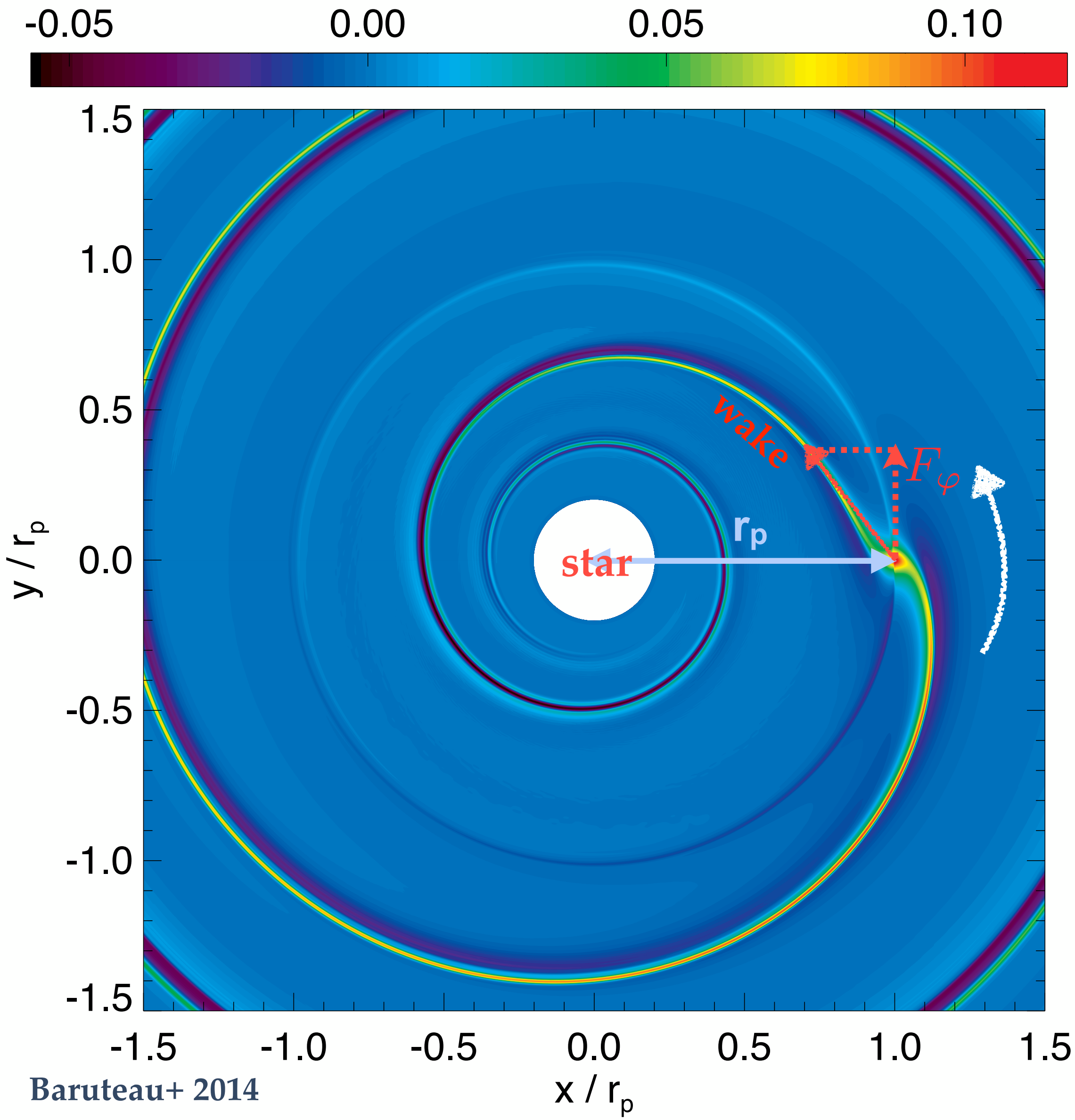

← gas density perturbation by a 5 Earth-mass planet

Baruteau+ 2014

Torque on planet by **inner** wake

$\sim r_\mathrm{p} \times F_\varphi > 0$

