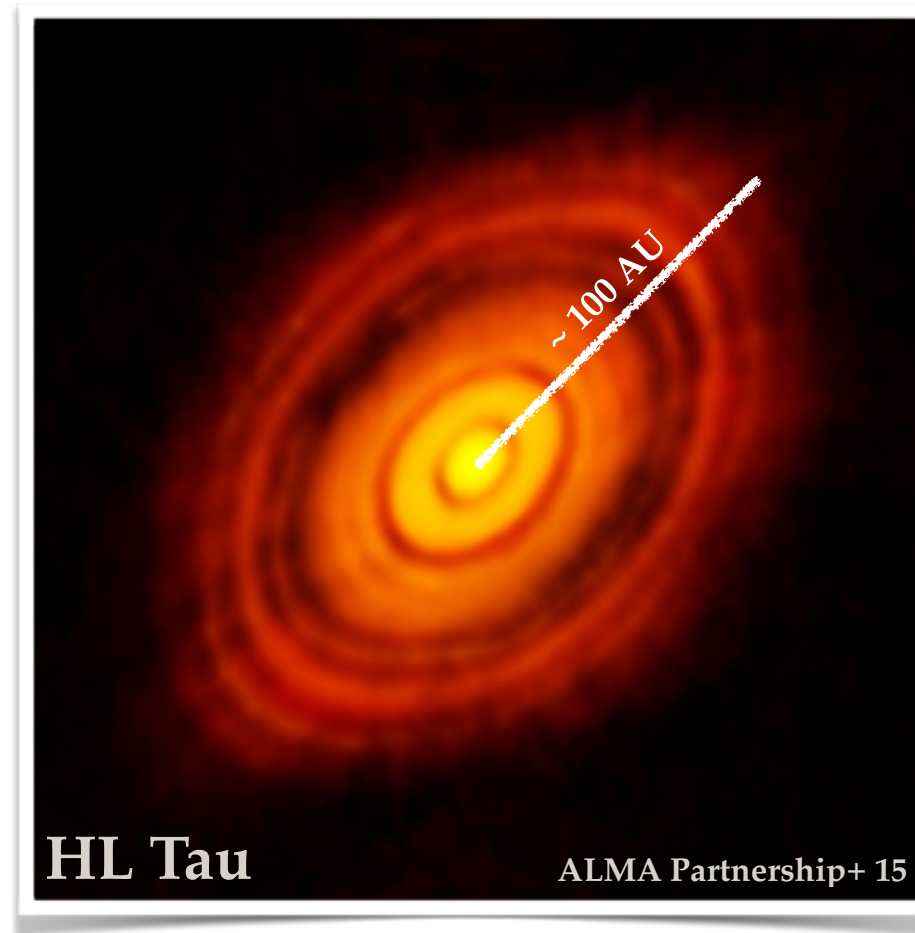


# Lecture 3: dust dynamics in protoplanetary disks

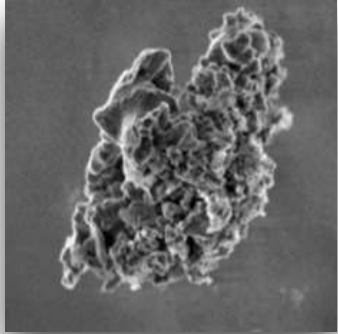


## Suggested references:

- Armitage 2011, **Lecture notes on the formation and early evolution of planetary systems**  
[arxiv.org/abs/astro-ph/0701485](https://arxiv.org/abs/astro-ph/0701485)
- Johansen et al. 2014, **The multifaceted planetesimal formation process**  
[arxiv.org/abs/1402.1344](https://arxiv.org/abs/1402.1344)

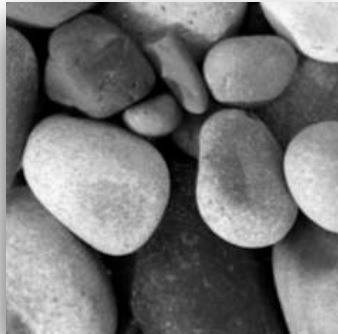
# From dust to planets

dust grains



$\mu\text{m}$

pebbles



cm

planetesimals



km

planet cores



$\sim 10^3$  km

giant planets



$\sim 10^5$  km



grains growth



planetesimals  
formation

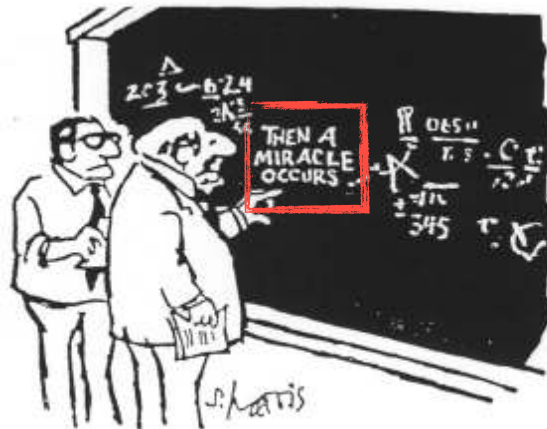


planetesimals growth  
to protoplanets



growth/accretion  
to gas giants

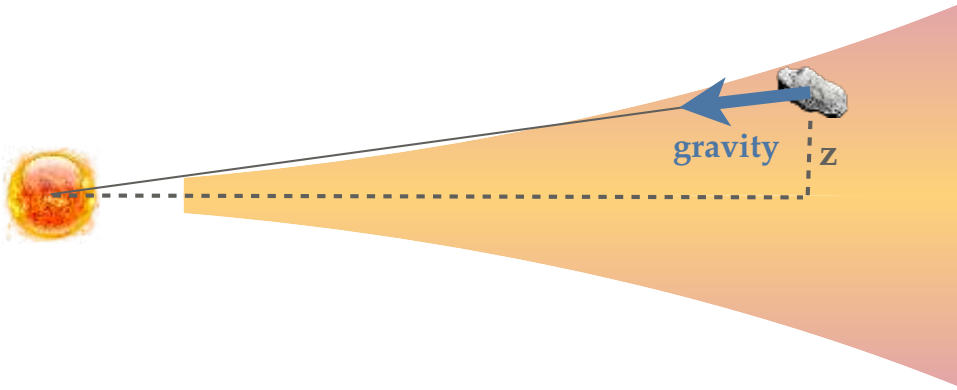
surface forces



I think you should be a little more specific, here in Step 2

gravity

# Vertical settling of dust particles



- Dust feels vertical stellar gravity and gas drag (but no pressure):

$$\ddot{z} = -\Omega_K^2 z - \frac{\dot{z} - v_{z,\text{gas}}}{\tau_{\text{stop}}} \quad [1]$$

→ damped harmonic oscillator

solution is  $z(t) = z_0 e^{-\frac{t}{2\tau_{\text{stop}}}} e^{\pm i\Omega_K t} \sqrt{1 - \frac{1}{4\text{St}^2}}$  if  $\text{St} = \tau_{\text{stop}}\Omega_K > 1/2$

→ large grains have a **settling time**  $\sim \tau_{\text{stop}} \equiv \frac{\text{St}}{\Omega_K}$

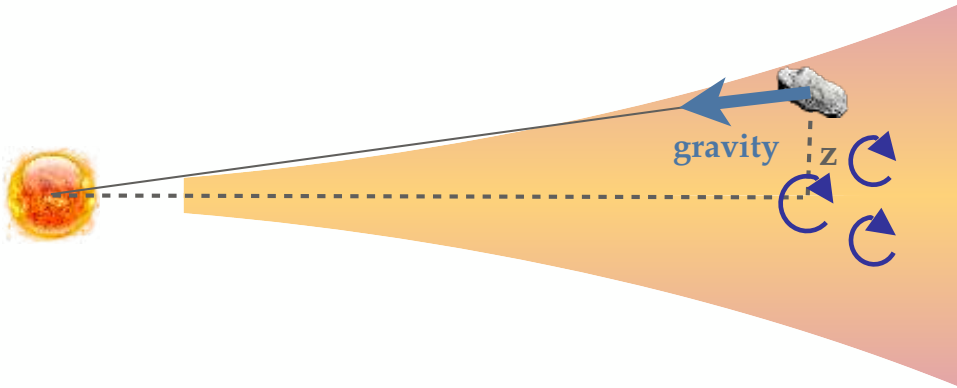
if  $\text{St} \ll 1$  approximate [1] as  $0 = -\Omega_K^2 z - \frac{\dot{z}}{\tau_{\text{stop}}} \rightarrow z(t) = z_0 e^{-\Omega_K^2 \tau_{\text{stop}} t}$

→ small grains have a **settling time**  $\sim (\Omega_K^2 \tau_{\text{stop}})^{-1} \equiv \frac{1}{\text{St}\Omega_K}$

- Solids **drift to the midplane** on a **settling timescale** such that  $\Omega_K \tau_{\text{sett}} \sim \text{St} + \text{St}^{-1}$

... which is fastest for  $\text{St} \sim 1$  solids and only of order the orbital period (1 yr at 1 AU)!

# Vertical settling of dust particles



- So how to keep dust up in the air?  
→ turbulence! (if really effective...)

**turbulent diffusion** prevents solids from settling indefinitely, and occurs on a timescale

$$\tau_{\text{diff}} \sim \frac{z^2}{D_{\text{dust}}} \quad \text{with} \quad D_{\text{dust}} \sim \frac{D_{\text{gas}}}{1 + \text{St}^2}$$

← gas turbulent viscosity

Youdin & Lithwick 2007

Settling proceeds down to a **height**  $H_d$  until turbulent diffusion starts acting against settling. That height is thus where the **settling** and **diffusion timescales** are roughly **equal**:

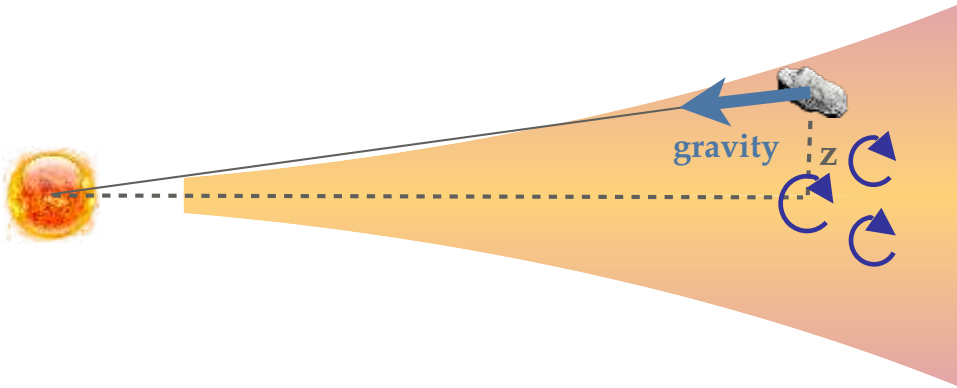
$$H_d \sim \sqrt{\frac{D_{\text{gas}}}{\Omega_K \text{St}}}$$

Writing  $D_{\text{gas}} \equiv \nu = \alpha c_s H = \alpha H^2 \Omega_K$  we obtain the dust-to-gas scale height ratio

$$\frac{H_d}{H} \sim \sqrt{\frac{\alpha}{\text{St}}}$$

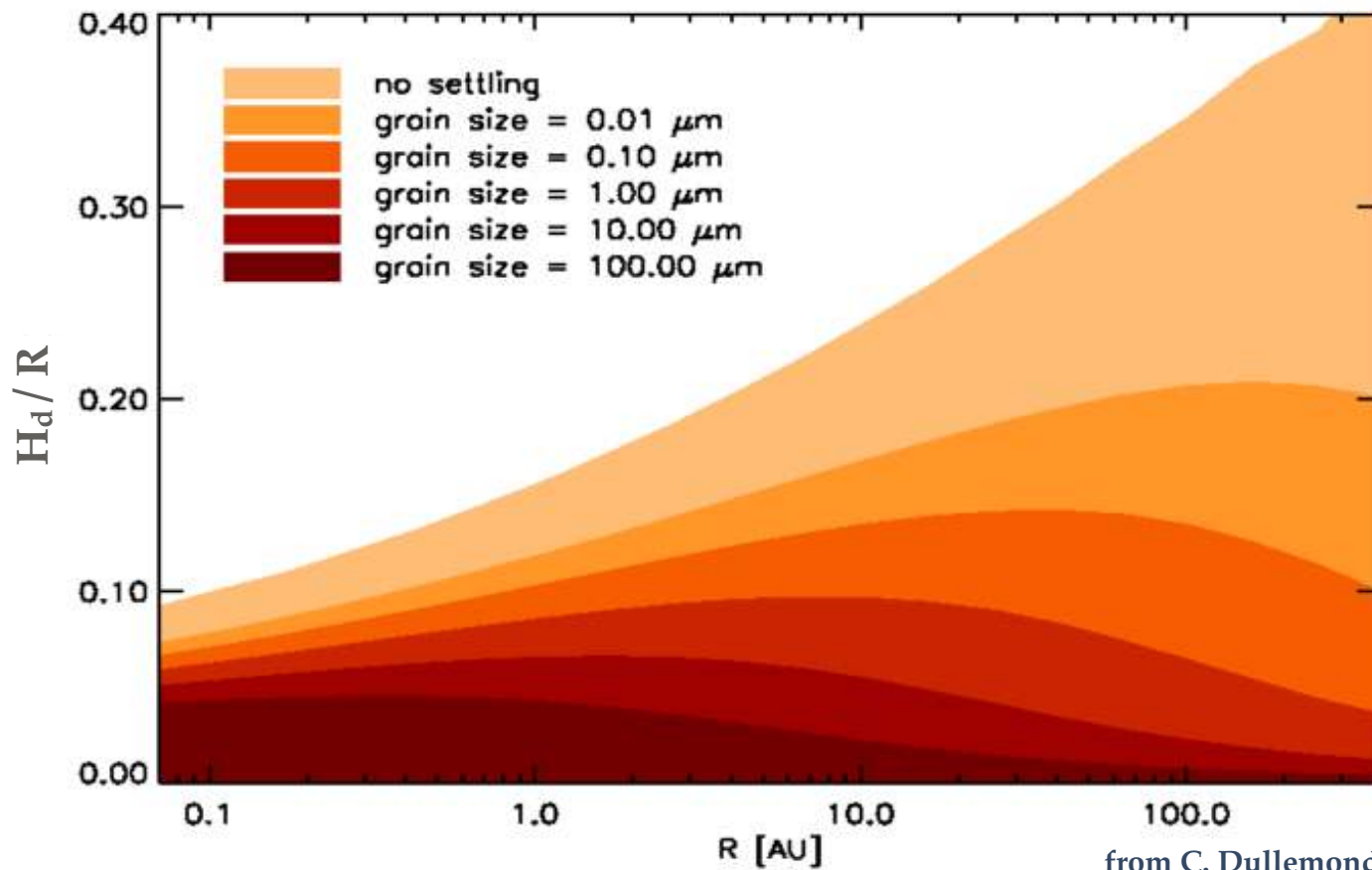
*NB:* calculation of  $H_d$  is actually a little more involved since  $\text{St}$  depends on  $z$ ...

# Vertical settling of dust particles



- So how to keep dust up in the air?  
→ turbulence! (if really effective...)

turbulent diffusion prevents solids from settling indefinitely, and occurs on a timescale



$$D_{\text{dust}} \sim \frac{D_{\text{gas}}}{1 + \text{St}^2}$$

← gas turbulent viscosity

Youdin & Lithwick 2007

acting against settling.  
roughly equal:

light ratio

$$\frac{H_d}{H} \sim \sqrt{\frac{\alpha}{\text{St}}}$$

depends on z...