→ moves planet further **out**

Disc migration of low-mass planets

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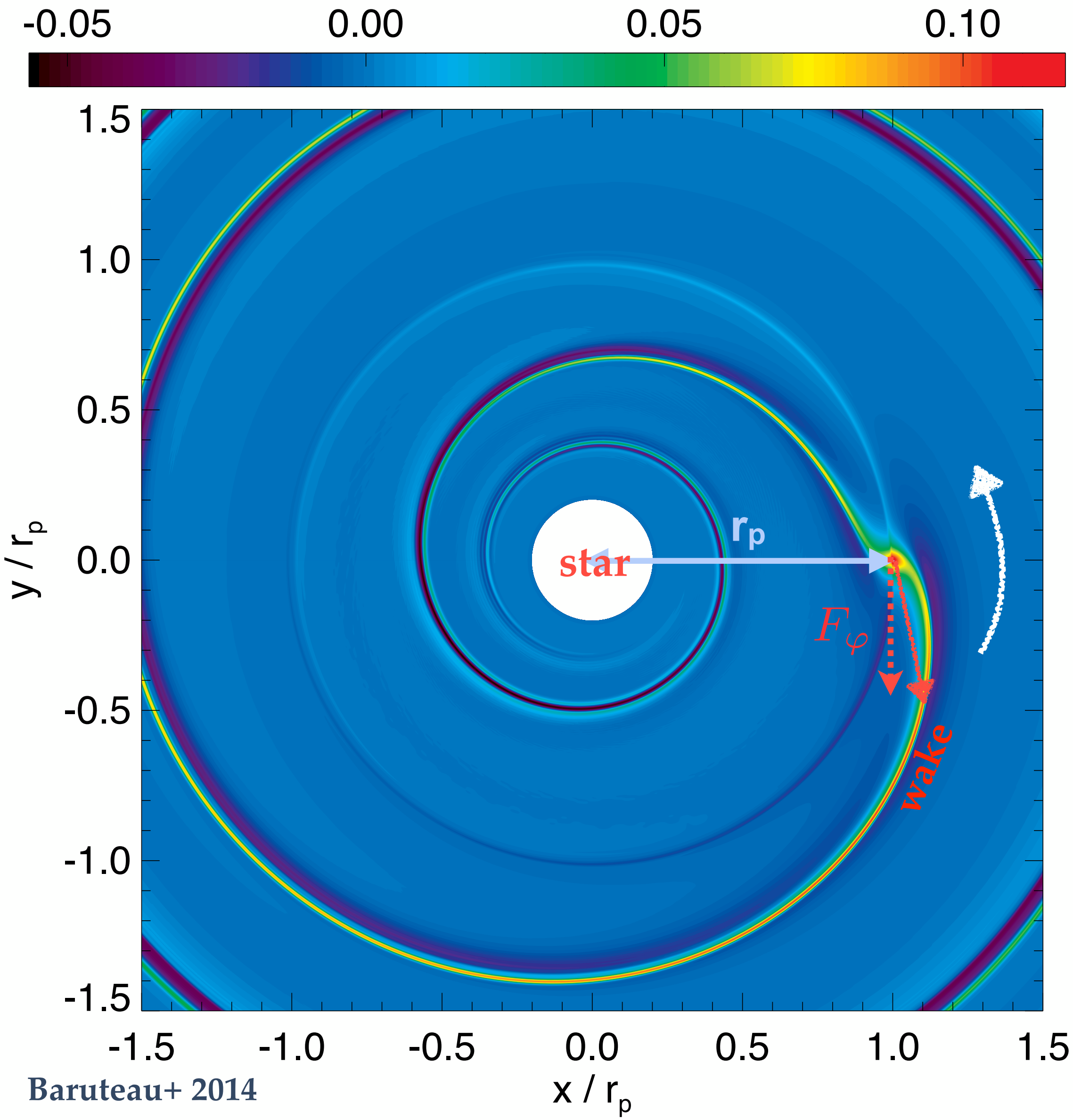


← gas density perturbation by a 5 Earth-mass planet

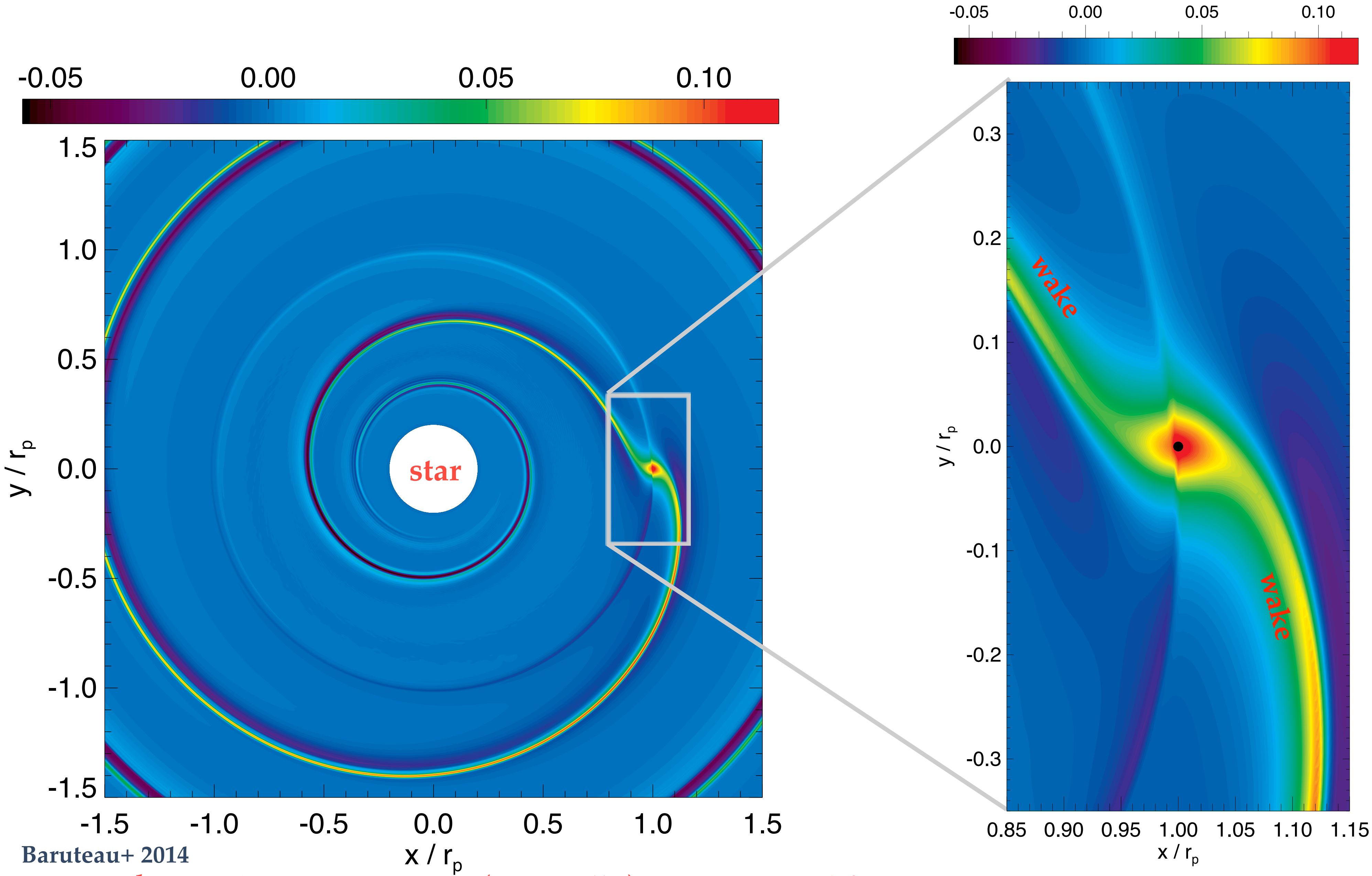
Baruteau+ 2014

Torque on planet by **outer** wake $\sim r_\mathrm{p} \times F_\varphi < 0$

→ moves planet further **in**!