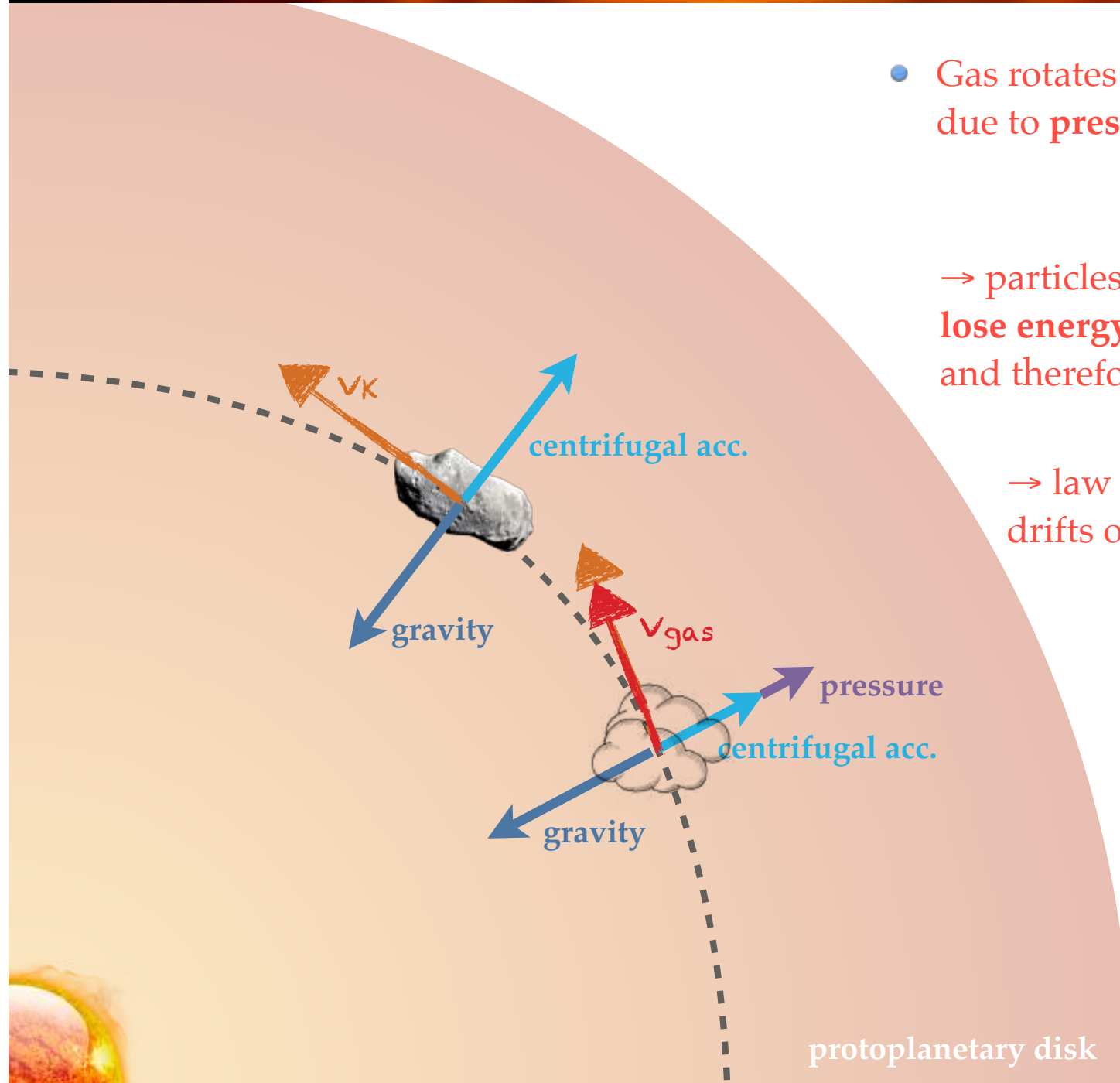
from C. Dullemond

# Radial drift of dust particles

- Gas rotates **slower** than solid particles due to **pressure support**

→ particles feel the gas like a **headwind**, **lose energy** and **angular momentum**, and therefore **drift towards the star**

→ law of action-reaction: gas slowly drifts outwards



- ❖ **drag acceleration on dust:**

$$a_{\text{drag}} = -\frac{v_{\text{dust}} - v_{\text{gas}}}{\tau_{\text{stop}}}$$

- ❖ **dust Stokes number:**

$$\text{St} = \tau_{\text{stop}} \Omega$$

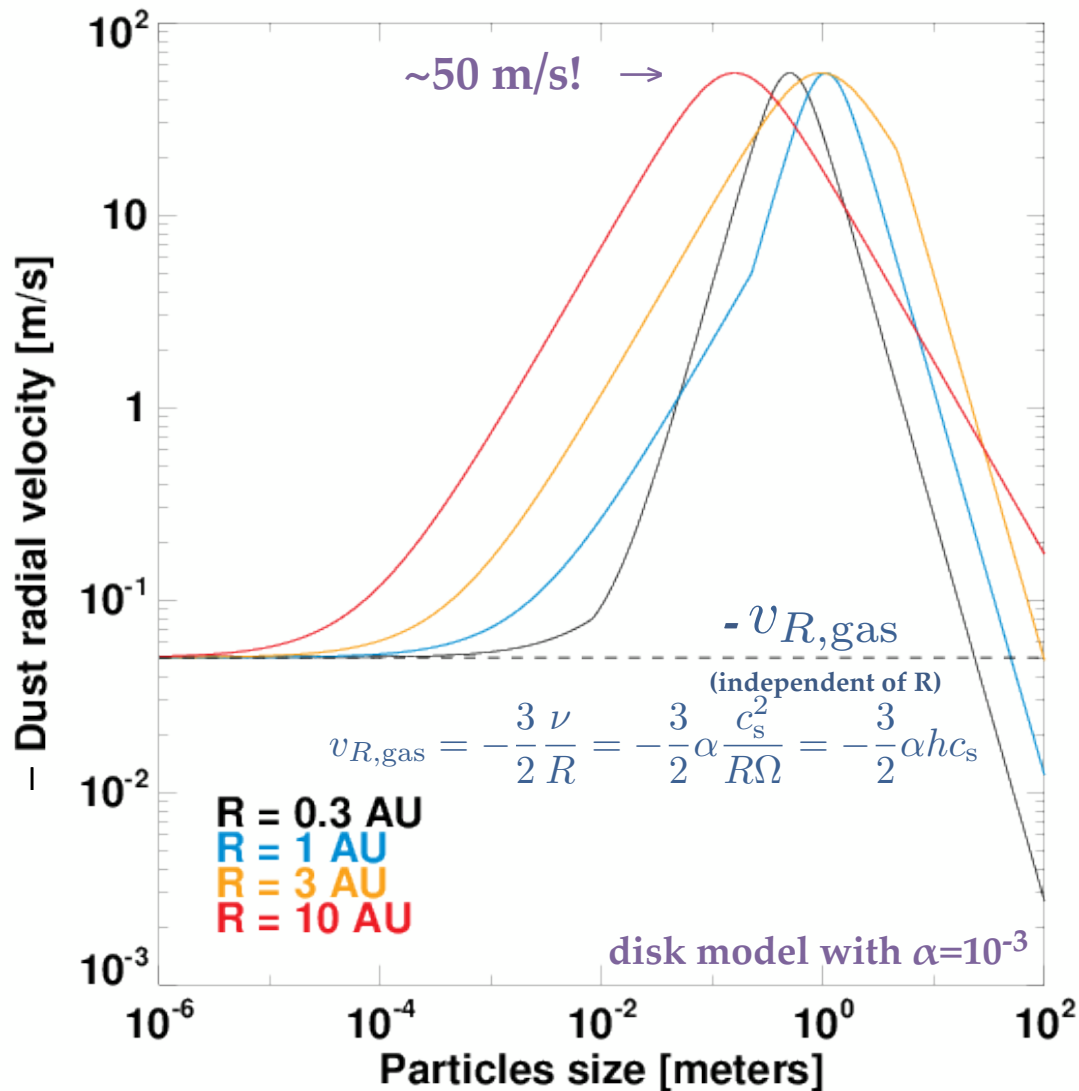
(depends on dust size, gas  $\Sigma$ )

# Radial drift of dust particles

For small dust-to-gas density ratios,  $v_{R,dust} = v_{R,gas} \frac{1}{1 + St^2} + h^2 v_K \left( \frac{\partial \log p}{\partial \log R} \right) \frac{1}{St + St^{-1}}$

means that the gas drags the solid particles, but we neglect the fact solid particles may drag the gas as well

maximum for  $St=1$  e.g., Armitage 2015

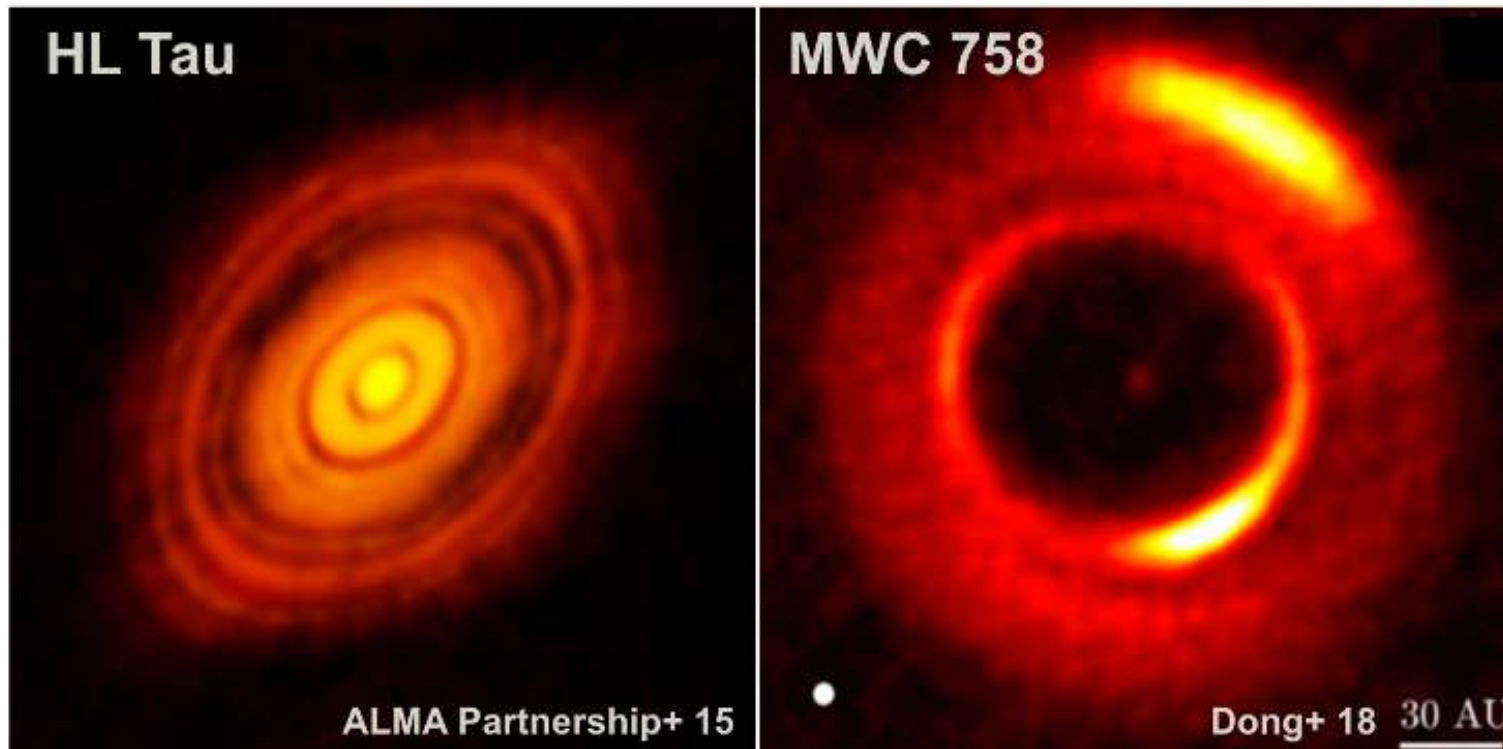
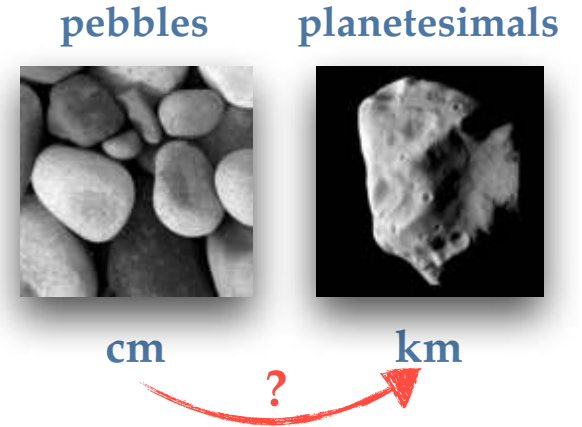


→ Meter-sized bodies only take ~100 years to reach the star from a distance of 1 AU!

→ severe constraint on the timescale for planetesimal formation!

# Radial drift of dust particles

- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?
    - ❖ invoke **pressure maxima** in the disk to trap dust?
- observational support?



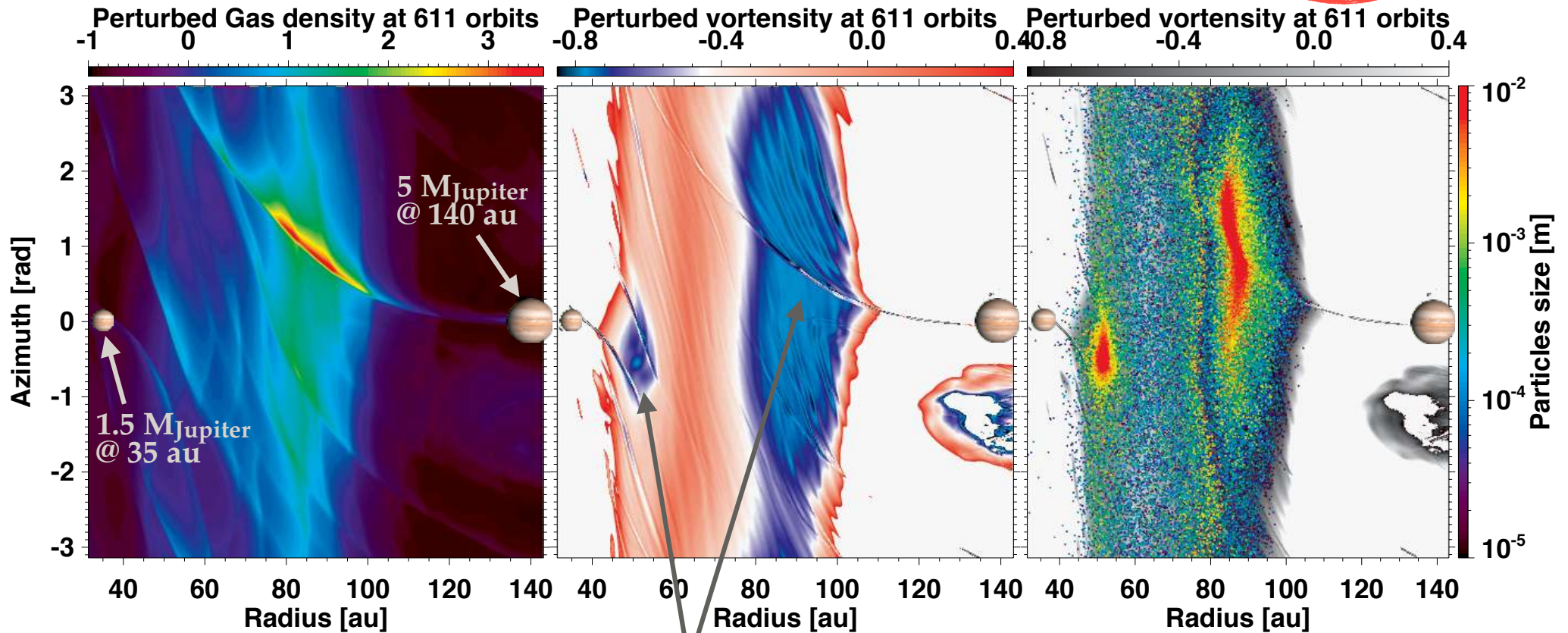
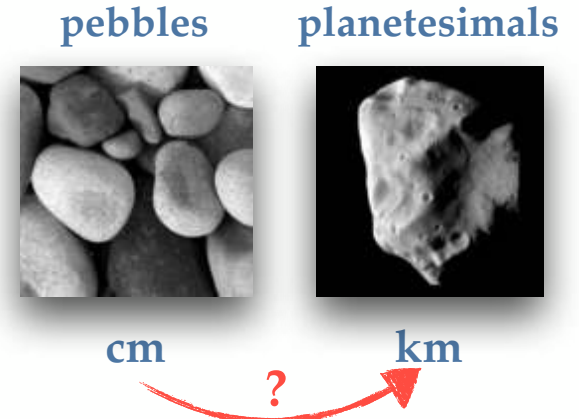


# Radial drift of dust particles

- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?

❖ invoke **pressure maxima** in the disk to trap dust?

→ observational support? a modelling example:

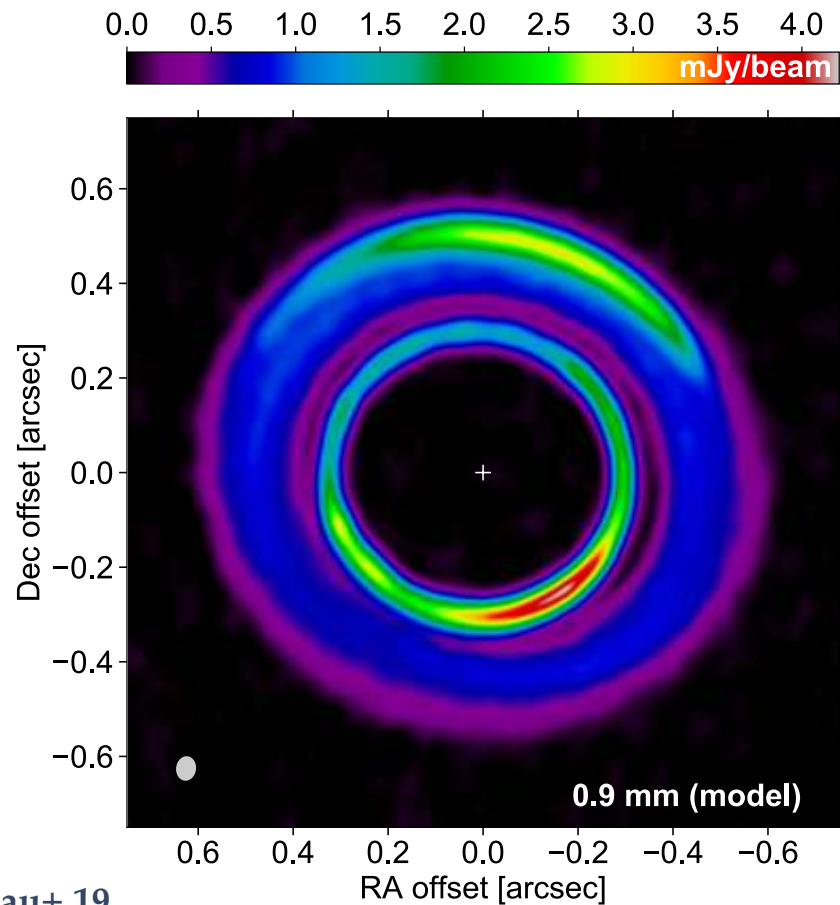
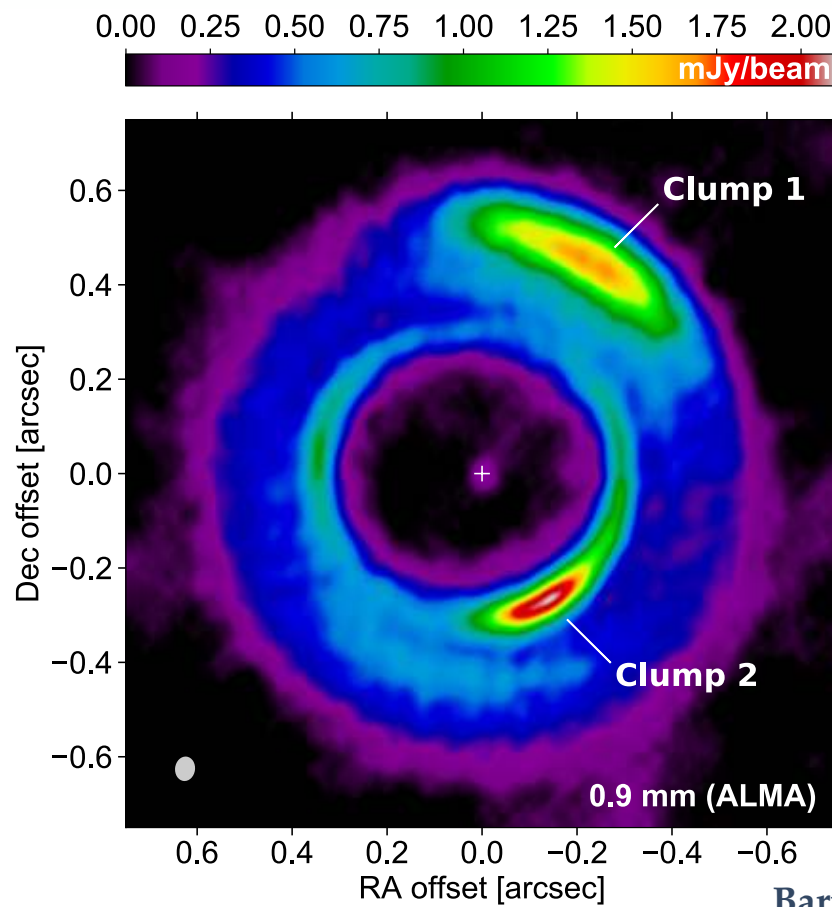
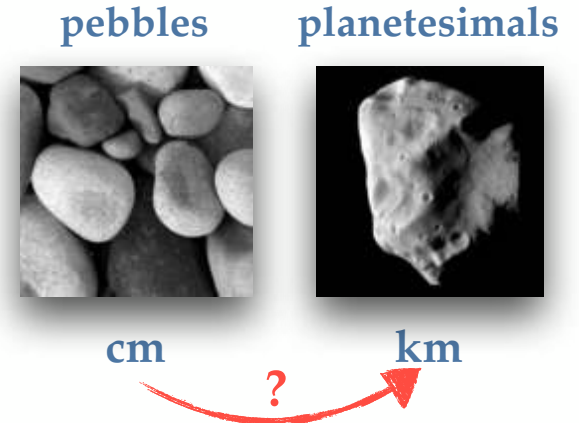