Disc migration of low-mass planets

(typically up to ~ 10 Earth masses)



Baruteau+ 2014

The **total wake torque** is (generally) **negative** and favors **inward migration**

Disc migration of low-mass planets

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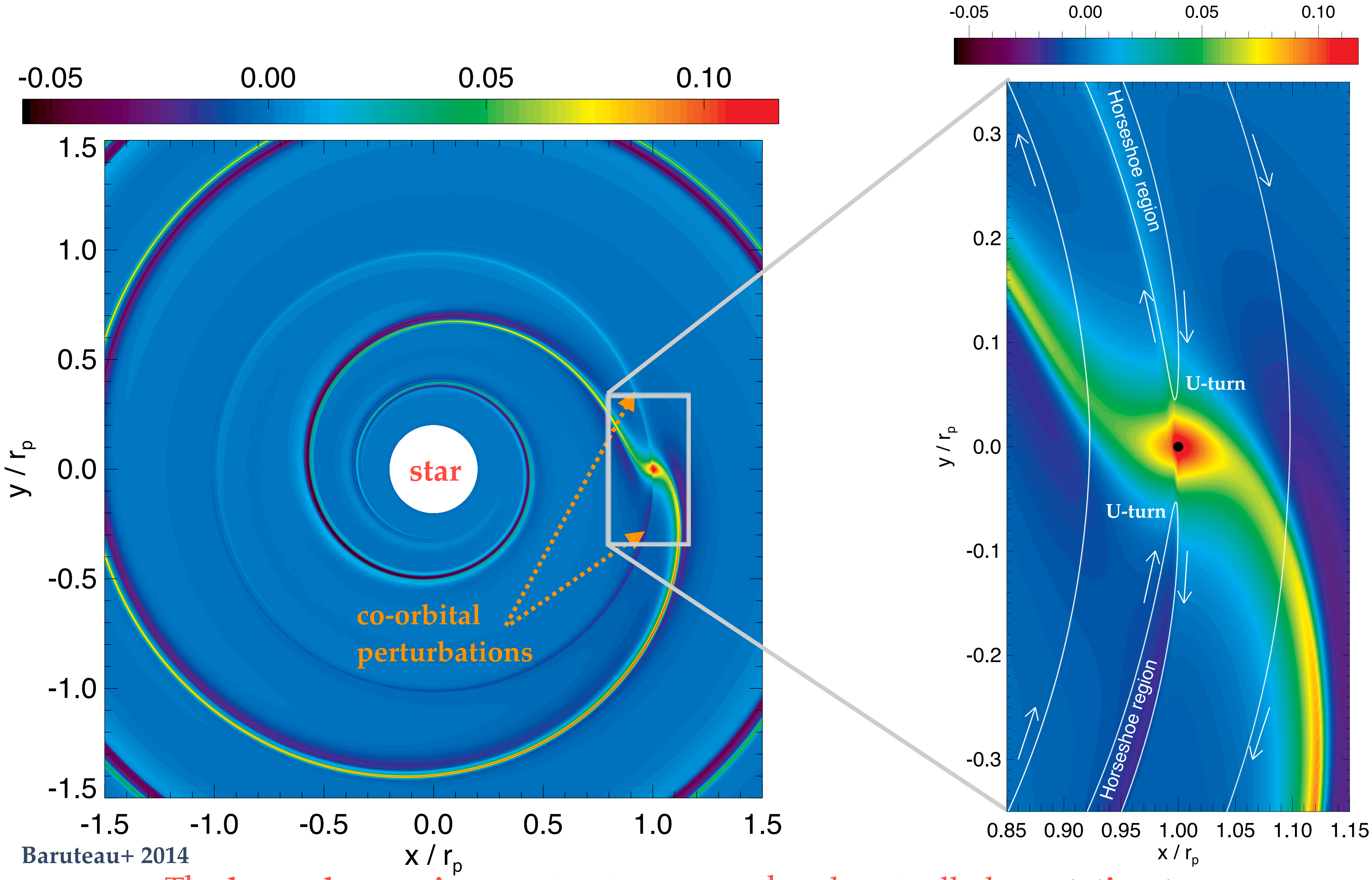

Baruteau+ 2014

The **horseshoe region** exerts a torque on the planet called **corotation torque**

Disc migration of low-mass planets

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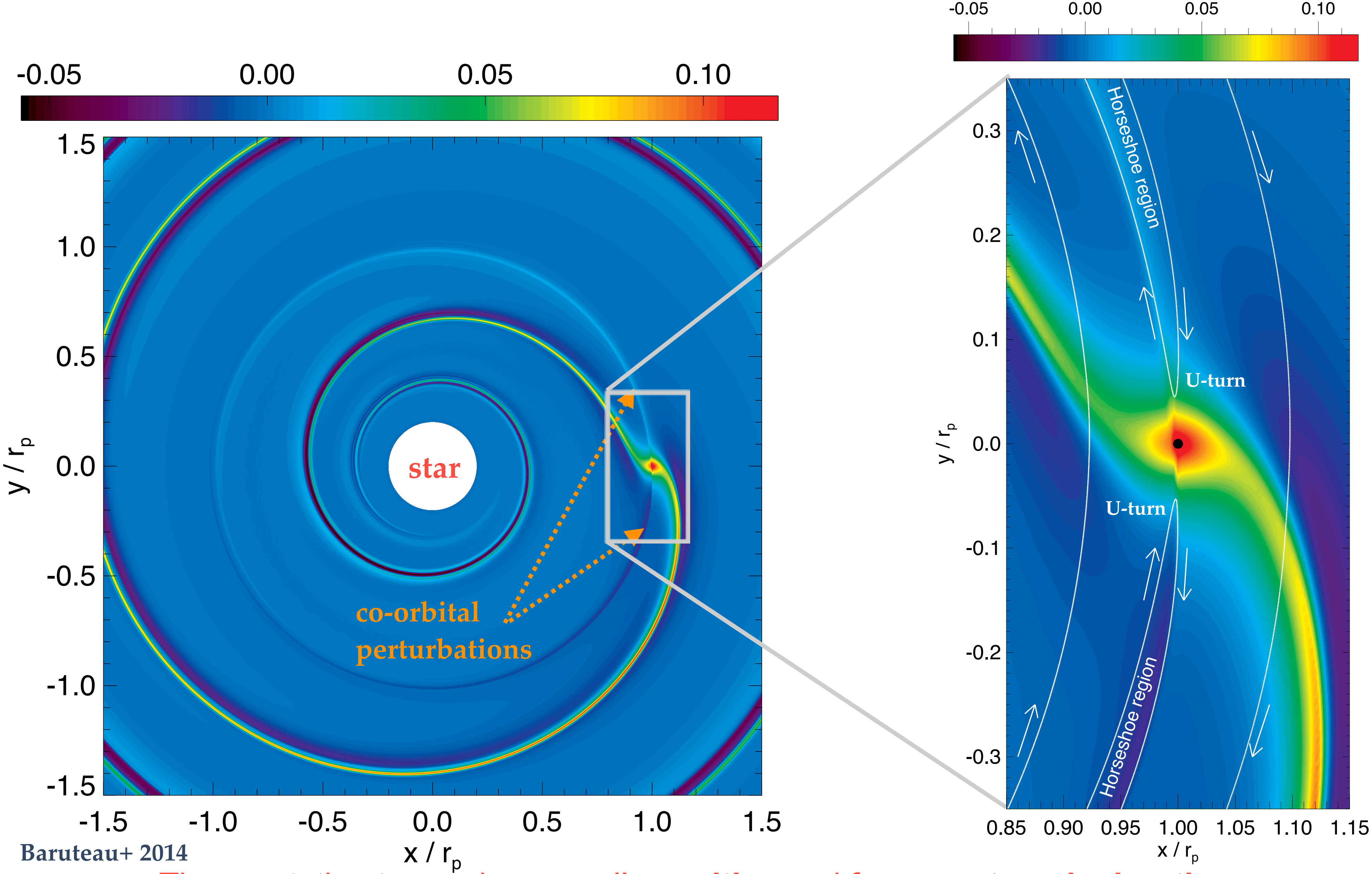