anticyclonic vortices formed by Rossby-Wave Instability

# Radial drift of dust particles

- So how do we not lose all the cm/meter-sized solids and build planetesimals?
  - ❖ invoke **pressure maxima** in the disk to trap dust?

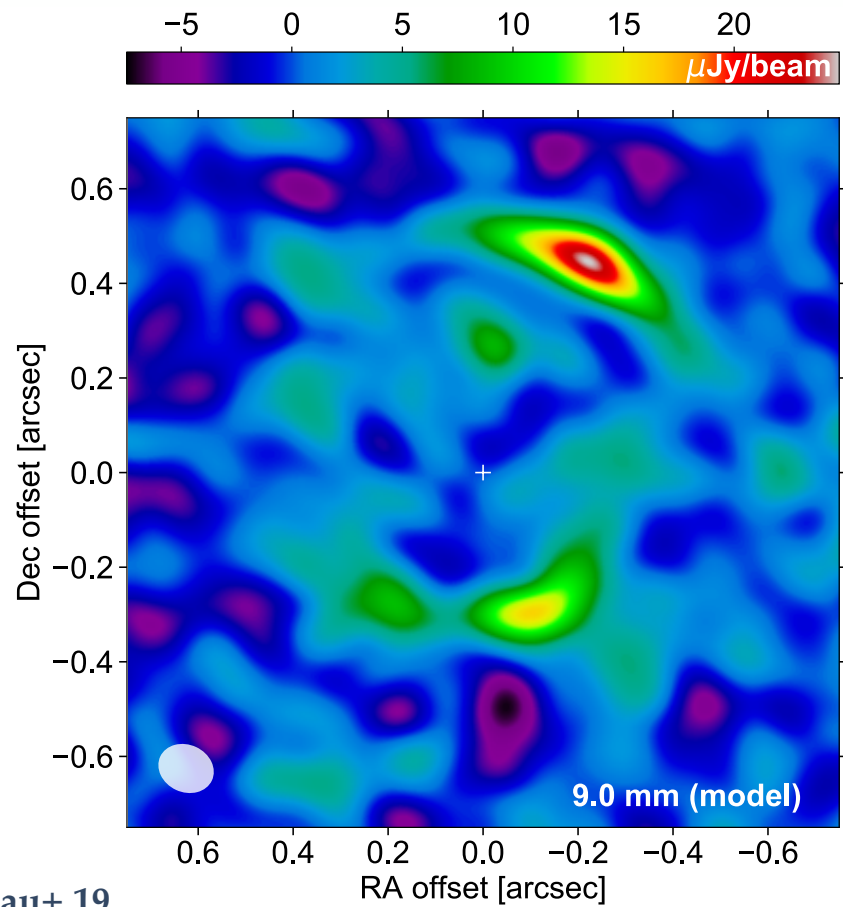
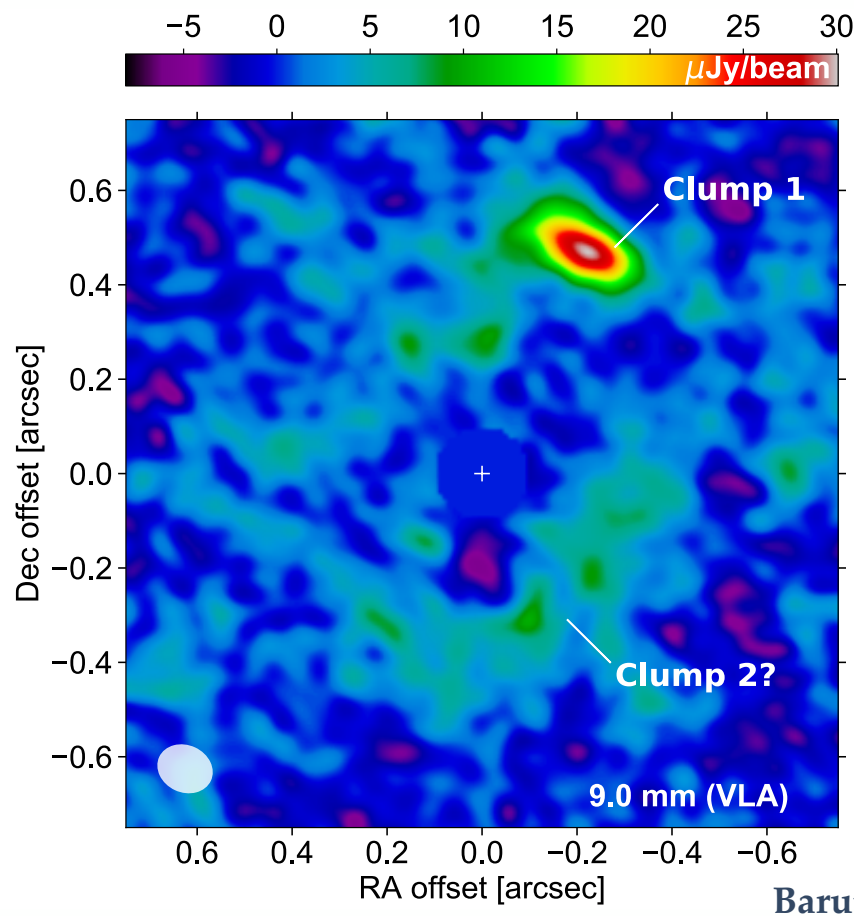
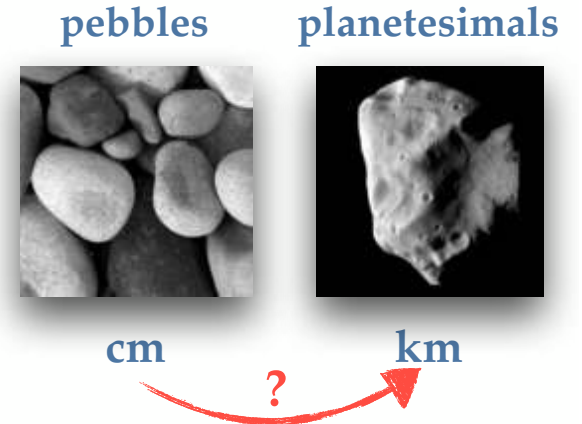
→ observational support? a modelling example:



# Radial drift of dust particles

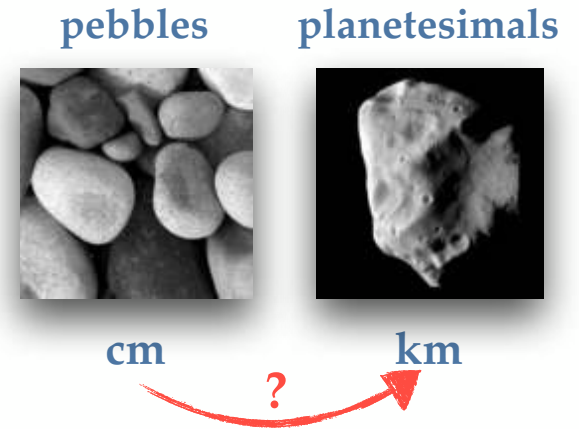
- So how do we not lose all the cm/meter-sized solids and build planetesimals?
- ❖ invoke pressure maxima in the disk to trap dust?