Baruteau+ 2014

The corotation torque is generally **positive** and favors **outward migration**

Focus on the planet wakes

- They are the superposition of spiral density waves emitted at **Lindblad** resonances
  - linear problem: **Lindblad resonances =** where the gas azimuthal velocity *relative* to the planet matches ±the phase velocity of acoustic waves in the azimuthal direction

Dopper-shifted wave frequency

- $v_\varphi - v_{\rm p} = \pm \frac{\omega}{k_\varphi} \rightarrow \Omega - \Omega_{\rm p} = \pm \frac{\omega}{m}$

azimuthal wavevector = m/R

For a gas disk without self-gravity, $\omega^2 = \kappa^2 + m^2 c_{\rm s}^2 / R^2$

horizontal epicyclic frequency $\kappa \approx \Omega$

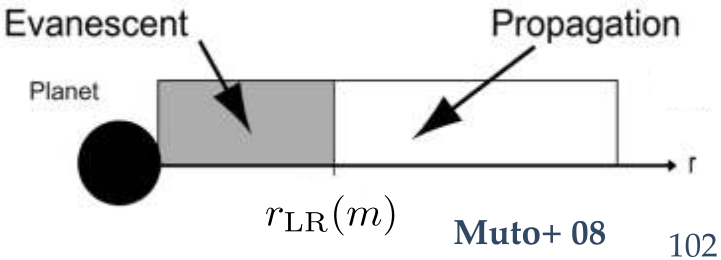
$$\Rightarrow \boxed{r_{\rm LR} = r_{\rm p}\left(1 \pm \sqrt{h^2 + m^{-2}}\right)^{2/3}}$$

disk's aspect ratio

- the 2/3 power implies that **outer** resonances lie slightly **closer** to the planet than **inner** resonances
- at large m, $v_{\rm phase} \rightarrow c_s$ and Lindlad resonances pile up at ± 2H/3 around the planet
- 2D + **WKB** approximation, the wave equation reduces to a Schrödinger-like equation **for every m:**

$$\frac{d^2 f}{dr^2} + V(r) f(r) = S(r)$$

f(r) ≡ gas perturbed enthalpy,
S(r) = forcing term due to planet's potential
V(r) defines regions of wave propagation:
V(r) > 0: waves can **propagate**
V(r) < 0: waves are **evanescent**
V(r) = 0 defines **resonances** location