→ observational support? a modelling example:



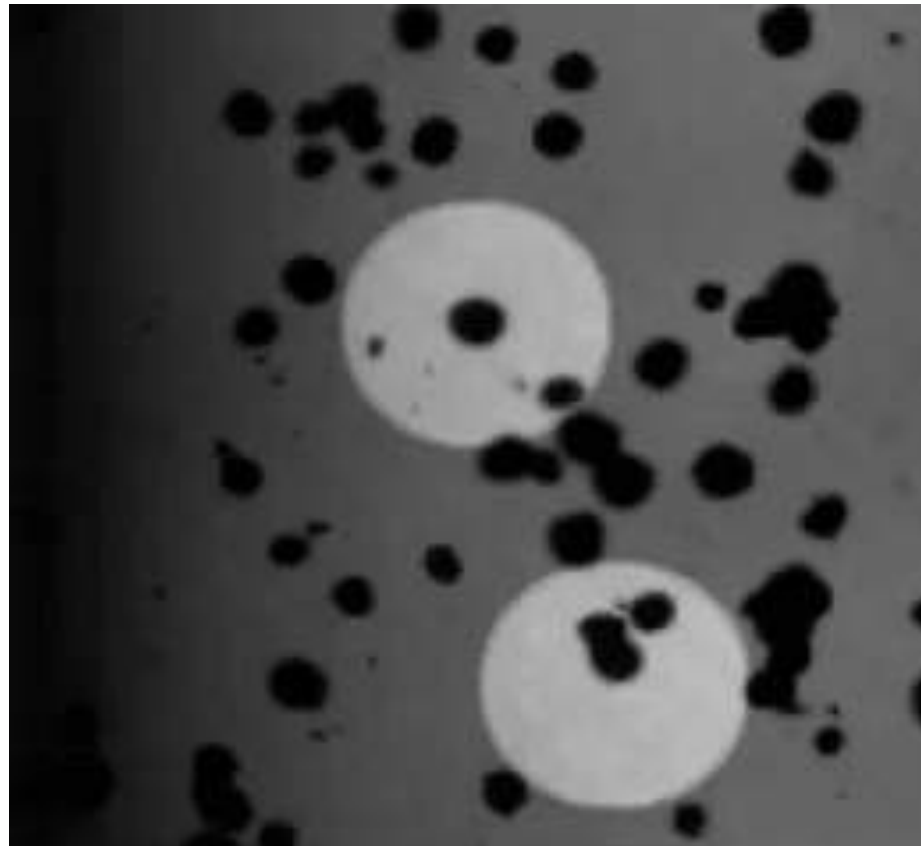
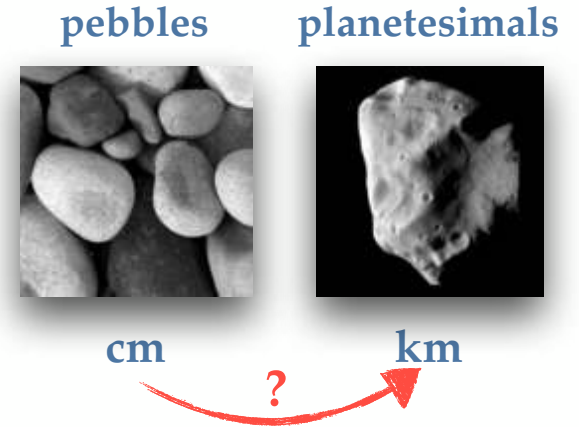
# Radial drift of dust particles

- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?
  - ❖ invoke **pressure maxima** in the disk to trap dust?
    - observational support? a modelling example:
    - but how efficient is dust growth at these locations?



# Radial drift of dust particles

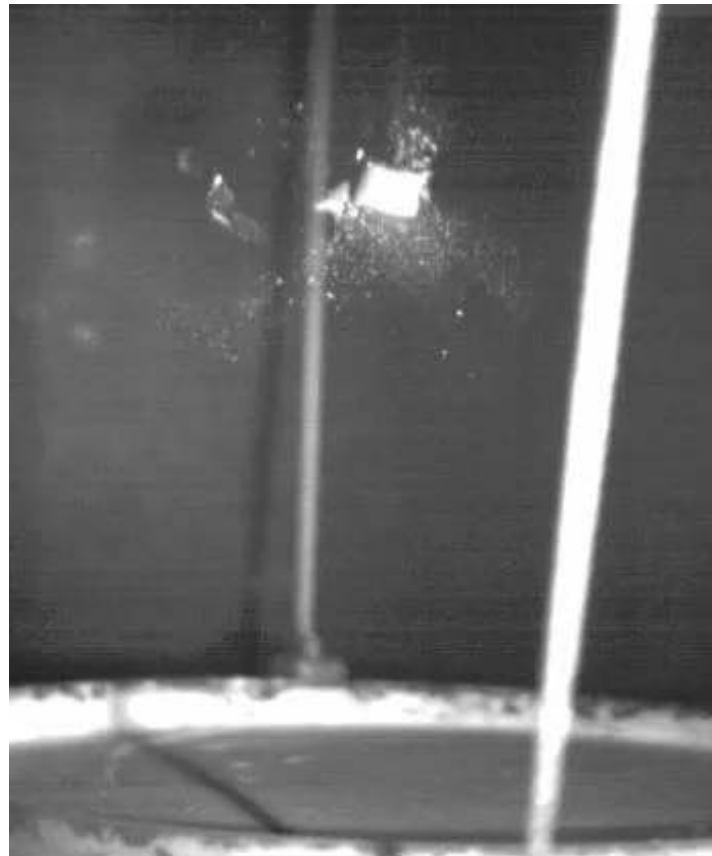
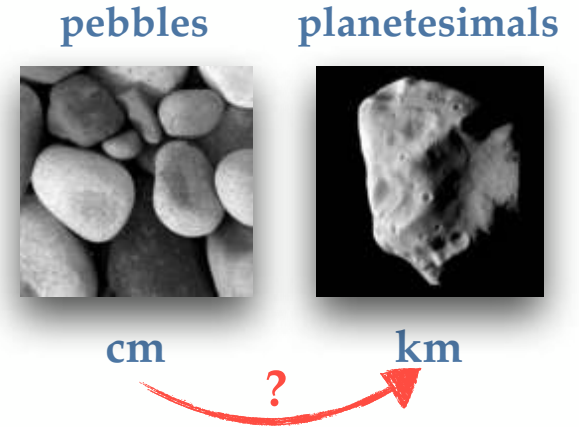
- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?
  - ❖ invoke **pressure maxima** in the disk to trap dust?
  - ❖ invoke very **efficient size growth**?
    - **bouncing** at low relative velocities



Weidling+ 2012 (mm-sized particles @ ~0.1 m/s)

# Radial drift of dust particles

- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?
  - ❖ invoke **pressure maxima** in the disk to trap dust?
  - ❖ invoke very **efficient size growth**?
    - **bouncing** at low relative velocities, **fragmentation** at large



Guettler+ 2010 (mm-sized particles @ ~40 m/s)

# Radial drift of dust particles

- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?
  - ❖ invoke **pressure maxima** in the disk to trap dust?
  - ❖ invoke very **efficient size growth**?
    - **bouncing** at low relative velocities, **fragmentation** at large
    - **mass transfert** due to repeated collisions with smaller dust?

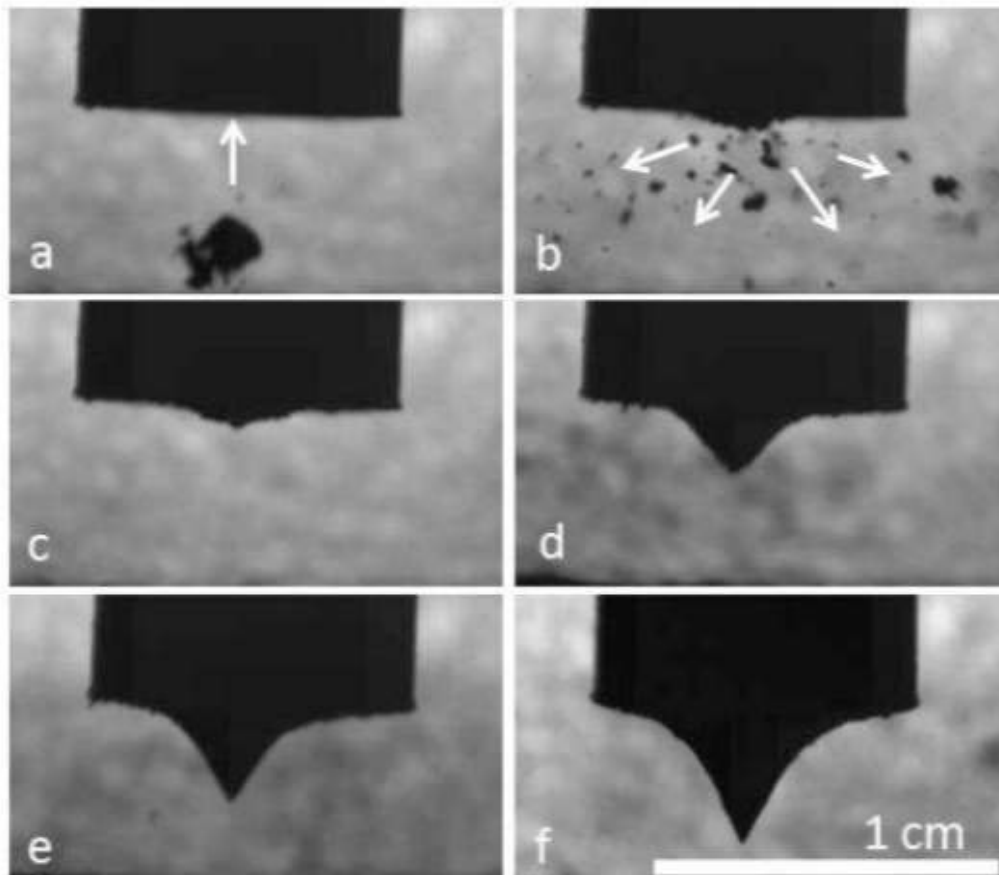
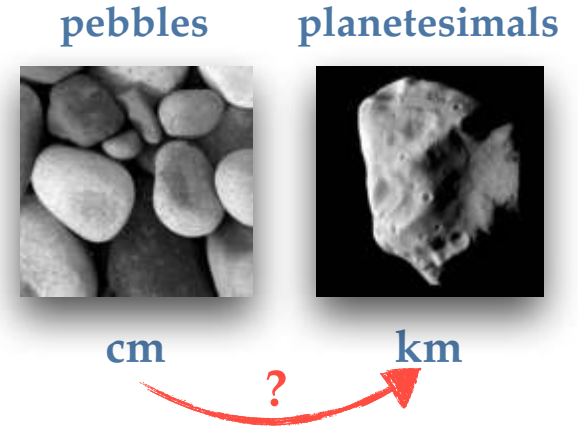
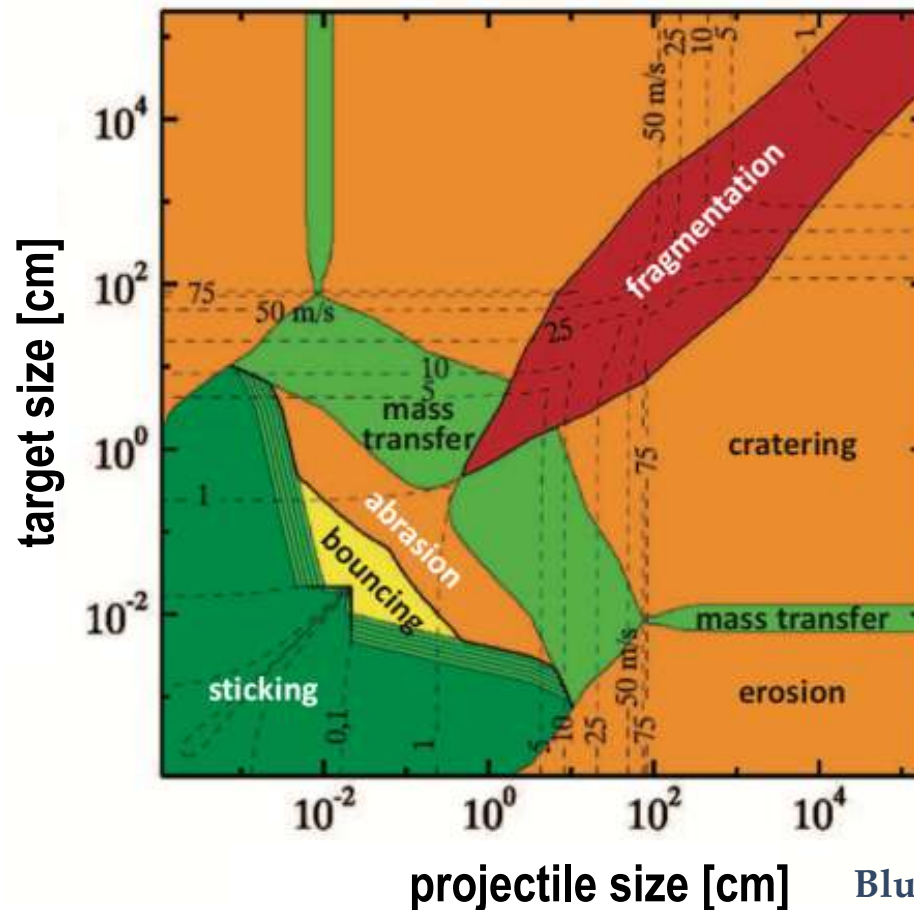
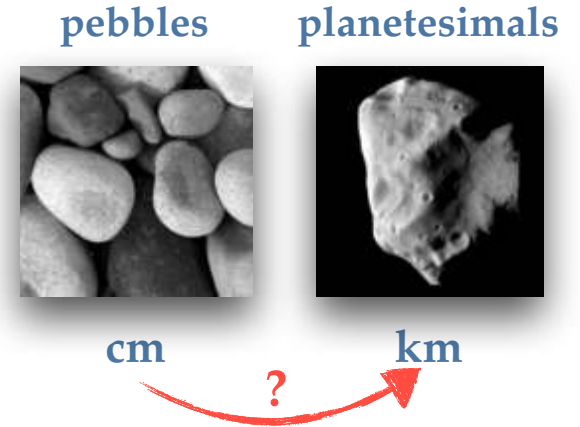


Fig. 10.— Experimental example of mass transfer in fragmenting collisions. All experiments were performed in vacuum. (a) A mm-sized fluffy dust aggregate is ballistically approaching the cm-sized dusty target at a velocity of 4.2 m/s. Projectile and target consist of monodisperse  $\text{SiO}_2$  spheres of  $1.5 \mu\text{m}$  diameter. (b) Shortly after impact, most of the projectile's mass flies off the target in form of small fragments (as indicated by the white arrows); part of the projectile sticks to the target. (c) - (e) The same target after 3 (c), 24 (d), 74 (e) and 196 (f) consecutive impacts on the same spot. Image credit: Stefan Kothe, TU Braunschweig.

# Radial drift of dust particles

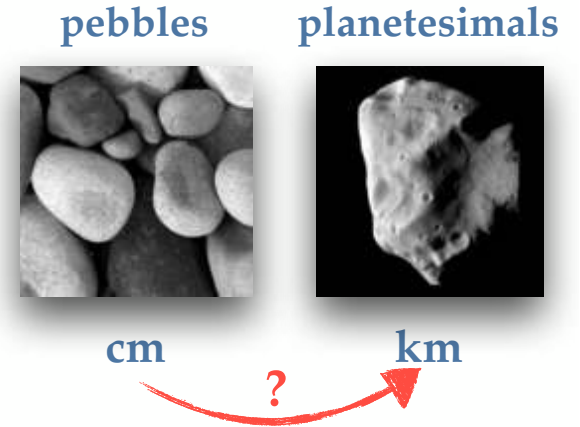
- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?
  - ❖ invoke **pressure maxima** in the disk to trap dust?
  - ❖ invoke very **efficient size growth**?
    - **bouncing** at low relative velocities, **fragmentation** at large
    - **mass transfer** due to repeated collisions with smaller dust?





# Radial drift of dust particles

- So how do we not lose all the cm/meter-sized solids and **build** planetesimals?
  - ❖ invoke **pressure maxima** in the disk to trap dust?
  - ❖ invoke very **efficient size growth**?
  - ❖ invoke effects of **dust drag onto the gas**?



→ for any **dust-to-gas density ratio  $\epsilon$** ,  $v_{R,\text{dust}} - v_{R,\text{gas}} \approx h^2 v_K \left( \frac{\partial \log p}{\partial \log R} \right) \frac{1}{\text{St} + \text{St}^{-1} (1 + \epsilon)^2}$   
Youdin & Goodman 2005

so  $v_{R,\text{dust}}$  becomes less negative as  $\epsilon = \rho_{\text{dust}}/\rho_{\text{gas}}$  increases. But can  $\epsilon \gg 1$ ?