Muto+ 08

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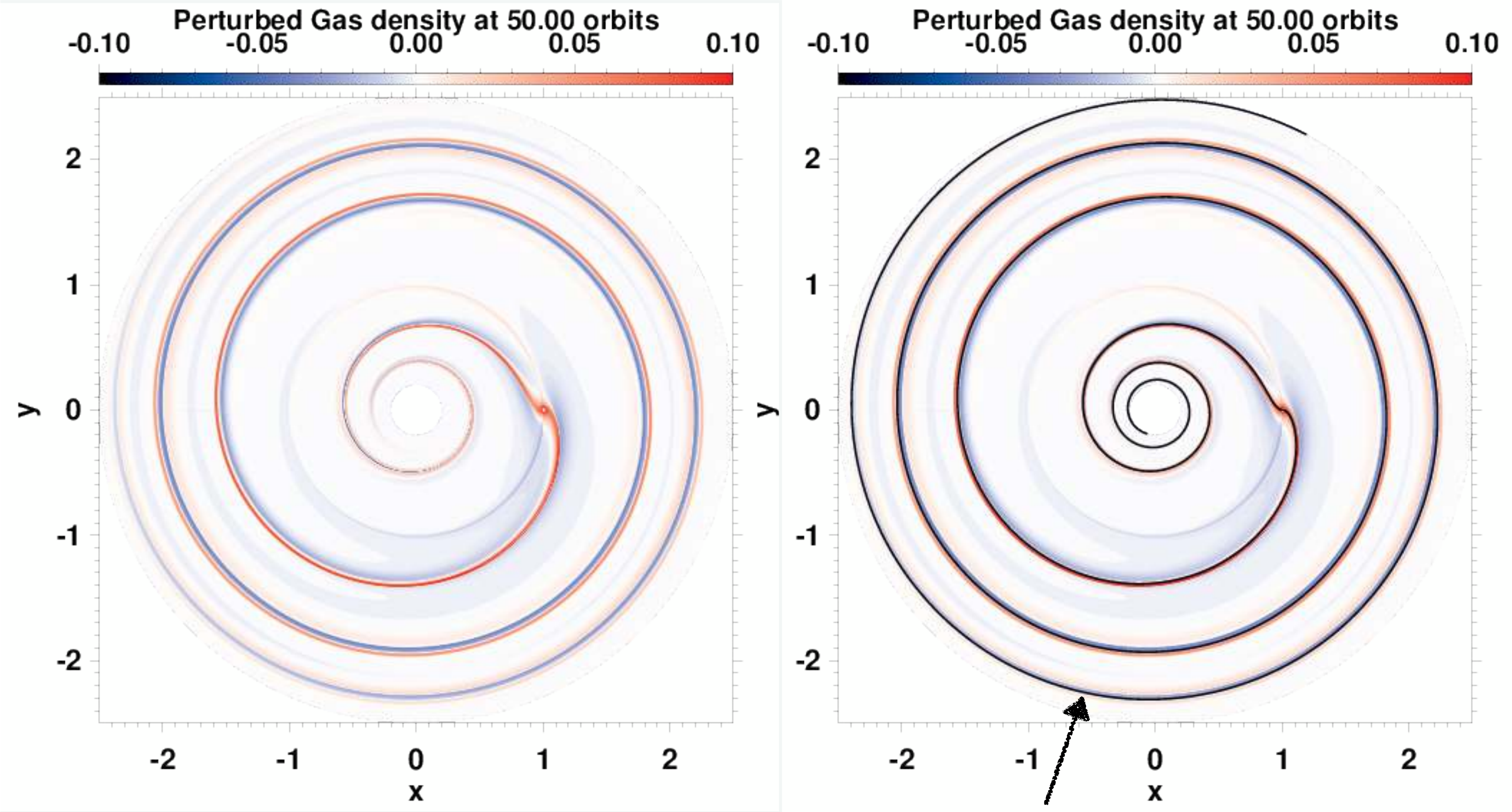
- the location of Lindblad resonances depends on whatever changes **Ω**, $\mathbf{\Omega_p}$ or the **waves dispersion relation**, like the disk's self-gravity, magnetic field…
- waves launched at Lindblad resonances **interfere constructively** into an one-armed spiral wave, called the planet's **wake**, which co-rotates with the planet

Ogilvie & Lubow 02

Focus on the planet wakes



waveform based on linear theory
(Ogilvie & Lubow 02)

Thanks for your attention!