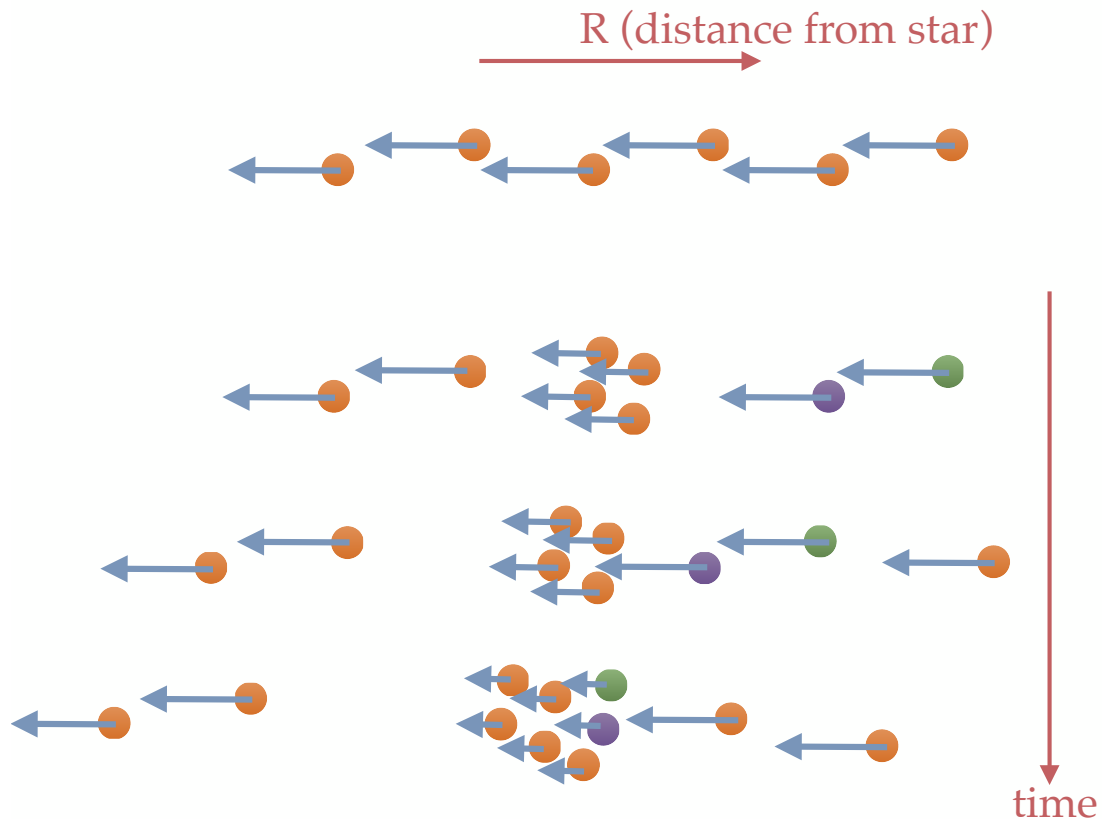
→ may trigger the **streaming instability**

# Streaming instability

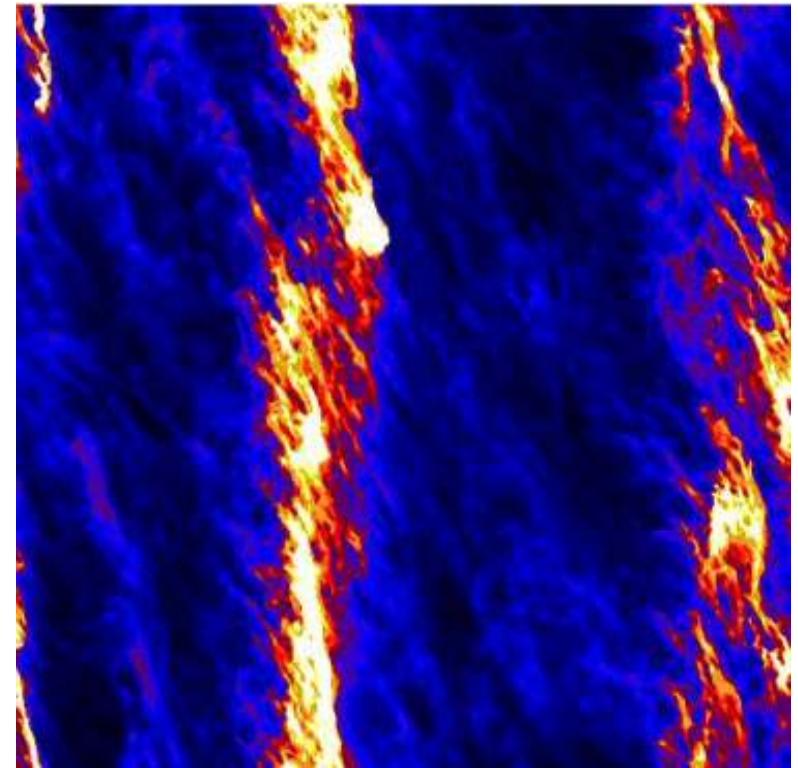
- **Linear instability** due to dust drag on the gas (“dust feedback”)

Youdin & Goodman 2005

→ forms of **dust filaments** with a very large concentration of solids



sort of dust traffic jam!



Formation of dust filaments by the streaming instability  
The dust-to-gas density ratio can reach a few x 1000

Johansen+ 2014

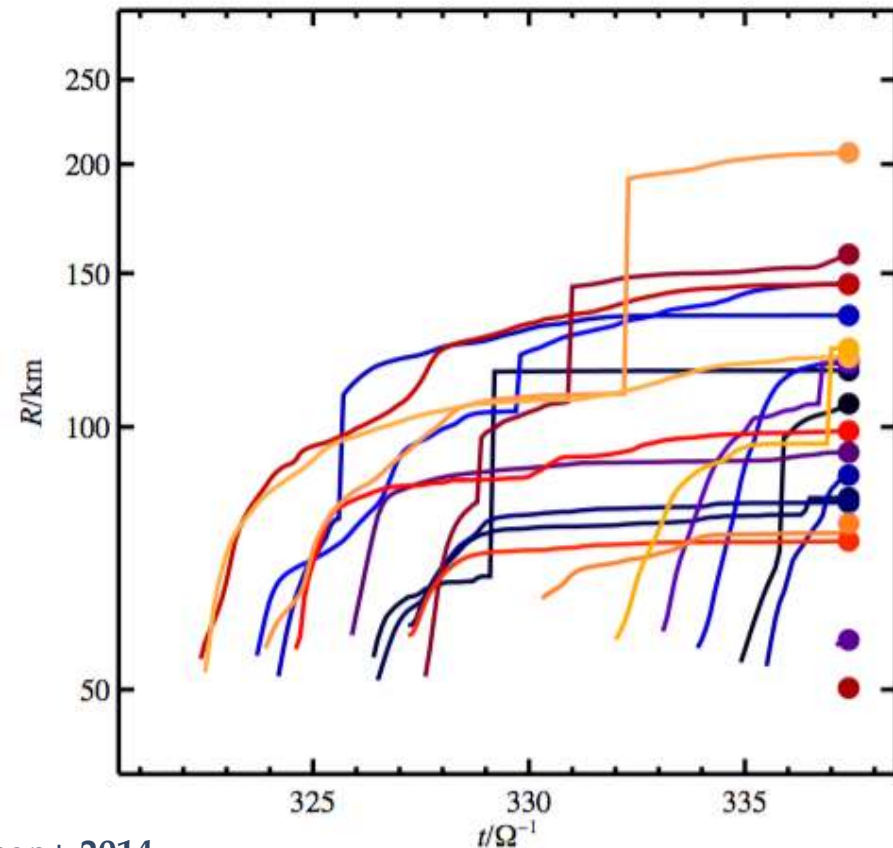
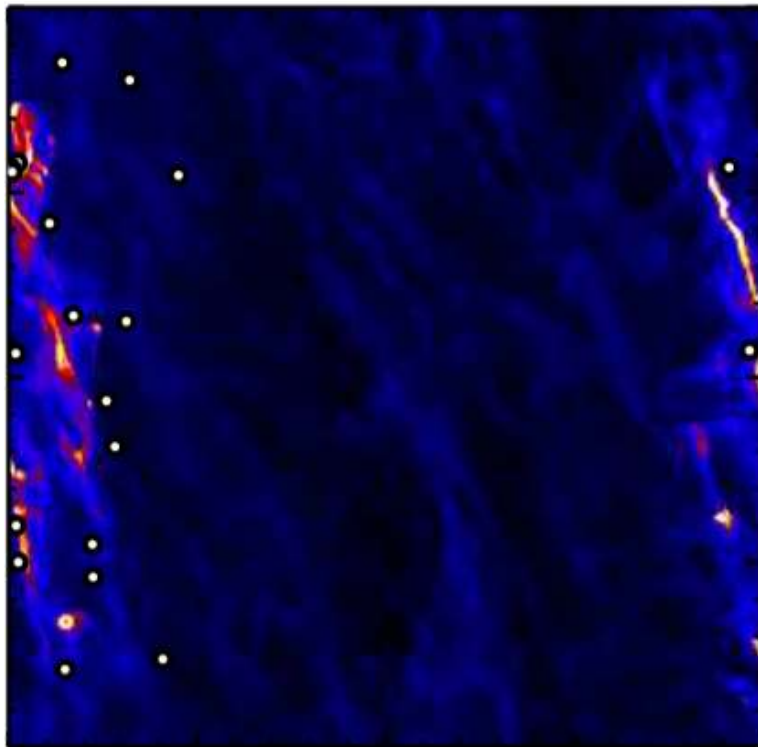
# Streaming instability

- **Linear instability** due to dust drag on the gas (“dust feedback”)

Youdin & Goodman 2005

→ forms of **dust filaments** with a very large concentration of solids

→ dust’s self-gravity causes the dust filaments to **collapse** which, with the help of collisions, can typically **form** ~100 km-sized **planetesimals**



Johansen+ 2014

→ very active field of research!

# A few summary points

- Solid particles **drift** towards **pressure maxima** (radial drift towards the star, vertical settling towards disk midplane, anticyclonic vortices...)
- Drift is **fastest** for dust **marginally** coupled to the gas (with Stokes number  $\sim 1$ ), which corresponds to cm-sized pebbles at a few AU from the star
- Collisions cause dust to **stick** up to pebble (cm) sizes; beyond, **bouncing** and **fragmentation** become dominant
- Formation of (km-sized) **planetesimals** is thought to occur in the regime of strong dust-gas interaction through the **streaming instability**